

# THE FAILURE OF THE TACOMA NARROWS BRIDGE

A Report to the  
Honorable John M. Carmody  
Administrator, Federal Works Agency  
Washington, D. C.

## BOARD OF ENGINEERS

Othmar H. Ammann  
Theodore von Kármán  
Glenn B. Woodruff

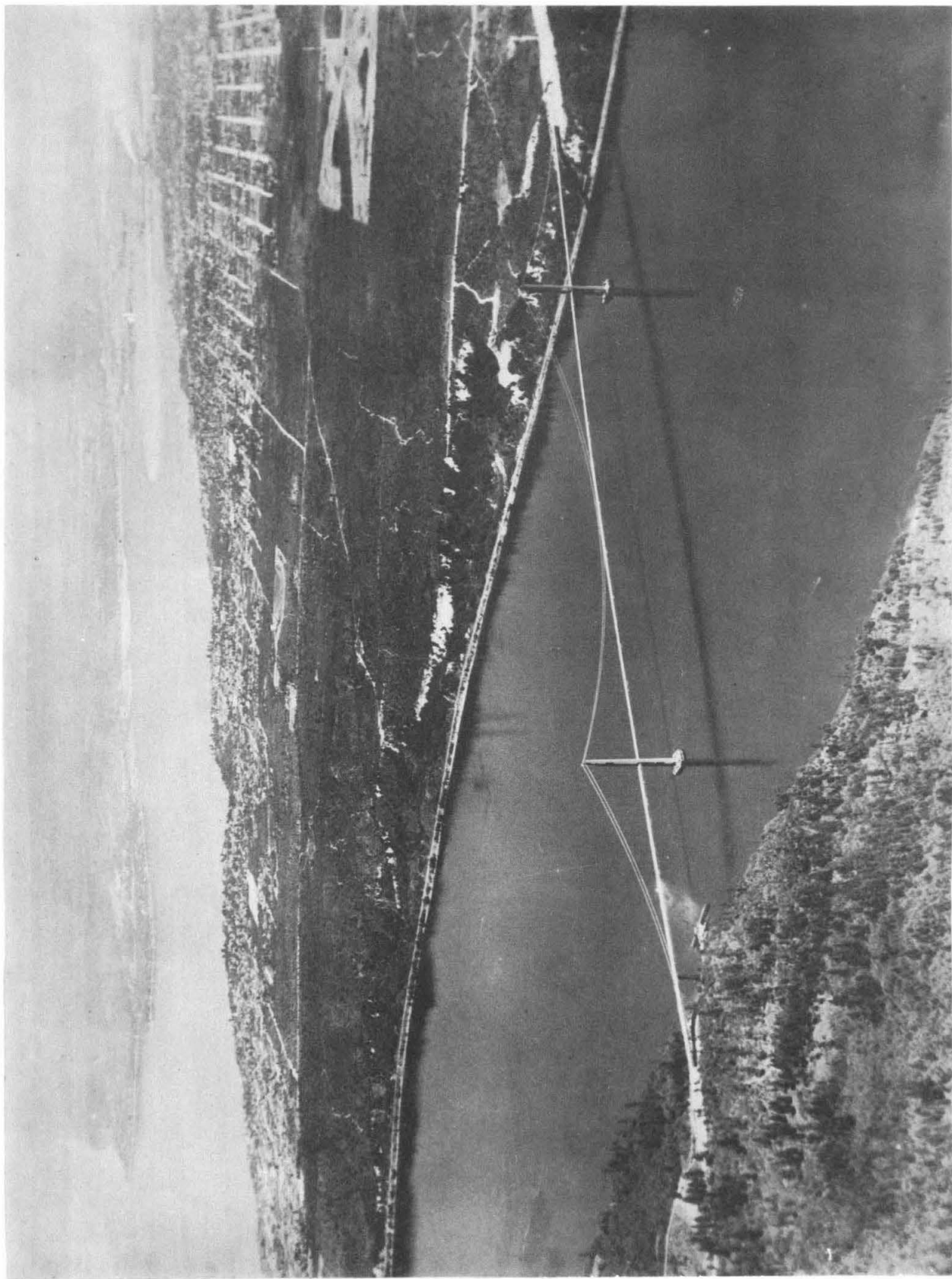
MARCH 28, 1941



Compliments of  
BOARD OF ENGINEERS

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Pasadena, California  
March 28, 1941

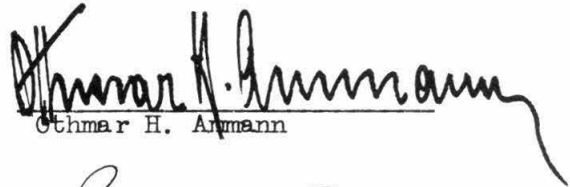
The Honorable John M. Carmody,  
Administrator, Federal Works Agency,  
Washington, D. C.

Sir:

The Board of Engineers appointed by you to  
report on the Failure of the Tacoma Narrows Bridge have  
made a complete investigation of the design, the behaviour  
after completion and the failure of the structure. Our  
report covering this investigation follows.

Respectfully submitted,

BOARD OF ENGINEERS

  
Othmar H. Ammann

  
Theodore von Kármán

  
Glenn B. Woodruff



## SUMMARY OF CONCLUSIONS

As a result of the investigations which are described in detail in this report, we have reached the following conclusions:

1. The Tacoma Narrows Bridge was well designed and built to resist safely all static forces, including wind, usually considered in the design of similar structures. Its failure resulted from excessive oscillations caused by wind action.
2. The excessive vertical and torsional oscillations were made possible by the extraordinary degree of flexibility of the structure and of its relatively small capacity to absorb dynamic forces. It was not realized that the aerodynamic forces which had proven disastrous in the past to much lighter and shorter flexible suspension bridges would affect a structure of such magnitude as the Tacoma Narrows Bridge, although its flexibility was greatly in excess of that of any other long span suspension bridge.
3. The vertical oscillations of the Tacoma Narrows Bridge were probably induced by the turbulent character of wind action. Their amplitudes may have been influenced by the aerodynamic characteristics of the suspended structure. There is, however, no convincing evidence that the vertical oscillations were caused by so-called aerodynamic instability. At the higher wind velocities torsional oscillations, when once induced, had the tendency to increase their amplitudes.
4. Vertical oscillations of considerable amplitudes were first observed during the erection of the suspended floor and continued, at intervals, until the day of failure. While, at times, the resulting stresses in the stiffening girders were high, there is no evidence that any structural damage resulted. Under certain observed conditions very high stresses were caused in the ties which connected the suspended floor structure to the cables at mid-span.
5. It appears reasonably certain that the first failure was the slipping of the cable band on the north side of the bridge to which the center ties were connected. This slipping probably initiated the torsional oscillations. These torsional movements caused breaking stresses at various points of the suspended structure and further structural damage followed almost immediately. The dropping of the greater part of the suspended structure of the center span was made possible by the failure of the suspenders.



This was followed by the sudden sagging of the side spans with resulting bending and overstressing of the towers and of the side spans.

6. The suspension type is the most suitable and the most economical that could have been selected for the Tacoma Narrows Bridge. No more satisfactory location would have been chosen.
7. Both the Public Works Administration and the Reconstruction Finance Corporation were entirely justified in assuming that, because of the experience and reputation of the consultants employed by the Washington Toll Bridge Authority, there could be no possible question as to the adequacy of the design. Both agencies exercised thorough and competent supervision during the construction of the bridge.
8. There can be no question that the quality of the materials in the structure, and the workmanship, were of a high order.
9. Certain parts of the towers were severely overstressed and permanently deformed during the failure. While there is no visual evidence of damage to the cables, except at the center of the north cable, it is probable that they were overstressed during the torsional oscillations and as a result of the sagging of the side spans. The main piers were not damaged, except locally, during the failure and can withstand considerably heavier tower reactions than they received from the bridge as it existed. The anchorages were not damaged and are safe for forces not greater than those imposed by the original construction.
10. The criteria usually considered for rigidity against static forces do not necessarily apply to dynamic forces.
11. The remedial installations in the bridge represented a rational effort to control the amplitudes of the oscillations. Further installations, including diagonal stay ropes from the top of the towers to the floor were being investigated when the failure occurred, and these would have increased the rigidity. It is doubtful that any measures of this nature would have been sufficient to compensate for the extreme flexibility of the structure.
12. The evidence as to whether the vertical oscillations of the bridge would have been affected by fairing (streamlining) is inconclusive. There is certain evidence that fairing would have had an unfavorable influence on the torsional stability.
13. Further experiments and analytical studies are desirable to investigate the action of aerodynamic forces on suspension bridges.

14. Pending the results of further investigations, there is no doubt that sufficient knowledge and experience exists to permit the safe design of a suspension bridge of any practicable span. The results of further research should furnish knowledge that will permit of more economical design.
15. This report has been restricted to the Tacoma Narrows Bridge, except that available information from other bridges has been considered.



# THE FAILURE OF THE TACOMA NARROWS BRIDGE

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## INTRODUCTION

Oscillations of considerable amplitudes, caused by wind were apparent during the erection of the Tacoma Narrows Bridge. In spite of certain remedial devices that were installed, these undulations continued after the completion of the structure and its opening to vehicular traffic on July 1, 1940. Previous to November 7, 1940, these oscillations, while of such magnitude that the structure could not be considered satisfactory for traffic, were not accompanied by torsion, and the resulting stresses in the suspended structure were within safe limits. About 10:00 A.M. on November 7th, a very severe torsional movement suddenly arose and the final failure, consisting of the dropping of most of the center suspended span, followed about an hour later.

This failure was the most notable in the history of bridge building since that of the first Quebec Bridge in 1907 and the dropping of the suspended span in the rebuilding of that structure in 1916. While there have been a number of bridge failures attributed to wind, that of the Tacoma Narrows Bridge brought the question of aerodynamic action on suspension bridges into greater prominence than ever before. It was the opinion of the Hon. John F. Carmody, Administrator, Federal Works Agency, that public interest warranted investigation of the entire matter. He therefore appointed a Board of Engineers consisting of Othmar H. Ammann, Consulting Engineer, of New York City; Theodore

von Karman, Director of the Daniel Guggenheim Aeronautical Laboratory at the California Institute of Technology, Pasadena, California; and Glenn B. Woodruff, Consulting Engineer, of San Francisco, for such purpose.

This Board has had the following meetings:

November 28, 1940, at Washington, D. C. Discussion of general procedure and scope of the investigation with Mr. Carmody and his staff. December 6-14, 1940, at Seattle and Tacoma, Washington. Inspection of the wrecked structure. Collection of data from the Washington Toll Bridge Authority. Visits to the University of Washington Laboratory to examine the models of the Tacoma Narrows Bridge and to collect available data from the tests. January 20-23, 1941, at New York City and Washington, D. C. Discussion of results to date with officials of P.W.A. February 24-26, 1941, at Pasadena. (Messrs. von Karman and Woodruff) Discussion of model tests. March 17-28, 1941, at Pasadena. Analysis of data and preparation of report.

Between the above meetings, the members of the Board made the calculations and assembled the data which form the basis for the report. In addition, Dr. von Karman directed the wind tunnel tests herein described (Appendix VIII).

The Washington Toll Bridge Authority also appointed a Board of Consultants, consisting of Messrs. Russell Cone, Chief Engineer of the Golden Gate Bridge, Francis Donaldson, Chief Engineer of the Mason-Hanger Company, General Contractors, and L. J. Sverdrup, Consulting Engineer of St. Louis, Missouri. It is our understanding that the principal assignment of this Board was to establish the amount of loss that might be collectible under the insurance coverage of the Bridge.

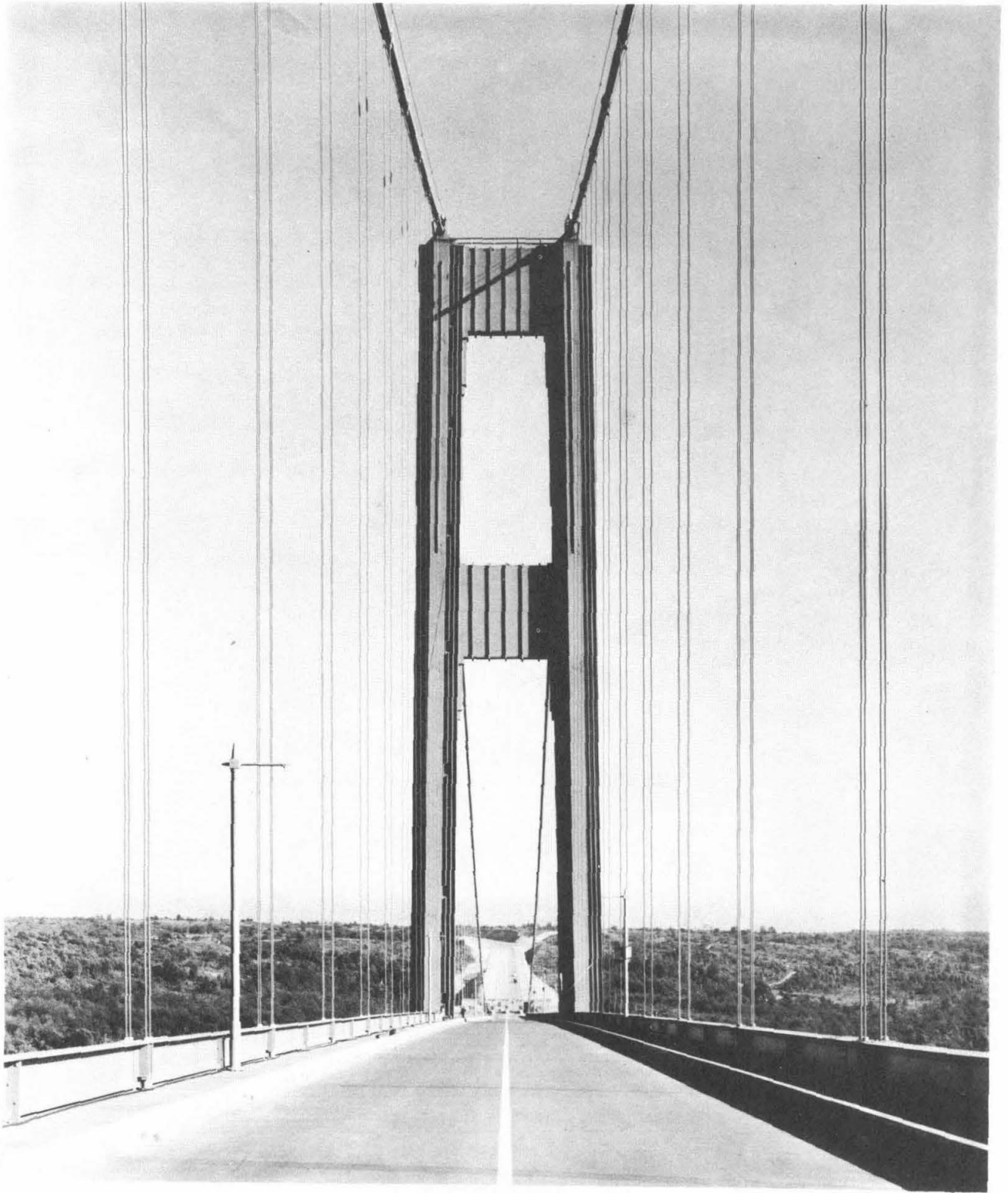


Fig. 1. Tacoma Narrows Bridge Along Roadway.



This project developed during a period when there was a very decided trend from private to public control of toll bridges as well as of other public utilities. The first active promotion of the bridge was by the Tacoma Narrows Bridge Company, which had secured a franchise and which in 1933 filed an application for financing with the Reconstruction Finance Corporation. A later development was an application to the Public Works Administration by Pierce County. In 1937, the State legislature created the Washington Toll Bridge Authority for the purpose of constructing and operating toll bridges within the State. The application of Pierce County was amended to show the Authority as the Applicant. On June 27, 1938, the P.W.A. made an offer of a grant of \$2,700,000 and a loan of \$3,300,000, which offer was accepted by the Authority. In October of 1938, after bids for the construction had been received, the grant and loan were respectively raised to \$2,880,000 and \$3,520,000, a total of \$6,400,000. The loan was made by the Reconstruction Finance Corporation, and was repaid by a refunding operation shortly after the bridge was opened to traffic.

Development of Plans. With the development of the project there was a corresponding evolution of plans, which may be briefly summarized as follows:

1. Cantilever - 1200' Center Span, 666' 8" Anchor Arms with 600' and 533' Truss Spans on each side. Trusses 30' c. to c.
2. Multiple Span Suspension - 3-1200' and 2-600' spans with tie cables.
3. Suspension - 2600' center span, 1000' side spans, 22' Roadway - 30' c. to c. cables.
4. Suspension - 2600' center span. 1300' side spans, 26' Roadway - 39' c. to c. Cables. Stiffening Trusses 25' c. to c. Chords.

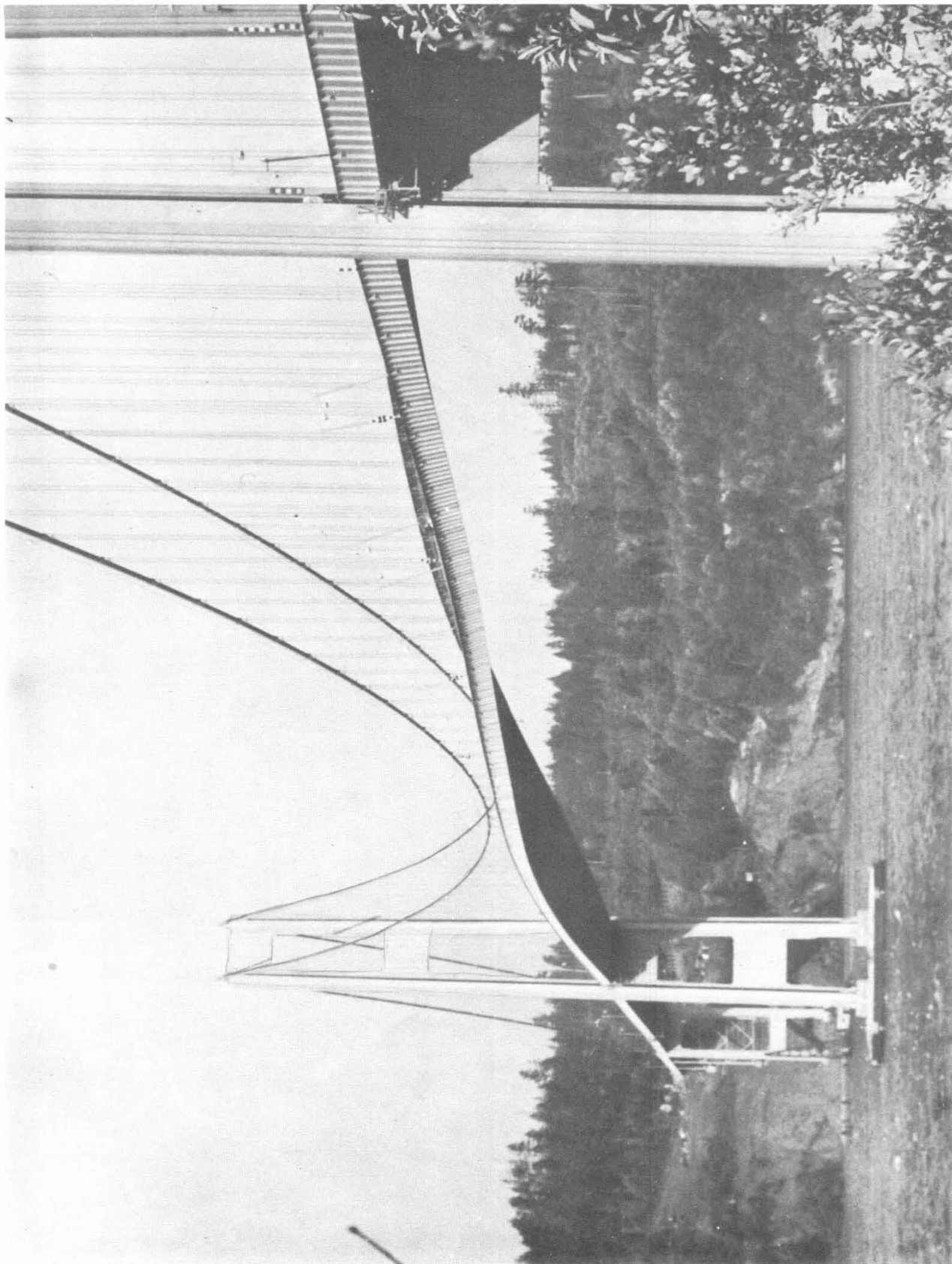


Fig. 2. One Node Torsional Oscillation of Bridge  
about one Hour before Final Failure.

5. Suspension Contract design - 2800' center span, 1100' side spans, 26' Roadway - 39' c. to c. Cables.  
Stiffening Girders 8 ft. deep.

Organization. The personnel of the various organizations connected with the project was as follows:

United States Federal Works Agency.

John M. Carmody, Administrator

Public Works Administration.

H. L. Ickes, Administrator of Public Works  
H. A. Gray, Asst. Administrator of Public Works  
Colonel E. W. Clark, Commissioner  
J. J. Madigan, Executive Officer  
Harry M. Brown, Director of Engineering  
Kenneth A. Godwin, Regional Director  
George A. Gregory, Project Engineer  
L. R. Durkee, Project Engineer  
D. L. Glenn, Chief Resident Engineer Inspector

Reconstruction Finance Corporation

Jesse Jones, Chairman  
Morton Macartney, Chief, Self Liquidating Division  
W. L. Drager, Chief, Engineering Division  
T. L. Condron, Advisory Engineer  
James A. Roper, Inspecting Engineer

Washington Toll Bridge Authority.

Governor Clarence D. Martin, Chairman  
Cliff Yelle, State Auditor  
Lacey V. Murrow, Director of Highways  
Olaf L. Olsen, Director of Finance  
P. H. Winston, Secretary of Authority

Engineering.

Lacey V. Murrow, Chief Engineer  
Chas. E. Andrew, Principal Consulting Engineer  
Luther E. Gregory, Consulting Engineer  
R. H. Thomson, Consulting Engineer  
R. B. McMinn, Consulting Engineer  
Moran, Proctor & Freeman, Consultants on Foundations  
Leon S. Moisseiff, Consultant on Superstructure  
C. H. Eldredge, Tacoma Narrows Bridge Engineer

General Contractor.

Pacific Bridge Co., General Construction Co.,  
Columbia Construction Co.  
Ralph Keenan, Project Manager  
Jack Graham, Construction Superintendent  
Theodore Kuss, Chief Engineer

Sub-Contractors.

Structural Steel  
Bethlehem Steel Company  
A. S. Holteman, Erection Engineer

East Anchorage and Roadway Packing  
Woodworth and Cornell

Electrical.

American Machinery and Electric Company

Contract Design. The final contract design was made during July and August of 1938. At this time, the center span was lengthened from 2600 to 2800 feet, and the change made from stiffening trusses to stiffening plate girders. The considerations leading to these changes are given in the report of Mr. Moisseiff to Mr. Murrow, dated July 18, 1938. This report and Mr. Moisseiff's letter of July 27th are attached as Appendix II.

We understand that the calculations for the superstructure were made by Mr. Moisseiff and his assistants, and that the details of the design were prepared under the direction of Mr. Eldredge and referred to Mr. Moisseiff for his approval. The contract drawings consisted of 39 sheets, dated August 6th, and September 7, 1938. They are signed by Mr. Murrow as State Highway Engineer and by Mr. Eldredge as Bridge Engineer. They also carry the approval signatures of Messrs. Moran, Proctor and Freeman, as Consultants on the substructure, and of Mr. Moisseiff, as Consultant on the superstructure.



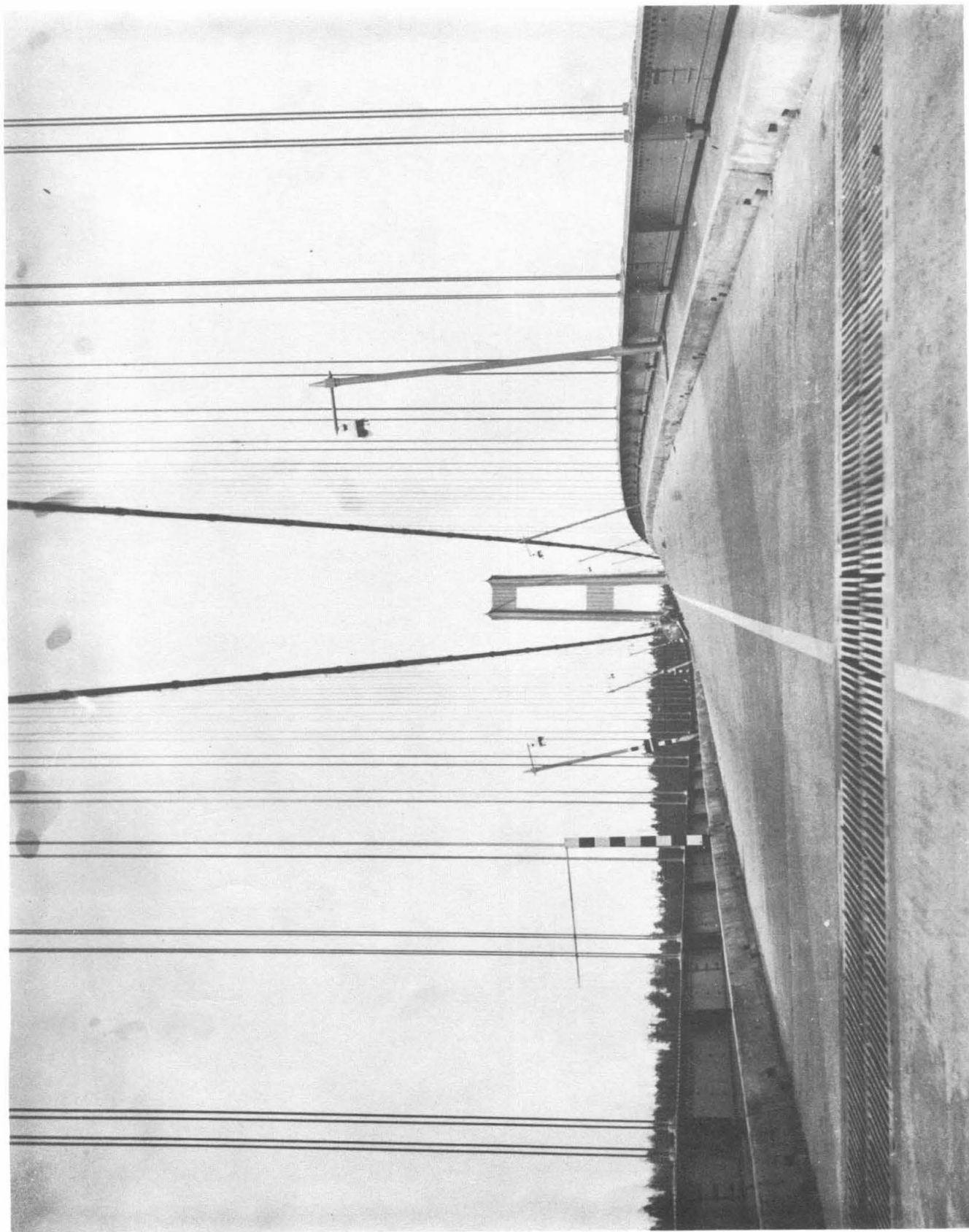


Fig. 3. Torsion of Floor Center Span seen from Tower.

Approval of Design. The approval of the project by the Public Works Administration, on June 27, 1938, came at a time when the design was being changed from one with stiffening trusses to the contract design with the stiffening girders. Appendix III, hereof, is a statement by Mr. J. J. Madigan, dated December 4, 1940, outlining the procedure of the Engineering Division P.W.A. in passing upon the engineering features of a project. It will be noted that, insofar as the detail design of any project was concerned, the P.W.A. relied almost exclusively on the reliability and integrity of the owner's engineers and architects, reserving the right of approval of such engineers and/or architects.

Such a course has been followed generally in all large undertakings. With the varied nature of the projects financed wholly or in part by the P.W.A., it would have been entirely impracticable for such organization to have had on its staff experts in all branches of design and construction. This project involved deep foundations with extreme currents and a very long, the third in the world, span suspension bridge. The owner had engaged consultants with well established reputations and wide experience in both of these major problems. It could not have been expected that any official of the Public Works Administration should have had any doubts as to a design recommended by such engineers.

To a limited extent, the design was given an independent review. The Reconstruction Finance Corporation, as the purchaser of the bonds, retained Mr. T. L. Condrón to review the plans and also to give general oversight to the construction of the foundations. In his engineering report dated September 21, 1938, excerpts from which are given in Appendix IV, Mr. Condrón made no attempt to go into details of design, but commented on the flexibility of the suspended structure. However,

in this respect, he deferred to the wider experience of Mr. Moisseiff. The design was also reviewed and approved by a Board of Consultants appointed by the Washington Toll Bridge Authority.

Description of Bridge. The bridge was a suspension structure with a center span of 2800 ft. and two side spans of 1100 ft. each (Drawing 2). Back stays of 497.4 ft. on the west side and 261.8 ft. on the east side made the total horizontal length of the bridge between strand shoes 4759.2 ft. The roadway was 26 ft. wide; two 4 ft. 9 in. sidewalks were provided; the width was 39 ft. center to center of cables and stiffening girders. In addition to the suspension structure, the plate girder approach spans, and the anchorages made the overall length of the project 5939 ft.

The main piers are 64'-6" x 117'-6" in plan. The west pier extends to El. - 170.0 and the east one to El. - 219; these elevations are at the cutting edge, the bottom of the excavation and seal being approximately 5 ft. lower. The caissons are of cellular construction, the seal extending about 20 ft. above the cutting edge. The cells are capped at the top with the concrete bases for the towers, in which bases the tower anchor grillages are embedded.

Except for the cable splay chambers and a chamber at the center line to reduce the weight, the anchorages consist of solid blocks of concrete. The east anchorage is 70 ft. x 170 ft. in plan, and an average of 50 ft. high. The west anchorage is 70 ft. x 160 ft. in plan and an average of 55 ft. high. Each anchorage contains approximately 20,000 cu. yards of concrete.

The main towers are 420 ft. in height and form rigid frames with-



Fig. 4. Failure of Concrete Floor near Mid Span, about 10:30 A.M.



out diagonal bracing. The tower shafts are of the cellular construction. The shafts are spaced at 39 ft. centers at the top and are battered to a spacing of 50 ft. at the base. Each shaft is of cruciform section. In the longitudinal direction, each shaft tapers from 13 ft. at the top to 19 ft. at the bottom. Transversely, a constant width of 13 ft. is maintained. Each tower weighs 1926 tons.

Each cable consists of 19 strands of 332-No. 6 cold drawn galvanized wires. The diameter of each cable under wrapping is 17 1/8 ins. and the net area of wire 190.3 sq. ins. It was constructed with a sag, under dead load of 232 ft. or a sag ratio of 1/12. The total weight of cable wire is 3817 tons.

The suspenders are spaced 50 ft. along each cable and consist of 4 parts of 1 1/4 in. diam. ropes.

The suspended structure consisted of two stiffening girders, spaced 39 ft. c. to c., transverse floorbeams at 25 ft. centers, five lines of longitudinal stringers spaced 5'-9" c. to c., the lateral system and the reinforced concrete roadway slabs, curbs and sidewalks. Each stiffening girder consisted of a web 96 in. x 1/2 in., 4 angles 8 in. x 6 in. x 1/2 in. and 2 covers 20 in. x 1/2 in. They were stiffened both horizontally and vertically. The plate girder floorbeams were 52 ins. deep. The stringers were 21 in. rolled beams. The roadway slab was 5 1/4 ins. and the sidewalk slab 2 1/2 ins. thick. Both slabs were heavily reinforced.

During the development of the details several innovations, including welded cable saddles and cable bands and new details for strand adjustment, were introduced.

Changes from Contract Plans. During the progress of the work, several

changes were made from the Contract Plans. Three of them were of sufficient importance to be noted.

1. The bottom of the seal of Pier 5 was lowered from El. - 200 to El. - 224. and 3217 c.y. of rip rap was placed around the pier. The original bottom at the site of Pier 5 sloped quite steeply towards the channel. As a preliminary to the successful landing of the caisson, it was necessary to level the area and in so doing the bottom was disturbed over a considerable area. As a result, there was a scour of 25 ft. at the west side of the pier. At this point, tidal velocities as high as 12.5 ft. per second were measured during construction.

The final depth is below the deepest point of the channel. With the precaution of taking soundings at frequent intervals, there should be no question of the safety of these piers against scour.

2. The design of both anchorages was changed to a considerable extent to conform to the soil conditions disclosed during the excavation.
3. The floor system of the bridge was redesigned with a saving in both structural and reinforcing steel.

Remedial Measures. With the experience at the Bronx-Whitestone and other bridges with comparatively shallow stiffening girders, the engineers of the Tacoma Narrows Bridge anticipated, before starting of erection of the suspended structure, the oscillations which later occurred. Accordingly, they studied means of reducing the amplitudes and designed the center

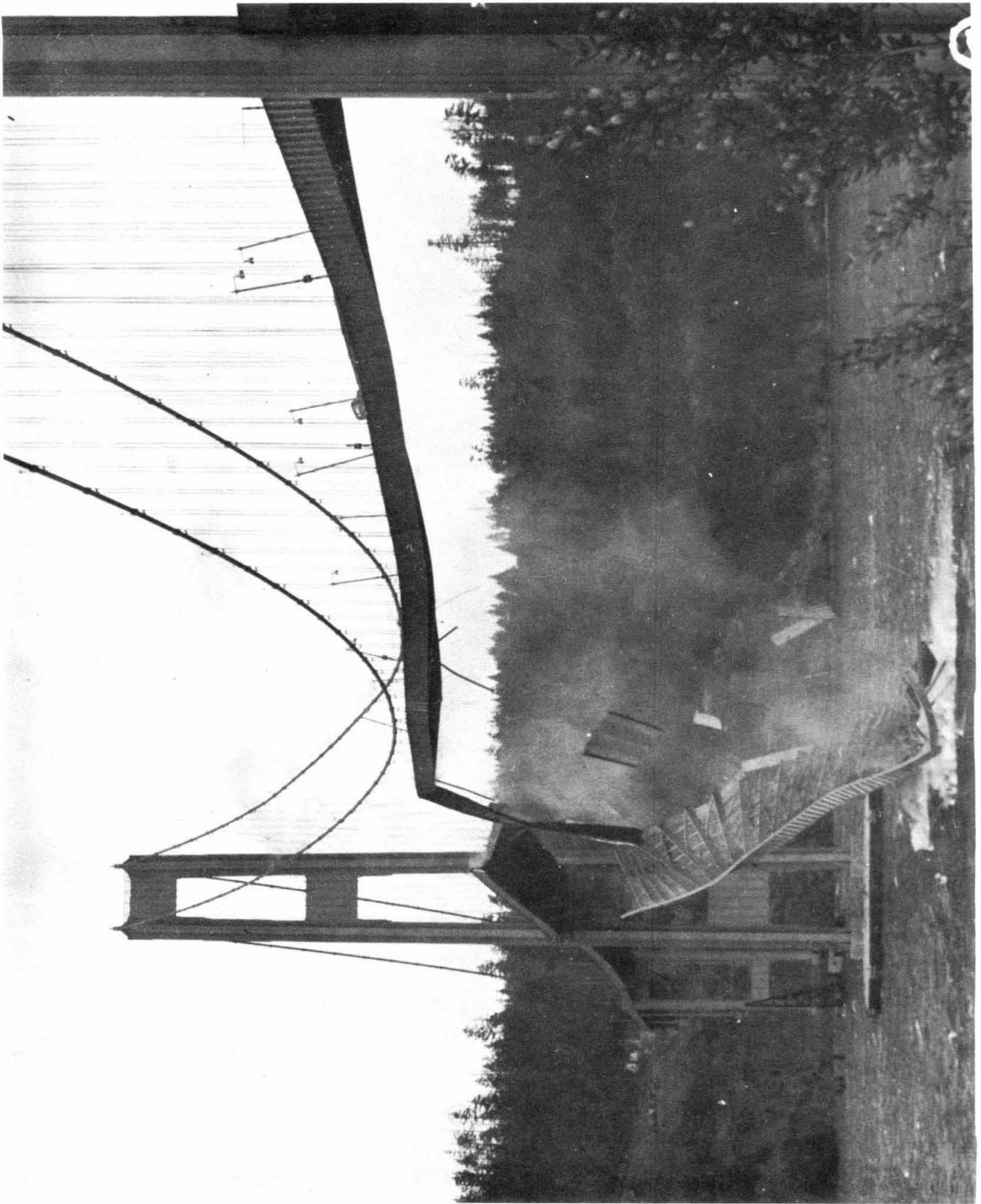


Fig. 5. Dropping of 600 ft. Section of Floor Structure at  
West Quarter Point of Center Span about 11:00 A.M.

ties and the hydraulic buffers at the towers. These devices were installed before opening the bridge to traffic (center ties about June 1st, the buffers about June 28th). The side span "hold-downs" were installed on October 4th and 7th, 1940, in the east and west side spans respectively.

The center ties (Drawing 3) consisted of diagonal stays between the mid-span cable bands and the stiffening girders. Each stay was 1 1/2 in. diam. bridge strand with an ultimate strength of 270,000 pounds. The stays were attached to the cable band by lugs welded thereto and to the girders by heavy clips. Turnbuckles were provided for adjustment.

The hydraulic jacks or buffers (Drawing 3) consisted of plungers connected to the floorsystem of the center span operating in cylinders attached to the tower. As the span moved longitudinally oil was forced through a pipe line, the flow being checked by a needle valve.

The side span hold-downs (Drawing 2), consisted of two 1 9/16 in. bridge strands connected to each girder of each side span and leading to heavy concrete blocks on the ground line for the east side span and supported on timber piles for the west side span.

Model Tests. The use of models has proven so advantageous in determining the erection procedure of suspension bridges that a similar one was constructed at the University of Washington under the direction of Prof. F. B. Farquharson (7)\* for a similar purpose. In the meantime information concerning the oscillations of the other bridges was secured and it was decided to construct a more elaborate model. As soon as the vertical oscillations were discovered and the need for remedial measures became

\*Throughout this report, figures within parentheses refer to the Bibliography, Appendix I.



apparent, tests were instituted to determine the relative efficiency of the different proposals. These tests have been continued since the failure of the bridge. The results are as yet unpublished but, through the courtesy of the sponsors, the data summarized in Appendix VII have been made available for our use. In addition, extensive wind tests were made at the University of Washington. Some of the results have been published (9) and are also considered in Appendix VII.

Construction. The following dates show the progress of construction:

|                             |                |
|-----------------------------|----------------|
| Bids received               | Sept. 27, 1938 |
| Contract awarded            | Nov. 23, 1938  |
| Construction commenced      | Nov. 23, 1938  |
| Main piers completed        | Sept. 11, 1939 |
| Towers completed            | Jan. 6, 1940   |
| Cables completed            | Mar. 9, 1940   |
| Suspended steel completed   | May 31, 1940   |
| Concrete Roadways completed | June 28, 1940  |
| Bridge opened for traffic   | July 1, 1940   |

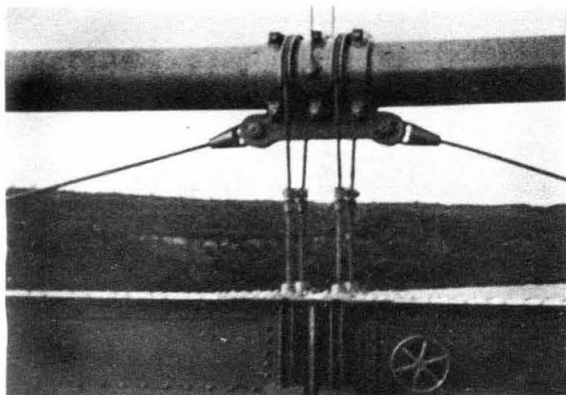
The bridge failed on Nov. 7, 1940.

Except for miscellaneous equipment and some minor construction items, the construction of the entire project was included in one contract. Two bids were received on September 27, 1938, as follows:

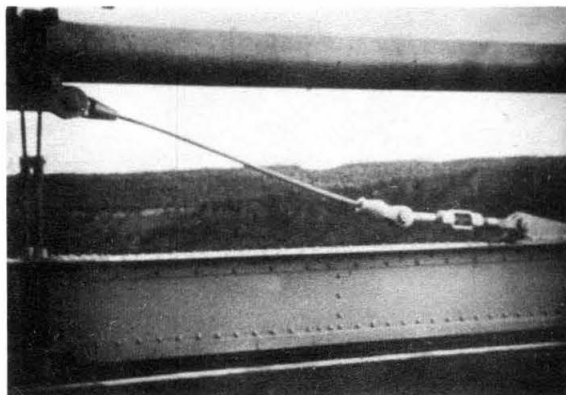
|   |                |
|---|----------------|
| Pacific Bridge Co., General Construction<br>Co., and Columbia Construction Co.<br>(Joint Venture) | \$5,949,730.40 |
| Columbia Bridge Co.   | \$6,038,560.44 |

A contract was executed with the low bidder on November 23, 1938, and these contractors began the clearing of the site on the same day. The general contractor constructed the foundations, including the west anchorage, with his own forces. He sublet the following work:

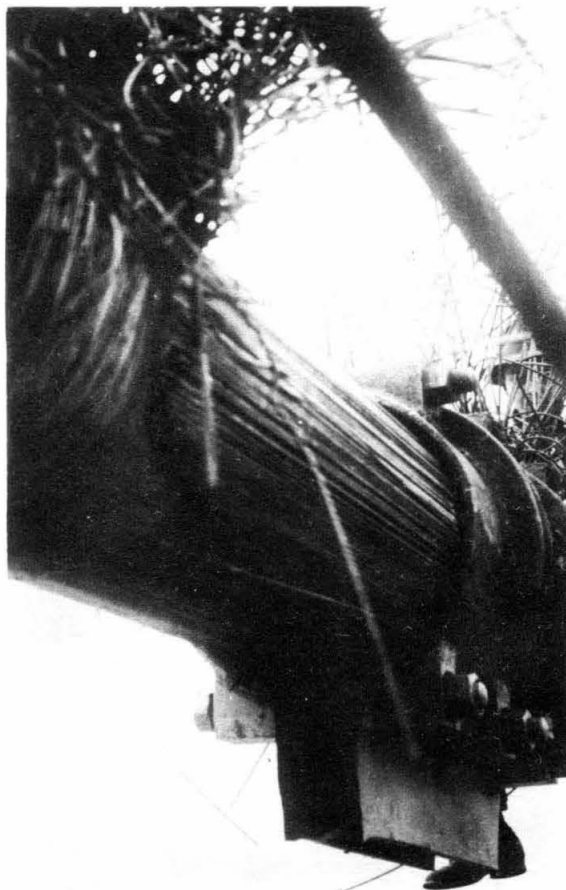
1. Fabrication and erection of structural steelwork and cables to: Bethlehem Steel Company;
2. East Anchorage and Roadway Paving to: Woodworth and Cornell;



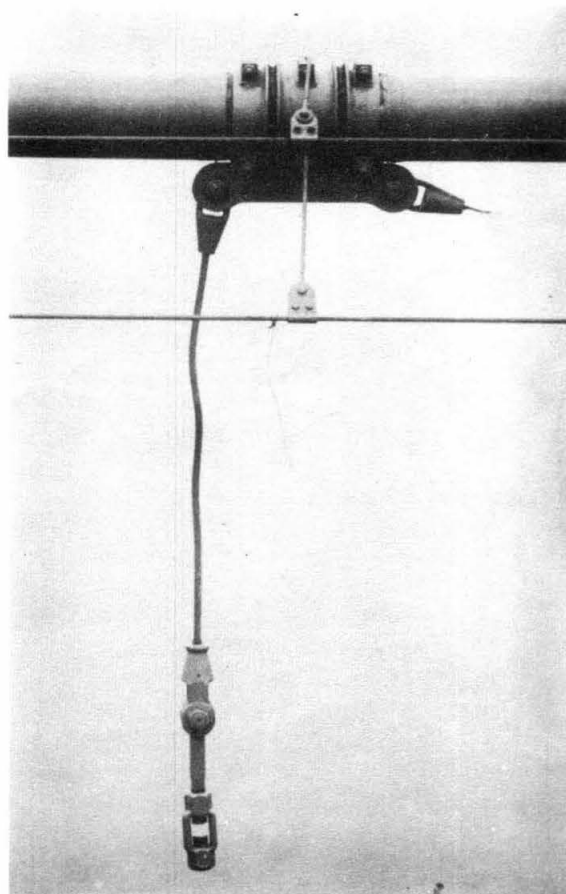
(a)



(b)



(c)



(d)

Fig. 6. Center Ties. (a) and (b) are of north cable taken at 9:30 A.M. on day of failure. (b) shows slackness of diagonal tie: (c) center of north cable after failure. About 500 wires are cut and longitudinal movement had amplitude of about 3-1/2 ft. Note missing bolts. (d) south cable after failure.

3. Electrical Work to:  
American Machinery and Electric Company.

The construction methods are fully described elsewhere (1-5).

The field inspection of the work was made by the owners' engineers under the direction of Mr. Eldredge. The mill and shop inspection of the structural steelwork was made by the Pittsburgh Testing Laboratories.

The Public Works Administration was represented on the work by Mr. George A. Gregory, Project Engineer, and by Mr. D. L. Glenn, Chief Resident Engineer Inspector, assisted by two Resident Engineer Inspectors. Mr. Gregory was relieved on January 25, 1940, by Mr. L. R. Durkee, Project Engineer on the Lake Washington Bridge, and on July 25, 1940, when the project was nearing completion, the remainder of Mr. Glenn's work was taken over by Mr. Durkee. The responsibilities of these P.W.A. inspectors is outlined in Appendix III. However, Mr. Glenn went considerably further than this. We have reviewed his reports and find that he made a careful and conscientious check of all engineering matters.

The R.F.C. was represented on the work by Mr. Condron, who arranged to be at the site at critical periods such as the sealing of the caissons and the uncovering of the anchorage foundations. In September of 1939, the R.F.C. assigned Mr. James H. Roper to the work as Inspecting Engineer on this work.

The Behaviour of the Bridge. The oscillations of the Tacoma Narrows Bridge were noticed during the erection of the suspended structure. No records of the modes nor of the amplitudes during this period are available, but observers have stated that the amplitudes were comparable to those in the completed structure.

After the completion of the bridge a systematic series of readings were taken under the direction of Mr. Eldredge. For the purpose of these observations a series of targets were mounted on the lamp standards on the south side of the bridge and readings taken through a transit located on the canopy at the toll house. The longitudinal spacing of the targets is indicated by points 1 to 9 on Drawing 5.

Tables 1 and 2 list the complete series of these observations as furnished us by Mr. Eldredge. The difference in form between the two tables arises from the fact that targets 3A, 4 and 5 were added subsequently to August 12. The analysis of this material indicates that in most cases one principal mode of oscillation prevailed over a certain period of time. However, the modes of oscillation frequently changed (according to Prof. Farquharson, more frequently than indicated by the tables). An attempt has been made to identify the actual oscillations of the bridge with the principal modes derived from theory and from observations made on the dynamically similar model of the bridge at the University of Washington. Drawing 4 shows the various modes of oscillations observed by Prof. Farquharson on the model and on the bridge. The theoretical calculations of frequencies and modes are given in Appendix VI. Drawing 5 shows typical examples of the modes of oscillation observed at the bridge. The numbers identify these particular modes with the types given on Drawing 4. The curves marked "T" (denoting Theoretical) represent the shape of the deflection obtained for the same modes by the mathematical calculation given, assuming interaction between the center and side spans, in Appendix VI. The numbers with the suffix "A" designate such oscillations which have the

TABLE 1

Recorded Oscillations - Tacoma Narrows Bridge - Aug. 1-12, 1940

| Date<br>1940 | Hour | Obser-<br>vation<br>No. | Wind             |                | Freq-<br>uency<br>per<br>Min. | Type | Double Amplitudes at Pt. |     |    |     |     |     |     |    | Remarks |
|--------------|------|-------------------------|------------------|----------------|-------------------------------|------|--------------------------|-----|----|-----|-----|-----|-----|----|---------|
|              |      |                         | Veloc.<br>m.p.h. | Direc-<br>tion |                               |      | 1                        | 2   | 3  | 6   | 7   | 8   | 9   |    |         |
| 8-1          | 2:15 | 1                       |                  | South          | 12                            | 5    | +6                       | +10 | +9 | -20 | -8  | +16 | +22 |    |         |
|              | 30   | 2                       |                  |                |                               | 5    | 4                        | 8   | 6  | 20  | 3   | 20  | 20  |    |         |
|              | 45   | 3                       |                  |                |                               | 5    | 8                        | 12  | 9  | 20  | 2   | 22  | 24  |    |         |
|              | 48   | 4                       |                  |                |                               | 5    | 2                        | 6   | 6  | 16  | 5   | 12  | 16  |    |         |
|              | 55   | 5                       |                  |                |                               | 5    | 2                        | 4   | 5  | 10  | 2   | 12  | 12  |    |         |
|              | 58   | 6                       |                  |                |                               | 5    | 2                        | 4   | 2  | 5   | 2   | 10  | 10  |    |         |
|              | 3:00 | 7                       |                  |                |                               | 5    | 1                        | 2   | 1  | 6   | 0   | 4   | 6   |    |         |
|              | 01   | 8                       |                  |                |                               | 5    | 0                        | 1   | 0  | 4   | 0   | 2   | 2   |    |         |
|              | 02   | 9                       |                  |                |                               | 5    | 0                        | 0   | 0  | 3   | 0   | 0   | 0   |    |         |
|              | 03   | 10                      |                  |                |                               | 5    | 0                        | 0   | 0  | 5   | 0   | 2   | 2   |    |         |
|              | 04   | 11                      |                  |                |                               | 5    | 0                        | 1   | 0  | 6   | 0   | 4   | 4   |    |         |
|              | 06   | 12                      |                  |                |                               | 5    | 1                        | 2   | 2  | 2   | 0   | 2   | 2   |    |         |
|              | 12   | 13                      |                  |                |                               |      | 0                        | 0   | 0  | 0   | 0   | 0   | 0   |    |         |
|              | 27   | 14                      |                  | North          |                               |      | 0                        | 2   | 1  | 0   | 0   | 1   | 2   |    |         |
|              | 30   | 15                      |                  |                |                               |      | 0                        | 3   | 2  | 0   | 0   | 4   | 4   |    |         |
|              | 32   | 16                      |                  |                |                               |      | 1                        | 2   | 2  | 0   | +2  | 2   | 2   |    |         |
|              | 34   | 17                      |                  |                |                               | 1    | 1                        | 2   | 2  | -2  | -4  | -4  | -4  |    |         |
|              | 40   | 18                      |                  |                |                               |      | 0                        | 0   | 0  | 0   | 0   | 0   | 0   |    |         |
|              | 5:30 | 19                      |                  |                |                               |      | 0                        | 0   | 0  | 0   | 0   | 0   | 0   |    |         |
|              | 8-2  | 8:53                    | 20               |                | South                         |      | 1                        | 0   | 1  | 1   | 2   | 2   | 4   | 4  |         |
|              |      | 58                      | 21               |                |                               |      | 1                        | 0   | 1  | 1   | 2   | 2   | 2   | 2  |         |
|              |      | 9:00                    | 22               |                |                               |      | 1                        | 1   | 3  | 3   | 2   | 3   | 4   | 4  |         |
|              |      | 03                      | 23               |                |                               |      | 1                        | 2   | 4  | 4   | 2   | 4   | 6   | 6  |         |
|              |      | 19                      | 24               |                |                               |      | 1                        | 1   | 4  | 3   | 2   | 8   | 10  | 12 |         |
|              |      | 25                      | 25               |                |                               |      | 1                        | 1   | 2  | 2   | 2   | 4   | 6   | 6  |         |
|              |      | 31                      | 26               |                |                               | 8    | 1                        | 1   | 1  | 1   | 2   | 3   | 4   | 4  |         |
| 35           |      | 27                      |                  |                |                               |      | 0                        | 0   | 0  | 0   | 0   | 0   | 0   |    |         |
| 10:58        |      | 28                      |                  |                |                               | 1    | 1                        | 2   | 2  | 2   | 3   | 4   | 4   |    |         |
| 11:00        |      | 29                      |                  |                |                               | 1    | 2                        | 4   | 4  | 4   | 6   | 6   | 6   |    |         |
| 02           |      | 30                      |                  |                | 8                             | 1    | 2                        | 4   | 4  | 6   | 8   | 12  | 12  |    |         |
| 04           |      | 31                      |                  |                |                               | 1    | 3                        | 5   | 5  | 4   | 6   | 12  | 12  |    |         |
| 05           |      | 32                      |                  |                |                               | 1    | 3                        | 6   | 6  | 6   | 10  | 16  | 16  |    |         |
| 09           |      | 33                      |                  |                | 8                             | 1    | 4                        | 7   | 7  | 6   | 12  | 18  | 18  |    |         |
| 15           |      | 34                      |                  |                |                               | 1    | 4                        | 7   | 7  | 8   | 12  | 20  | 20  |    |         |
| 18           |      | 35                      |                  |                | 8                             | 1    | 6                        | 12  | 12 | -10 | -14 | -20 | -22 |    |         |
| 32           |      | 36                      |                  |                | 8                             | 1    | 4                        | 6   | 6  | -4  | -8  | -12 | -12 |    |         |
| 38           |      | 37                      |                  |                |                               | 1    | 1                        | 2   | 2  | 2   | 6   | 10  | 12  |    |         |
| 46           |      | 38                      |                  |                |                               | 1    | 1                        | 2   | 2  | 2   | 4   | 4   | 4   |    |         |
| 51           |      | 39                      |                  |                |                               | 1    | 2                        | 4   | 4  | 3   | 8   | 10  | 10  |    |         |
| 12:13        |      | 40                      |                  |                |                               | 1    | 1                        | 2   | 2  | 1   | 2   | 4   | 4   |    |         |
| 16           |      | 41                      |                  |                |                               | 8    | 1                        | 1   | 1  | -2  | +2  | +3  | -4  |    |         |
| 25           |      | 42                      |                  |                |                               | 1    | 2                        | 2   | 2  | -4  | -4  | -2  | -4  |    |         |
| 29           |      | 43                      |                  |                |                               | 1    | 2                        | 4   | 4  | 4   | 6   | 8   | 8   |    |         |
| 34           |      | 44                      |                  |                | 8                             | 1    | 1                        | 2   | 2  | 0   | 4   | 4   | 4   |    |         |
| 46           |      | 45                      |                  |                |                               | 0    | 0                        | 0   | 0  | 0   | 0   | 0   | 0   |    |         |



TABLE 1 (Continued)

Recorded Oscillations - Tacoma Narrows Bridge - Aug. 1-12, 1940

| Date<br>1940 | Hour  | Observation<br>No. | Wind             |                | Freq-<br>uency<br>per<br>Min. | Type | Double |    | Amplitudes at Pt. |    |    |     |     | Remarks    |
|--------------|-------|--------------------|------------------|----------------|-------------------------------|------|--------|----|-------------------|----|----|-----|-----|------------|
|              |       |                    | Veloc.<br>m.p.h. | Direc-<br>tion |                               |      | 1      | 2  | 3                 | 6  | 7  | 8   | 9   |            |
| 8-2          | 2:36  | 46                 |                  |                | 8                             | 1    | +1     | +1 | +3                | -2 | -4 | -8  | -8  |            |
|              | 2:54  | 47                 |                  | N.W.           | 8                             | 1    | +2     | +4 | +4                | -2 | -9 | -6  | -8  |            |
|              | 3:01  | 48                 |                  |                | 8                             | 1    | 4      | 8  | 8                 | 6  | 12 | 16  | 20  |            |
|              | 12    | 49                 |                  |                | 8                             | 1    | 4      | 6  | 6                 | 4  | 10 | 18  | 18  |            |
|              | 25    | 50                 |                  |                | 8                             | 1    | 3      | 8  | 8                 | 6  | 12 | 18  | 20  | Slight     |
|              | 31    | 51                 |                  |                | 8                             | 1    | 3      | 6  | 7                 | 8  | 10 | 14  | 16  | Breeze     |
|              | 41    | 52                 |                  |                | 8                             | 1    | 6      | 9  | 10                | 8  | 12 | -20 | -20 | Fresh      |
|              | 52    | 53                 |                  |                | 5                             | 5    | 1      | 3  | 3                 | 1  | 0  | +4  | +5  | Breeze     |
|              | 55    | 54                 |                  |                | 9                             | 5    | 2      | 2  | 2                 | 3  | -2 | 2   | 2   |            |
|              | 4:00  | 55                 |                  |                |                               |      | 0      | 0  | 0                 | 0  | 0  | 0   | 0   |            |
|              | 04    | 56                 |                  |                | 12                            | 5    | 1      | 1  | 1                 | 6  | 0  | +4  | +6  |            |
|              | 09    | 57                 |                  |                | 12                            | 5    | 0      | 1  | 1                 | 4  | 0  | 6   | 6   | Slight     |
|              | 16    | 58*                |                  |                | 12                            | 6    | 0      | 0  | 0                 | 5  | 0  | 3   | 5   | Breeze     |
|              | 17    | 59                 |                  |                |                               | 5    | 2      | 6  | 6                 | 12 | 0  | 18  | 18  |            |
|              | 21    | 60                 |                  |                | 12                            |      | 4      | 4  | 4                 | 12 | -2 | 12  | 12  |            |
|              | 30    | 61                 |                  |                | 12                            |      | +1     | +1 | +1                | -4 | 0  | 4   | 4   |            |
|              | 31    | 62                 |                  | N.W.           |                               |      | 0      | 0  | 0                 | +2 | 0  | +2  | +2  | Slight     |
|              | 34    | 63                 |                  |                |                               |      | 0      | 0  | 0                 | 0  | 0  | 0   | 0   | Breeze     |
|              | 37    | 64                 |                  |                | 12                            |      | +1     | +2 | +2                | -8 | 0  | +12 | +12 |            |
|              | 44    | 65                 |                  |                | 12                            |      | 2      | 4  | 4                 | 12 | -2 | 14  | 14  |            |
|              | 48    | 66                 |                  |                | 12                            |      | 3      | 5  | 5                 | 20 | 1  | 20  | 20  | Fresh      |
|              | 54    | 67                 |                  |                | 12                            |      | 1      | 2  | 2                 | 10 | 1  | 8   | 8   | Breeze     |
|              | 59    | 68                 |                  |                | 12                            |      | 2      | 2  | 2                 | 8  | -1 | 4   | 4   |            |
|              | 5:04  | 69                 |                  |                | 12                            |      | 3      | 3  | 3                 | 12 | 0  | 6   | 6   | Fresh Br.  |
|              | 15    | 70                 |                  |                | 12                            |      | 3      | 3  | 3                 | 12 | 0  | 12  | 12  | Gusty      |
|              | 30    | 71                 |                  |                | 12                            |      | 1      | 2  | 2                 | 8  | 0  | 8   | 8   | Fresh Wind |
|              | 33    | 72                 |                  | N.W.           | 12                            |      | 1      | 1  | 1                 | -4 | 0  | +3  | +3  | Fresh Br.  |
|              | 40    | 73                 |                  |                |                               |      | 0      | 0  | 0                 | 0  | 0  | 0   | 0   |            |
| 8-3          | 11:32 | 74                 | 1                | N.W.           | 12                            | 6    | 0      | 0  | 0                 | -4 | 0  | +4  | +4  |            |
|              | 40    | 75                 | 2                |                | 12                            | 5    | +1     | +2 | +2                | -4 | -2 | +2  | +4  |            |
|              | 55    | 76                 |                  |                |                               |      | 0      | 0  | 0                 | 0  | 0  | 0   | 0   |            |
| 8-5          | 8:00  | 77                 |                  |                |                               |      | 0      | 0  | 0                 | 0  | 0  | 0   | 0   |            |
|              | 58    | 78                 | 5.7              | 30°L           | 8                             | 1    | +1     | +2 | +2                | -1 | -2 | -2  | -3  |            |
|              | 9:55  | 79                 |                  |                |                               |      | 0      | 0  | 0                 | 0  | 0  | 0   | 0   |            |
|              | 56    | 80                 | 4.3              | 20°L           | 12                            |      | +1     | +1 | +1                | -2 | 0  | +2  | +2  |            |
|              | 59    | 81                 |                  |                |                               |      | 0      | 0  | 0                 | 0  | 0  | 0   | 0   |            |
|              | 10:04 | 82                 | 2.7              | 45°L           | 8                             |      | +1     | +2 | +2                | -2 | -1 | -2  | -2  |            |
|              | 30    | 83                 |                  |                |                               |      | 0      | 0  | 0                 | 0  | 0  | 0   | 0   |            |
|              | 11:26 | 84                 | 5.7              | 60°L           | 12                            | 5    | +1     | +2 | +2                | -6 | -2 | +7  | +7  |            |
|              | 32    | 85                 | 4.5              | 60°L           | 12                            | 5    | 2      | 4  | 4                 | 14 | 2  | 12  | 16  |            |
|              | 45    | 86                 | 4.0              | 60°L           | 12                            | 5    | 1      | 2  | 2                 | 4  | 0  | 6   | 6   |            |
|              | 47    | 87                 | 5.7              | 15°            | 12                            | 5    | 0      | 0  | 0                 | 0  | 0  | 0   | 0   |            |
|              | 12:33 | 88                 | 5.1              | 55°L           | 12                            | 5    | +1     | +3 | +2                | -4 | -2 | +8  | +8  |            |
|              | 42    | 89                 | 4.2              | 50°L           | 12                            | 5    | 1      | 3  | 3                 | 11 | 2  | 10  | 12  |            |

TABLE 1 (Continued)

Recorded Oscillations - Tacoma Narrows Bridge - Aug. 1-12, 1940

| Date<br>1940 | Hour  | Observation<br>No. | Wind             |                | Freq-<br>uency<br>per<br>Min. | Type | Double Amplitudes at Pt. |    |    |     |    |     |     |   | Remarks |
|--------------|-------|--------------------|------------------|----------------|-------------------------------|------|--------------------------|----|----|-----|----|-----|-----|---|---------|
|              |       |                    | Veloc.<br>m.p.h. | Direc-<br>tion |                               |      | 1                        | 2  | 3  | 6   | 7  | 8   | 9   |   |         |
| 8-5          | 12:51 | 90                 | 5.6              | 70°L           | 12                            | 5    | +2                       | +4 | +4 | -13 | -1 | +16 | +20 |   |         |
|              | 56    | 91                 | 3.4              | 80°L           | 12                            | 5    | 3                        | 7  | 8  | 18  | 2  | 18  | 22  |   |         |
|              | 1:02  | 92*                | 3.7              | 75°L           | 12                            | 5    | 4                        | 9  | 8  | 22  | 3  | 22  | 26  |   |         |
|              | 22    | 93                 | 3.4              | 60°L           | 12                            | 5    | 3                        | 10 | 9  | 18  | 2  | 18  | 22  |   |         |
|              | 35    | 94                 | 5.0              | 75°L           | 12                            | 5    | +2                       | +4 | +3 | -11 | -2 | +14 | +13 |   |         |
|              | 2:10  | 95                 | 3.8              | 35°L           | 12                            | 5    | 3                        | 9  | +8 | 22  | 2  | 20  | 24  |   |         |
|              | 26    | 96                 | 5.3              | 35°L           | 12                            | 5    | 3                        | 5  | +4 | 12  | 1  | 11  | 13  |   |         |
|              | 37    | 97                 | 3.5              | 30°L           | 12                            | 5    | 1                        | 2  | +2 | 5   | 1  | 8   | 10  |   |         |
|              | 46    | 98                 | 3.4              | 25°L           | 12                            | 5    | 0                        | 1  | 1  | -2  | -2 | +2  | +2  |   |         |
|              | 51    | 99                 |                  | 25°L           |                               |      | 0                        | 0  | 0  | 0   | 0  | 0   | 0   |   |         |
|              | 54    | 100                | 4.5              | 25°L           | 8                             | 1    | 0                        | +1 | +1 | -2  | -2 | -2  | -2  |   |         |
|              | 3:07  | 101                | 4.7              | 30°L           | 8                             | 1    | +1                       | 2  | 2  | 2   | 2  | 2   | 2   |   |         |
|              | 10    | 102                | 2.8              | 25°L           | 8                             | 1    | 1                        | 3  | 2  | 2   | 4  | 8   | 8   |   |         |
|              | 24    | 103                | 5.2              | 50°L           | 8                             | 1    | 3                        | 5  | 4  | 3   | 9  | 12  | 13  |   |         |
|              | 28    | 104                | 3.0              | 30°L           | 8                             | 1    | 5                        | 11 | 10 | 8   | 12 | 14  | 16  |   |         |
|              | 37    | 105                | 4.0              | 30°L           | 8                             | 1    | 2                        | 4  | 3  | 2   | 5  | 10  | 10  |   |         |
|              | 48    | 106                | 1.9              | 85°L           | 8                             | 1    | 4                        | 10 | 9  | 8   | 10 | 12  | 16  |   |         |
|              | 4:03  | 107                | 1.4              | 70°L           | 8                             | 1    | 2                        | 3  | 2  | 2   | 4  | 7   | 8   |   |         |
|              | 12    | 108                | 2.0              | 30°L           | 8                             | 1    | 0                        | 1  | 1  | 1   | 3  | 4   | 5   |   |         |
|              | 8-6   | 4:00               | 109              |                |                               |      |                          | 0  | 0  | 0   | 0  | 0   | 0   | 0 |         |
| 27           |       | 110                | 0                |                |                               |      | 0                        | 0  | 0  | 0   | 0  | 0   | 0   |   |         |
| 47           |       | 111                |                  |                |                               |      | 0                        | 0  | 0  | 0   | 0  | 0   | 0   |   |         |
| 49           |       | 112                |                  |                | 8                             |      | +2                       | +4 | +2 | -2  | -3 | -4  | -4  |   |         |
| 53           |       | 113                |                  |                |                               |      | 0                        | 0  | 0  | 0   | 0  | 0   | 0   |   |         |
| 4:45         |       | 114                |                  |                |                               |      | 0                        | 0  | 0  | 0   | 0  | 0   | 0   |   |         |
| 5:24         |       | 115                | 5.1              | 120°R          | 12                            | 5    | +1                       | +3 | +2 | -5  | -1 | +6  | +8  |   |         |
| 29           |       | 116                | 5.1              | 105°R          | 12                            | 5    | 3                        | 6  | 5  | 12  | 1  | 16  | 20  |   |         |
| 33           |       | 117                | 5.7              | 110°R          | 12                            | 5    | 3                        | 10 | 8  | 20  | 2  | 18  | 20  |   |         |
| 6:25         |       | 118                | 5.1              | 105°R          | 12                            | 5    | 3                        | 10 | 10 | 20  | 1  | 18  | 20  |   |         |
| 50           |       | 119                | 4.5              | 105°R          | 12                            | 5    | 3                        | 8  | 6  | 14  | 2  | 20  | 26  |   |         |
| 7:10         |       | 120                | 5.7              | 120°R          | 12                            | 5    | 3                        | 12 | 10 | 22  | 4  | 16  | 20  |   |         |
| 20           |       | 121                | 6.3              | 105°R          | 13                            | 5    | 4                        | 12 | 10 | 26  | 6  | 26  | 30  |   |         |
| 33           |       | 122                | 5.7              | 105°R          | 12                            | 5    | 5                        | 14 | 10 | 28  | 5  | 26  | 32  |   |         |
| 45           | 123   | 6.3                | 100°R            | 12             | 5                             | 4    | 12                       | 11 | 24 | 4   | 24 | 30  |     |   |         |
| 8-7          | 8:00  | 124                |                  |                |                               |      | 0                        | 0  | 0  | 0   | 0  | 0   | 0   |   |         |
|              | 54    | 125                | 1.0              | 90°R           | 7.6                           | 1    | +2                       | +5 | +4 | -1  | -3 | -4  | -4  |   |         |
|              | 9:02  | 126                | 2.0              | 105°R          | 8.6                           | 1    | 3                        | 8  | 7  | 3   | 8  | 8   | 10  |   |         |
|              | 22    | 127                |                  |                |                               |      | 0                        | 0  | 0  | 0   | 0  | 0   | 0   |   |         |
|              | 24    | 128                | 2.2              | 90°R           | 7.6                           | 1    | +1                       | +3 | +3 | -2  | -4 | -6  | -8  |   |         |
|              | 36    | 129                |                  |                |                               |      | 0                        | 0  | 0  | 0   | 0  | 0   | 0   |   |         |
|              | 11:20 | 130                | 4.7              | 60°R           | 8.4                           | 1    | +2                       | +6 | +5 | -3  | -6 | -8  | -10 |   |         |
|              | 27    | 131                | 5.1              | 60°R           | 7.5                           | 1    | 4                        | 12 | 11 | 6   | 13 | 15  | 20  |   |         |
| 33           | 132   | 3.7                | 75°R             | 8.0            | 1                             | 6    | 16                       | 14 | 6  | 18  | 22 | 24  |     |   |         |

TABLE 1 (Continued)

Recorded Oscillations - Tacoma Narrows Bridge - Aug. 1-12, 1940

| Date<br>1940 | Hour  | Observation<br>No. | Wind             |           | Frequency<br>per<br>Min. | Type | Double |     | Amplitudes at Pt. |     |     |     |     | Remarks |
|--------------|-------|--------------------|------------------|-----------|--------------------------|------|--------|-----|-------------------|-----|-----|-----|-----|---------|
|              |       |                    | Veloc.<br>m.p.h. | Direction |                          |      | 1      | 2   | 3                 | 6   | 7   | 8   | 9   |         |
| 8-7          | 11:43 | 133                | 4.5              | 100°R     | 7.5                      | 1    | +3     | +6  | +5                | -3  | -6  | -10 | -16 |         |
|              | 54    | 134                | 1.4              | 75°R      | 8.6                      | 1    | 1      | 2   | 2                 | 0   | 0   | 2   | 3   |         |
|              | 12:00 | 135                | 4.2              | 80°R      | 7.5                      | 1    | 3      | 5   | 4                 | -2  | -5  | 5   | 7   |         |
|              | 10    | 136                | 1.0              | 90°R      |                          |      | 0      | 0   | 0                 | 0   | 0   | 0   | 0   |         |
|              | 30    | 137                | 3.0              | 90°R      | 7.5                      | 1    | +2     | +4  | +2                | -2  | -5  | -8  | -10 |         |
|              | 1:00  | 138                | 4.7              | 105°R     | 7.5                      | 1    | 4      | 8   | 6                 | 5   | 8   | 14  | 18  |         |
|              | 15    | 139                | 2.0              | 75°R      | 8.4                      | 1    | 1      | 3   | 2                 | 2   | 3   | 4   | 4   |         |
|              | 17    | 140                | 2.0              | 100°R     |                          |      | 0      | 0   | 0                 | 0   | 0   | 0   | 0   |         |
| 8-12         | 5:44  | 141                | 6.3              | 120°R     | 20.1                     | 8    | +8     | +12 | +14               | -10 | +26 | -6  | -28 |         |
|              | 6:00  | 142                | 5.6              | 110°R     | 20.7                     | 8    | 7      | 10  | 14                | 4   | 26  | 10  | 30  |         |
|              | 05    | 143                | 6.2              | 120°R     | 20.7                     | 8    | 6      | 10  | 5                 | 8   | 30  | 12  | 36  |         |
|              | 11    | 144                | 8.0              | 130°R     | 20.4                     | 8    | 2      | 3   | 4                 | 4   | 26  | 12  | 34  |         |
|              | 17    | 145                | 6.6              | 125°R     | 20.4                     | 8    | 2      | 2   | 4                 | 10  | 24  | 16  | 34  |         |
|              | 24    | 146                | 7.1              | 120°R     | 21.0                     | 8    | 3      | 4   | 2                 | -6  | +26 | -12 | -34 |         |
|              | 30    | 147                | 5.8              | 120°R     | 20.7                     | 8    | 4      | 4   | 8                 | +8  | -28 | +12 | +32 |         |
|              | 42    | 148                | 9.2              | 120°R     | 20.1                     | 8A   | 2      | 4   | 6                 | 2   | 12  | 6   | 20  |         |
|              | 48    | 149                | 9.5              | 135°R     | 17.8                     |      | 6      | 12  | 10                | 10  | 20  | +1  | 10  |         |
|              | 7:10  | 150                | 6.8              | 120°R     | 19.4                     |      | 8      | 24  | 18                | 2   | 24  | -24 | +3  |         |
|              |       |                    |                  |           | 17.7                     |      |        |     |                   |     |     |     |     |         |
|              | 24    | 151                | 7.2              | 120°R     | 19.7                     |      | 6      | 20  | 16                | 20  | 30  | 30  | 0   |         |
|              |       |                    |                  |           | 17.4                     |      |        |     |                   |     |     |     |     |         |
|              | 42    | 152                | 7.5              | 120°R     | 19.7                     |      | 4      | 8   | 8                 | +22 | -30 | -30 | 0   |         |
|              |       |                    |                  |           | 17.7                     |      |        |     |                   |     |     |     |     |         |
|              | 8:15  | 153                | 7.2              | 120°R     | 19.4                     |      | +10    | 20  | 16                | -30 | +30 | +20 | +4  |         |
|              |       |                    |                  |           | 18.0                     |      |        |     |                   |     |     |     |     |         |
|              | 45    | 154                | 8.0              | 100°R     | 17.7                     |      | 10     | 26  | 20                | 0   | -30 | -26 | 0   |         |
|              |       |                    |                  |           | 16.8                     |      |        |     |                   |     |     |     |     |         |
|              | 9:15  | 155                | 8.5              | 100°R     | 17.7                     |      | 5      | 12  | 10                | -8  | +24 | +26 | -2  |         |
|              | 30    | 156                | 9.2              | 105°R     | 16.8                     | 7    | 8      | 16  | 12                | 20  | 20  | 24  | -6  |         |
|              |       |                    |                  |           | 16.6                     |      |        |     |                   |     |     |     |     |         |
|              | 45    | 157                | 7.1              | 100°R     | 16.8                     | 7    | 8      | 12  | 12                | 20  | 10  | 10  | 0   |         |
|              |       |                    |                  |           | 17.4                     |      |        |     |                   |     |     |     |     |         |
|              | 10:05 | 158                | 5.6              | 100°R     | 16.8                     | 7    | 8      | 12  | 12                | 0   | 24  | +24 | 0   |         |
|              | 10    | 159                | 7.0              | 90°R      | 19.7                     | 8    | 6      | 22  | 20                | 0   | 26  | 0   | -24 |         |
|              |       |                    |                  |           | 17.2                     |      |        |     |                   |     |     |     |     |         |
|              | 20    | 160                | 7.0              | 90°R      | 22.2                     |      | 10     | 24  | 20                | -22 | 20  | -18 | +20 |         |
|              |       |                    |                  |           | 17.3                     |      |        |     |                   |     |     |     |     |         |
|              | 27    | 161                | 6.2              | 90°R      | 20.7                     | 8    | 8      | 20  | 20                | 0   | 24  | 0   | -24 |         |
|              |       |                    |                  |           | 16.8                     |      |        |     |                   |     |     |     |     |         |
|              | 35    | 162                | 8.5              | 90°R      | 17.2                     |      | 6      | 20  | 18                | -20 | 12  | -18 | 0   |         |
|              |       |                    |                  |           | 16.8                     |      |        |     |                   |     |     |     |     |         |
|              | 42    | 163                | 5.0              | 90°R      | 17.2                     | 7    | 6      | 12  | 6                 | -18 | +14 | +16 | 0   |         |
|              |       |                    |                  |           | 17.0                     |      |        |     |                   |     |     |     |     |         |
|              | 47    | 164                | 7.8              | 90°R      | 19.1                     |      | 12     | 16  | 20                | +24 | 0   | -24 | 0   |         |
|              |       |                    |                  |           | 16.8                     |      |        |     |                   |     |     |     |     |         |

TABLE 1 (Continued)

Recorded Oscillations - Tacoma Narrows Bridge - Aug. 1-12, 1940

| Date<br>1940 | Hour  | Observation<br>No. | Wind             |                | Frequency<br>per<br>Min. | Type | Double Amplitudes at Pt. |     |     |     |     |     |     |            | Remarks |
|--------------|-------|--------------------|------------------|----------------|--------------------------|------|--------------------------|-----|-----|-----|-----|-----|-----|------------|---------|
|              |       |                    | Veloc.<br>m.p.h. | Direc-<br>tion |                          |      | 1                        | 2   | 3   | 6   | 7   | 8   | 9   |            |         |
| 8-12         | 11:00 | 165                | 6.8              | 100°R          | 16.5<br>16.8             |      | +8                       | +20 | +20 | +12 | +10 | 0   | -4  |            |         |
|              | 10    | 166                | 6.2              | 100°R          | 16.2<br>16.5             | 7A   | 6                        | 16  | 8   | 10  | -20 | -18 | 0   |            |         |
|              | 25    | 167                | 5.4              | 110°R          | 16.5<br>17.6             | 7A   | 8                        | 18  | 8   | 20  | 0   | 18  | 0   |            |         |
|              | 35    | 168                | 6.2              | 110°R          | 17.7<br>16.2             | 7A   | 8                        | 18  | 14  | 20  | 0   | 20  | 0   |            |         |
|              | 50    | 169                | 6.0              | 100°R          | 16.2<br>16.8             | 7A   | 5                        | 14  | 8   | 20  | -20 | 22  | 0   |            |         |
|              | 55    | 170                | 7.8              | 100°R          | 17.4<br>16.0             | 7A   | 4                        | 8   | 8   | 24  | 0   | 20  | 0   |            |         |
|              | 12:00 | 171*               | 5.0              | 100°R          | 16.2<br>16.8             | 7A   | 6                        | 20  | 18  | 28  | -8  | -20 | 0   |            |         |
|              | 20    | 172                | 6.2              | 90°R           | 16.5<br>16.8             |      | 6                        | 28  | 24  | 20  | +26 | +8  | -14 |            |         |
|              | 31    | 173                | 4.8              | 90°R           | 16.5<br>16.8             |      | 8                        | 22  | 18  | 20  | +16 | +8  | 0   |            |         |
|              | 40    | 174                | 4.2              | 90°R           | 17.0<br>16.2             | 7A   | 4                        | 24  | 16  | +28 | 0   | -14 | 0   |            |         |
|              | 1:04  | 175                | 2.0              | 90°R           | 17.0<br>16.5             | 7    | +3                       | +4  | 3   | -6  | +12 | +12 | 0   |            |         |
|              | 07    | 176                |                  |                |                          |      | 0                        | 0   | 0   | 0   | 0   | 0   | 0   |            |         |
|              | 12    | 177                | 5.2              | 105°R          | 12.0                     | 5    | +3                       | +10 | +8  | -10 | -6  | +10 | +20 |            |         |
|              | 44    | 178                | 3.1              | 80°R           | 12.0                     | 5    | +2                       | 3   | 3   | 10  | 2   | 10  | 12  |            |         |
|              | 2:00  | 179                | 2.5              | 15°R           |                          |      | 0                        | 0   | 0   | 0   | 0   | 0   | 0   |            |         |
|              | 2:30  | 180                | 8.0              | 75°L           | 24.6<br>21.7             |      | +10                      | +6  | -18 | -4  | -6  | +16 | -16 |            |         |
|              | 35    | 181                | 7.6              | 70°L           | 20.4<br>22.0             | 8A   | 7                        | 0   | 14  | 0   | +16 | 0   | -20 |            |         |
|              | 45    | 182*               | 8.9              | 70°L           | 11.6<br>24.8             |      | 20                       | 0   | 24  | -2  | 0   | +2  | 0   | Very Gusty |         |
|              | 48    | 183                | 8.2              | 70°L           | 21.7                     | 8    | 3                        | 0   | 6   | +2  | -20 | +20 | +30 | Gusty      |         |
|              | 52    | 184                | 9.5              | 70°L           | 21.0                     | 8A   | 10                       | +6  | 10  | -6  | +20 | -8  | -30 | Gusty      |         |
|              | 3:05  | 185                | 6.0              | 80°L           | 20.8                     |      | 4                        | 4   | -6  | 4   | 20  | 4   | +20 | Gusty      |         |
|              | 11    | 186                | 5.2              | 90°L           | 21.0                     | 8    | 2                        | 4   | 0   | 4   | 16  | +2  | -16 | Very Gusty |         |
|              | 25    | 187                | 4.0              | 45°L           | 17.0<br>20.7             |      | +12                      | +20 | +24 | +10 | -20 | 0   | +20 | Very Gusty |         |
|              | 40    | 188                | 4.0              | 90°L           | 16.7                     |      | 4                        | 10  | 6   | +20 | 10  | -20 | -6  | Gusty      |         |
|              | 4:05  | 189                | 3.2              | 90°L           | 16.7                     | 5    | 4                        | 12  | 10  | -14 | +20 | +20 | 0   | Gusty      |         |
|              | 21    | 190                |                  |                |                          |      | 0                        | 0   | 0   | 0   | 0   | 0   | 0   |            |         |

Note: Where two values are given for frequency, the upper value is for the center span and the lower value is for the east side span. \*Indicates that oscillations are plotted on Drawing 5.

TABLE 2

## Recorded Oscillations - Tacoma Narrows Bridge - Aug. 14-Oct. 10, 1940

| Date<br>1940 | Hour  | Obser-<br>vation<br>No. | Wind<br>Veloc.<br>m.p.h. | Direc-<br>tion<br>per<br>Min. | Freq-<br>uency | Type | Double Amplitudes at Point |     |     |    |     |     |     |     |     | Remarks |   |
|--------------|-------|-------------------------|--------------------------|-------------------------------|----------------|------|----------------------------|-----|-----|----|-----|-----|-----|-----|-----|---------|---|
|              |       |                         |                          |                               |                |      | 1                          | 2   | 3   | 3A | 4   | 5   | 6   | 7   | 8   |         | 9   |
| 8-14         | 8:00  | 200                     | 11.4                     | 90°R                          | 16.7           | 7    | 12                         | 20  | 18  | 8  | -7  | -15 | -24 | +6  | +12 | 0       | Very Gusty<br>#5 Varies from<br>12" to 26"<br>#5 Varies from<br>8" to 30" |
|              | 10    | 201                     | 11.2                     | 90°R                          | 16.5           | 7A   | 8                          | 20  | 18  | 6  | +4  | +14 | +20 | -20 | -14 | 0       |   |
|              | 30    | 202*                    | 10.6                     | 90°R                          | 16.7           | 7    | 8                          | 20  | 18  | 6  | -6  | -15 | 0   | +30 | +20 | -4      |   |
|              | 50    | 203                     | 9.1                      | 90°R                          | 17.6           |      | 9                          | 20  | 16  | 8  | +8  | +30 | -14 | -30 | 0   | +16     |   |
| 9-15         | 9:15  | 204                     | 9.0                      | 90°R                          | 16.7           |      | 6                          | 24  | 18  | 9  | +12 | +24 | +24 | -24 | 0   | +18     | Very Gusty<br>Gusty<br>Gusty<br>Gusty<br>Gusty                            |
|              | 30    | 205                     | 9.0                      | 90°R                          | 16.7           |      | 10                         | 28  | 22  | 8  | +5  | +24 | +24 | -30 | 0   | +20     |   |
|              | 40    | 206                     | 8.2                      | 120°R                         | 20.7           | 8    | 4                          | 14  | 12  | 5  | -12 | -30 | 0   | +24 | 0   | -24     |   |
|              | 10:00 | 207                     | 12.3                     | 120°R                         | 20.7           | 8    | 8                          | 22  | 20  | 8  | +5  | +1  | +20 | +24 | 0   | -20     |   |
|              | 20    | 208*                    | 13.0                     | 120°R                         | 20.7           |      | 8                          | 24  | 20  | 5  | -8  | -16 | +20 | -20 | 0   | +24     |   |
| 11-00        | 35    | 209*                    | 11.4                     | 120°R                         | 18.0           |      | 12                         | 30  | 20  | 9  | +6  | +16 | +28 | +32 | 0   | -18     | Side Sway 18"   |
|              | 45    | 210                     | 13.2                     | 120°R                         | 21.0           | 8A   | +6                         | +28 | +14 | 7  | +9  | +30 | +6  | -32 | 0   | +30     |   |
|              | 52    | 211                     | 10.2                     | 120°R                         | 21.0           | 8    | +4                         | +8  | 0   | 0  | -14 | -30 | 0   | +30 | -4  | -36     |   |
| 12-15        | 15    | 212                     | 11.5                     | 120°R                         | 21.0           |      | 0                          | 2   | +3  | +1 | +8  | +24 | +2  | -12 | +4  | +20     | Showers   |
|              | 52    | 213                     | 9.6                      | 120°R                         | 21.0           |      | +6                         | 20  | 12  | 6  | +6  | +1  | +20 | +12 | 0   | -14     |   |
|              | 59    | 214                     | 11.4                     | 120°R                         | 22.0           | 8A   | 3                          | 6   | 4   | +2 | +14 | +20 | 0   | -24 | +6  | +30     |   |
|              | 12:15 | 215*                    | 13.3                     | 120°R                         | 20.7           | 8    | 1                          | 1   | 1   | 0  | -13 | -36 | +10 | +32 | 0   | -32     |   |
| 1-00         | 20    | 216                     | 13.3                     | 120°R                         | 20.7           | 8A   | 1                          | 3   | 1   | +1 | +10 | +15 | +2  | -10 | 0   | +12     | Very Gusty<br>Heavy Shower  |
|              | 1:00  | 217*                    | 14.0                     | 120°R                         | 21.0           | 8A   | +1                         | +3  | +4  | 3  | +15 | +34 | 0   | -24 | +10 | +36     |   |
|              | 15    | 218                     |                          |                               |                |      | 0                          | 0   | 0   | 0  | 0   | 0   | 0   | 0   | 0   | 0       |   |
|              | 3:35  | 219                     | 10.5                     | 120°R                         | 21.0           | 8A   | +1                         | +2  | +3  | +2 | +18 | +36 | +10 | -28 | +12 | +34     |   |
| 4-10         | 4:10  | 220                     | 8.0                      | 120°R                         | 16.5           | 7A   | +5                         | +16 | +12 | +6 | +8  | +24 | +24 | 0   | -12 | 0       | Very Gusty<br>Heavy Shower  |
|              | 15    | 221                     | 7.0                      | 120°R                         | 16.5           | 7A   | +7                         | +20 | +18 | +6 | +5  | +16 | +16 | -20 | -20 | 0       |   |



TABLE 2 (Continued)

## Recorded Oscillations - Tacoma Narrows Bridge - Aug. 14-Oct. 10, 1940

| Date<br>1940 | Hour  | Observation<br>No. | Wind<br>Veloc.<br>m.p.h. | Direction | Frequency<br>per<br>Min. | Type | 1   | 2   | 3   | 3A  | 4   | 5   | 6   | 7   | 8   | 9   | Remarks      |
|--------------|-------|--------------------|--------------------------|-----------|--------------------------|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|--------------|
| 8-22         | 3:15  | 222                | 2.0                      | 90°R      | 7.6                      | 1    | +2  | +4  | +3  | 0   | 0   | -2  | -2  | -4  | -6  | -6  |              |
|              | 4:30  | 223                |                          |           |                          |      | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |              |
| 8-23         | 1:25  | 224                | 2.0                      | 115°R     | 7.6                      | 1    | +6  | +8  | +8  | +7  | 0   | -4  | -6  | -12 | -20 | -22 |              |
|              | 35    | 225*               | 3.0                      | 135°R     | 7.6                      | 1    | +8  | +14 | +11 | +5  | -1  | -3  | -8  | -14 | -20 | -22 |              |
|              | 1:50  | 226                |                          |           |                          |      | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |              |
| 8-24         | 1:07  | 227                | 3.0                      | 60°L      | 7.6                      | 1    | +4  | +8  | +7  | +3  | 0   | 0   | -4  | -6  | -9  | -10 |              |
|              | 15    | 228                | 4.0                      | 90°L      | 7.6                      | 1    | +8  | +12 | +8  | +4  | -1  | -2  | -8  | -13 | -18 | -20 |              |
|              | 40    | 229                |                          |           |                          |      | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |              |
| 8-27         | 7:30  | 230*               | 9.7                      | 120°R     | 16.7                     | 3    | +10 | +25 | +20 | +8  | +6  | +18 | +28 | +16 | +5  | 0   |              |
|              | 45    | 231                | 10.1                     | 120°R     | 17.0                     |      | +10 | +20 | +20 | +12 | -7  | -24 | +22 | 0   | -12 | -6  |              |
|              | 8:00  | 232                | 8.0                      | 120°R     | 16.5                     | 7A   | +8  | +18 | +10 | +5  | +6  | +12 | 0   | -14 | -12 | 0   |              |
|              | 16    | 233                | 6.6                      | 120°R     | 12.0                     | 5    | +2  | +4  | +2  | +1  | -2  | -12 | -10 | 0   | +12 | +14 |              |
|              | 22    | 234                | 5.1                      | 120°R     | 11.6                     | 5    | +3  | +8  | +6  | +3  | -4  | -12 | -14 | 0   | +14 | +20 |              |
|              | 9:00  | 235                | 4.0                      | 105°R     | 12.0                     | 5    | +4  | +10 | +8  | +3  | -5  | -16 | -20 | -2  | +24 | +30 | Light Rain   |
|              | 30    | 236                | 4.2                      | 120°R     | 12.0                     | 5    | +4  | +6  | +4  | +2  | -2  | -10 | -12 | -2  | +12 | +18 | Rain         |
| 8-30         | 1:10  | 237                | 17.0                     | 120°R     | 21.0                     | 8A   | +14 | -3  | -14 | -8  | -12 | -26 | -6  | +28 | -6  | -28 |              |
|              | 30    | 238                | 12.0                     | 120°R     | 17.0                     | 7    | +8  | +20 | +18 | +10 | -12 | -30 | -24 | +30 | +28 | +6  |              |
|              | 4:30  | 239                | 10.7                     | 120°R     | 21.3                     | 8A   | +5  | +12 | +8  | +5  | +10 | +30 | +4  | -28 | +6  | +30 |              |
| 9-10         | 9:00  | 240*               | 18.0                     | 120°R     | 22.0                     | 10A  | -1  | +3  | +5  | +1  | +10 | +28 | -5  | +20 | -8  | -30 | Rain         |
|              | 20    | 241*               |                          |           | 25.0                     | 11   | -1  | +1  | +3  | +1  | +18 | +35 | -10 | +18 | -22 | -5  | " Very Gusty |
|              | 30    | 242*               |                          |           | 25.0                     | 9    | 0   | 0   | 0   | 0   | -7  | -25 | +11 | +10 | -20 | 0   | Hvy Rain     |
|              | 10:15 | 243                |                          |           | 21.0                     |      | -1  | +3  | +3  | +2  | +10 | +20 | -3  | +15 | -10 | -20 | Rain - Gusty |
|              | 11:00 | 244                | 15.0                     |           | 21.0                     | 8A   | 0   | -3  | -5  | -2  | +10 | +15 | +10 | -10 | -10 | +5  |              |
|              | 15    | 245                | 15.0                     |           | 19.4                     | 8A   | +1  | -7  | -12 | -5  | -10 | -20 | 0   | +24 | -10 | -22 |              |
|              | 45    | 246                | 25.0                     |           | 25.0                     | 9    | 0   | 0   | 0   | 0   | +10 | +25 | -10 | -8  | +28 | 0   |              |
|              | 1:00  | 247                | 15.0                     |           | 21.0                     | 8A   | +2  | -8  | -15 | -8  | -12 | -30 | -8  | +28 | -10 | -30 |              |
|              | 2:20  | 248                | 15.0                     |           | 21.0                     | 8A   | +3  | -6  | -25 | -10 | -10 | -28 | -10 | +25 | -15 | -25 |              |
|              | 45    | 249                |                          |           |                          |      | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |              |

Note: Where two values are given for frequency, the upper value is for the center span and the lower value is for the east side span. \*Indicates that oscillations are plotted on Drawing 5.

same character as the calculated modes; however, with an apparently inverted phase relation from those indicated on Drawing 4, between the main span and the side span. The only physical change in the structure during the periods of the measurements was the installation of the "hold-downs" in the east and west side spans on October 4th and 7th, respectively. The observations do not include the complete history, nor do they necessarily include the greatest amplitudes.

The following is quoted from a letter of Mr. L. R. Durkee, Project Engineer, dated November 4, 1940:

- " (1) Motions of considerable magnitude, having amplitudes as high as 48" with frequencies of 16 per minute, have been observed with wind velocities as low as three or four miles per hour.
- (2) Motions of varying degrees of violence have been noted in winds up to 48 miles per hour. The violence of motion is not necessarily proportional to the velocity of wind.
- (3) The bridge has remained motionless at times in wind velocities varying from zero to 35 miles per hour.
- (4) There appears to be no difference in the motion whether the wind is steady or gusty.
- (5) Traffic loads appear to have no measurable effect on the motion.
- (6) Altogether, seven different motions have been definitely identified on the main span of the bridge, and likewise duplicated on the model. These different wave actions consist of motions from the simplest, that of no nodes, to the most complex, that of seven nodes. The seven-node motion has been observed but once and had a frequency of 30 cycles per minute.
- (7) Amplitudes as high as 60" (over all motion plus to minus), have been observed."

Excerpts of other reports from Mr. Durkee follow:

Sept. 18, 1940. "Movement appears to be caused by wind or air currents and exist from wind velocities of one or two miles per hour up to the highest observed velocities of thirty-

five miles an hour. These movements are vertical and have approached a maximum magnitude of about fifty inches and a maximum rate of twenty-seven complete oscillations per min. The most common type appears to be that which is designated No. 3 (Type 5 on Drawing 4). These movements do not occur continuously but for irregular intervals of time from a few moments up to six or eight hours. While no definite data is available, it is estimated that of the total time, these movements are probably occurring 10 to 15% of the time."

"Traffic has and is using the bridge continuously without other than the unusual sensation experienced. Close observations have been kept to observe any structural effect from these movements but none has been noted."

Attached to that report was a diagram showing the types of motions, their amplitudes and frequency. These data may be tabulated as follows. It will be noted that the amplitudes are, in some cases, larger than shown on Tables 1 and 2.

| Type<br>(Dwgs. 4 and 5) | Double Amplitude<br>Ins. | Frequency<br>per min. |
|-------------------------|--------------------------|-----------------------|
| 1                       | 26                       | 8                     |
| 2                       | 36                       | 13                    |
| 3                       | 34                       |                       |
| 5                       | 36                       | 12                    |
| 5A                      | 38                       |                       |
| 7                       | 38                       | 17                    |
| 7A                      | 28                       |                       |
| 8                       | 30                       | 21                    |

Sept. 19, 1940. "On August 26, particularly turbulent wave motions were observed having a frequency approximating 27 per minute with a total amplitude, trough to crest, of nearly four feet. This was a multiple node wave movement somewhat similar to that shown as No. 10 on the chart (Type 8A) although the exact number of nodes was not definitely determined."

Nov. 7, 1940 - After failure of the bridge. "Violent undulations and vibrations occurring during the early forenoon caused traffic to be shut down across the bridge. The wind velocity was clocked at 42 miles per hour by an anemometer on or near the structure about 9:00 a.m. Professor Farquharson of the University of Washington was present at the time of failure and he has stated that he believed the velocity was somewhat higher than this at the time the bridge went down."

On the morning of the failure, the center span was oscillating with

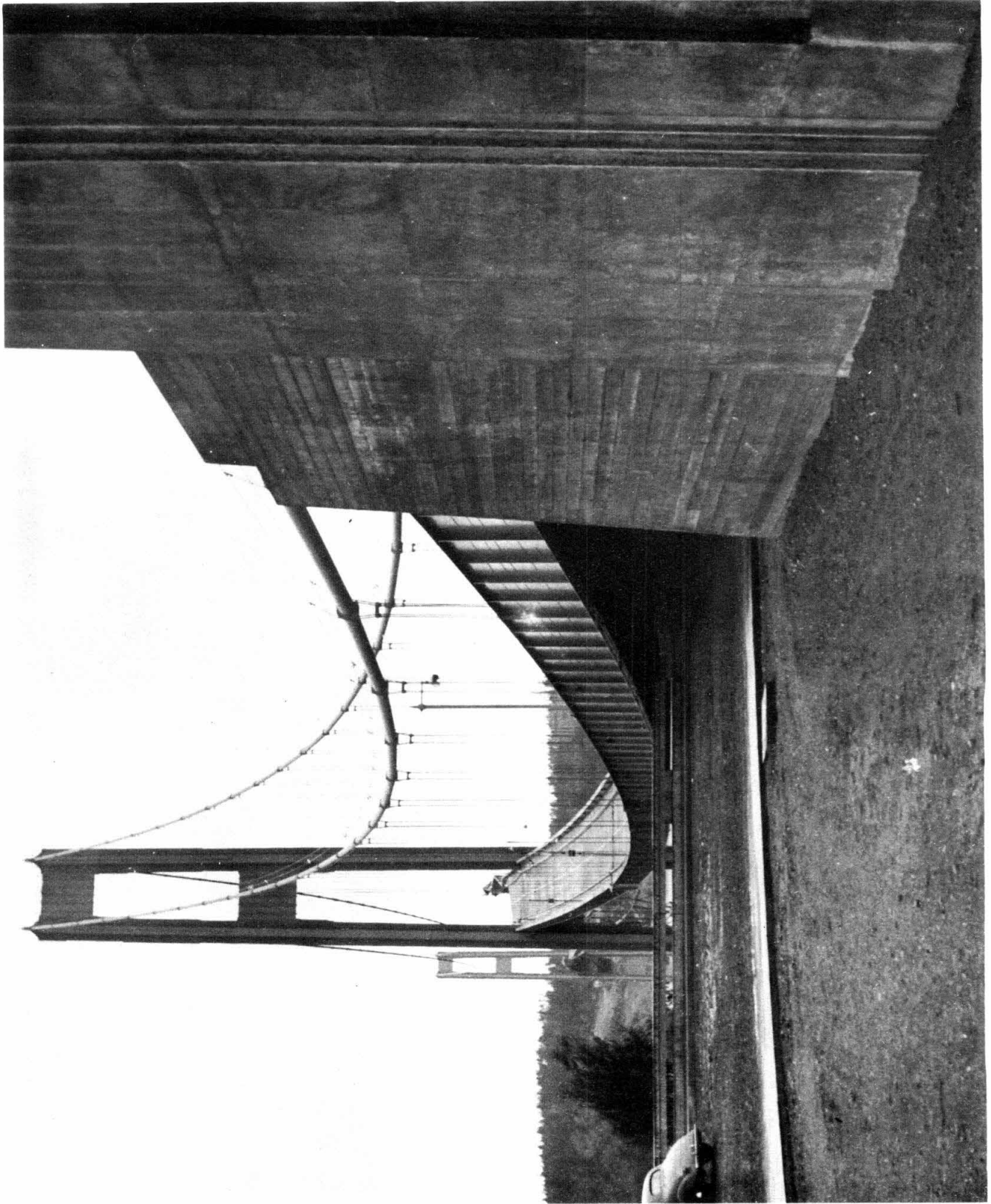


Fig. 7. Sagging of East Side Span after Dropping of  
Center Span of Suspended Structure.

either 8 or 9 nodes with a frequency of 36 to 38 per minute. There appears to have been no instrumental measurement of the amplitudes. The opinion of witnesses (Appendix V) is that they were no larger than had previously occurred. Figs. 6(a) and (b) are enlargements of single frames of motion pictures taken by Walter Miles of the Pacific Bridge Company at 9:30 A.M. on the morning of the failure and are of the north center tie. These pictures show, the complete film much better, a decided longitudinal motion of the center of the cable in relation to the center of the suspended span. From these pictures, it is evident that the diagonal guys became alternately slack and, therefore, partially ineffective. This slackness permitted greater amplitudes of the relative motions between the cable and the suspended structure; and, motions of these amplitudes had a material effect on the stresses in the center tie. The pictures show that the motions, largely as a result of the slackness, were accompanied by severe impact.

Failure of the Bridge. Prior to 10:00 A.M. on the day of failure, there were no recorded instances of the oscillations being otherwise than with the two cables in phase and with no torsional motions. During the morning of November 7th, the velocity of the wind was approximately 42 m.p.m., one of the most severe storms that had been experienced. Statements of witnesses indicate the following sequence of happenings.

Suddenly, at approximately 10:00 A.M., the center span developed a torsional movement with a node at mid span. The frequency suddenly changed from 37 to 14. At the quarter points, the angle of rotation was observed as nearly 45 degrees each way, the amplitude of the cable movement being approximately 28 feet. Shortly after the start of this violent

motion, a differential motion between the concrete sidewalk slab and the girders was observed. The torsional movement at times changed to a different mode but soon returned to the single noded motion. About 10:30, a section of the concrete roadway slab dropped from the center of the bridge and photographs show a panel of laterals hanging from their center connection.

The first major failure, the dropping of about 600 ft. of the suspended structure at the Gig Harbor (West) quarter point, occurred at about 11:00 A.M. With this failure, the vibrations quieted down, but soon restored themselves. At 11:10, the final failure, consisting of the dropping of most of the remainder of the suspended structure in the center span, occurred. With this release of weight in the center span the side spans sagged and the towers were suddenly deflected shorewards, the maximum deflection being about twenty-five feet.

During the violent motion of the center span, the east side span, presumably also the west side span, was relatively quiet. After the failure of the center span, a few torsional oscillations occurred in the east side span. During the failure, there was a considerable slipping of the north cable through the center cable band which was held by the center stays. The evidence supporting this statement may be summarized as follows:

1. Statement of Lt. Hogan.- Appendix V.
2. Figs. 6(a) and (b), above described, was taken at approximately 9:30 A.M. on the day of the failure. The motion pictures show that there was no slipping of the band at this time. Fig. 6(c) shows the cable and band at this point taken after the accident. About 500 wires were literally worn or cut through by the movements of the band. The amplitude of the motion was about 42 inches.



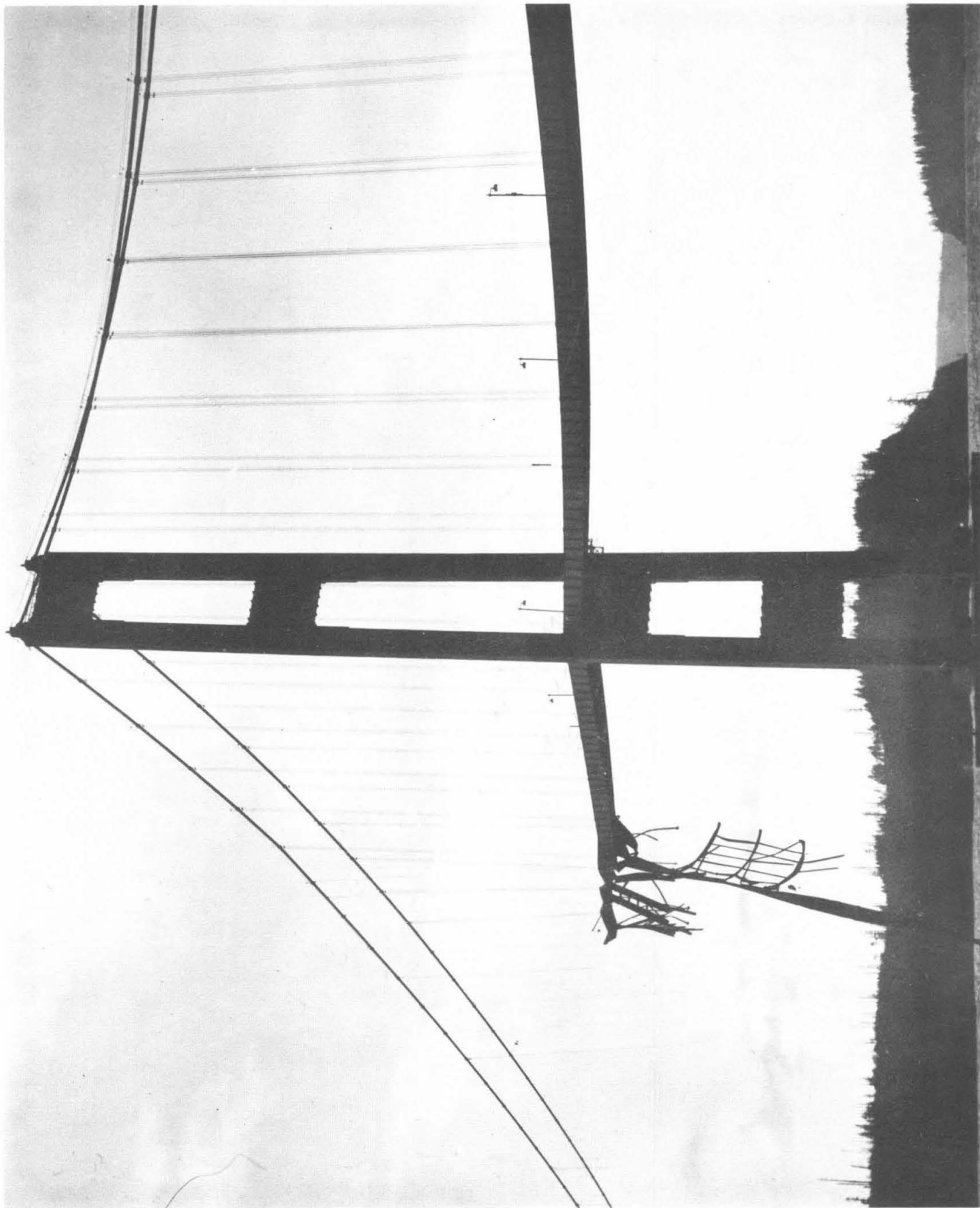


Fig. 8. Conditions Near West Tower After Failure.

A more detailed account, Appendix V, of the failure is given by the observations of five eyewitnesses. These statements were secured shortly after the accident by Project Engineer L. R. Durkee. Also attached to the same Appendix is a statement by Lt. Hogan of the United States Coast Guard.

Physical Condition of Remaining Parts of the Structure. As stated in the introduction, the Washington Toll Bridge Authority appointed a Board of Engineers to survey the physical condition of the remaining parts of the structure and to determine the amount of loss under the insurance on the structure. Our assignments did not include this question. On our visits to the site, we made a superficial examination of the remaining parts. The results are summarized below.

Anchorage. There is no apparent damage to the anchorages. Mr. Eldredge stated that a careful survey had indicated no movement.

Main Piers. Aside from a small amount of spalling of the concrete surrounding the channel connections to the tower anchorage grillage, there appears no damage to the main piers. Mr. Eldredge advised us that a survey had shown no displacements. We were further informed that the Toll Bridge Authority engineers are making periodic soundings to be certain that no undue amount of scour is taking place.

Towers. The cover plates of the shoreward sections of all four tower shafts exhibit buckles throughout their entire height, indicating that these plates have been stressed beyond their yield strength. It is probable that some yielding has also taken place on adjacent parts. The splices on the channelward sections have yielded sufficiently to allow the joints to open. At the base the channelward sections lifted off the base plates. The top struts failed at their center. Considering the severe bending of these towers during the failure of the bridge, there is a remarkably small amount of damage.

Cables. At the center of the north cable approximately 500 wires have been cut by the chafing of the center cable band. The cable rolled off the south hold-down saddle at the end of the west side span with a considerable torsional movement. The amount of damage at this point is not known. At other points superficial

examination discloses no evidence of failure, but this is no proof that the cables are undamaged other than above set forth.

Suspended Structure. In the side spans the stiffening girders have buckled beyond repair at certain points easily discernible. At these points the stringers and laterals have failed by buckling.

Approach Spans. The approach spans were practically undamaged during the failure.

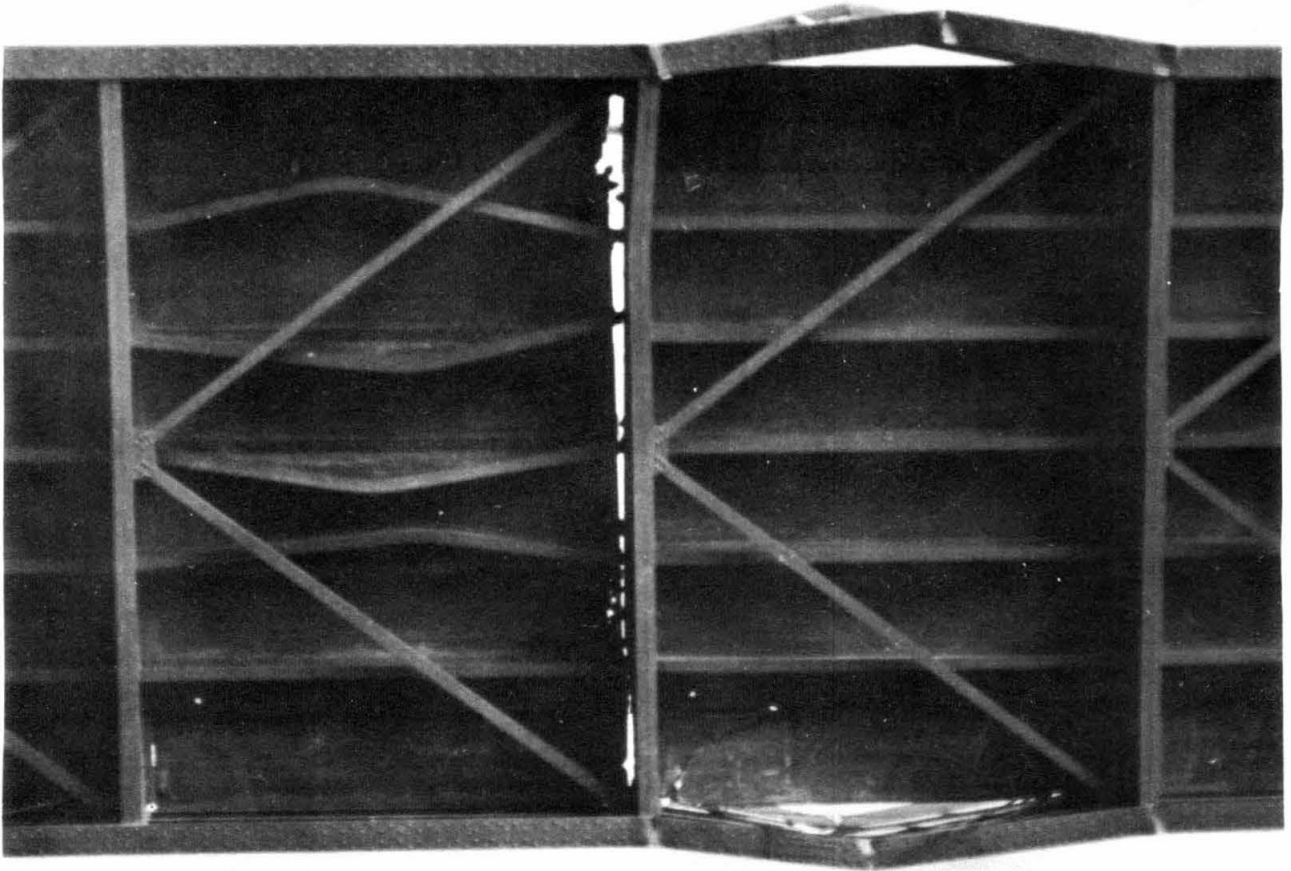
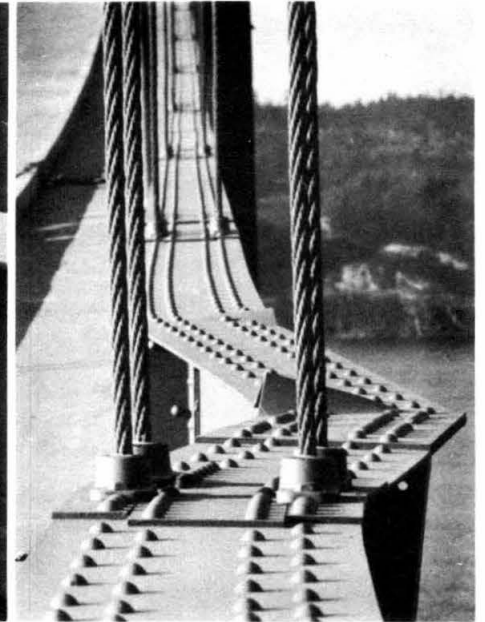


Fig. 9. Buckling of Suspended Floor between Floor beams  
near Center of Side Spans.

## CHAPTER II

### REVIEW OF THE DESIGN OF THE TACOMA NARROWS BRIDGE

The behavior of the Tacoma Narrows Bridge during the hour before the final failure of its center span floor was a severe test of the strength of the structure as a whole, as well as of its major component parts. Under the violent dynamic motions and during the final failure certain parts of the floor structure, the suspenders, the cables and the towers were subjected to stresses far beyond the safe stresses for which these parts were designed, and at many points to stresses under which failure had to result. That under these severe stresses the main carrying members, in particular the towers, did not actually fail attests to the excellence of their design, quality of materials and workmanship, and to the ample margin of safety which the structure would have had except for the severe dynamic motions. A review of the design of the structure on the basis of accepted requirements for strength and other structural qualities confirms that conclusion.

This review is based upon the design of the structure as actually built, and upon our independent calculations and judgment in respect to the intensity of the assumed loads and forces, the manner of their application and the permissible stresses in the various parts of the structure.

The review is limited to the more important parts of the bridge. In a structure of this type and size there are numerous details, the efficiency of which is subject to differences of opinion. Structural details which had evidently no bearing upon the failure of the structure

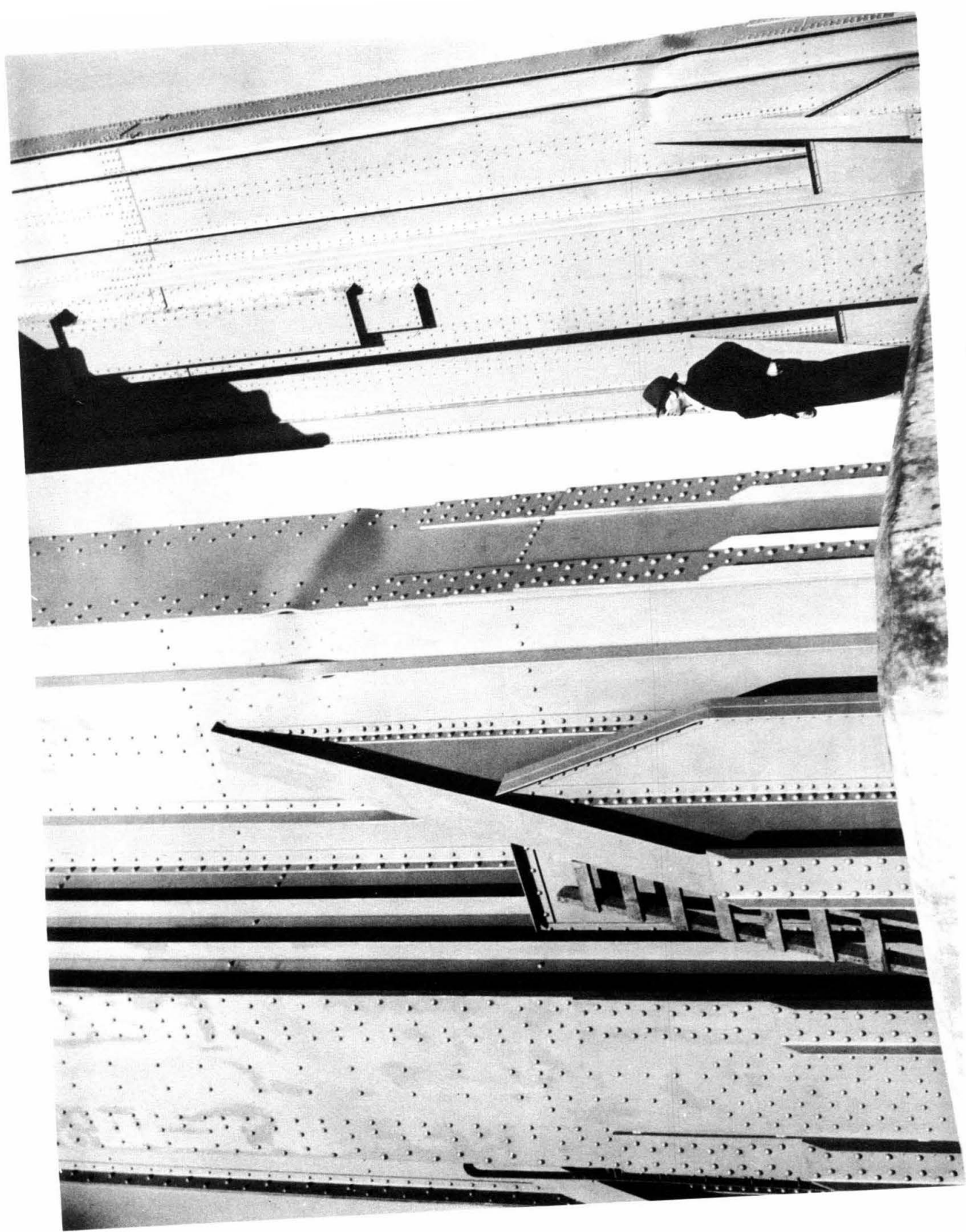


Fig. 10. Buckles on Shoreward Side of Tower near Base.



require no comment.

Besides the calculation of stresses produced in the structure by the assumed static forces, the major deformations, vertical, horizontal and torsional, from these forces have been determined with a view to examine their effect on the structure and on the grades, alignment and tilting of the floor and to compare them with similar deformations in other long span suspension bridges.

In order to explain the rupture of the floor and suspenders, and apparent overstressing of other parts, particularly the towers, an attempt has been made to evaluate the stresses which were developed at various critical points of the structure at the time of its failure. These are necessarily approximations based on the rather scant observations on the deformations of the bridge at the time of failure and on approximate evaluations of the accompanying dynamic effects.

#### TYPE OF STRUCTURE, SPAN ARRANGEMENT AND TRAFFIC CAPACITY

The suspension type of bridge was unquestionably the most suitable and most economical type for the purpose and the locality.

The arrangement of spans and the location of the anchorages were evidently chosen after careful study of the geological and topographical conditions for the pier and anchorage foundations and the economics of the structure, and could not be materially improved. Shorter side spans, and avoidance of the extension of the cables beyond them, would have furnished greater rigidity. It would not have been advisable, however, to effect such a change by moving the anchorages closer to the banks of the river because of danger of sliding, and a material increase in center span would have been more costly.

The capacity of the bridge, a two-lane roadway and two footwalks, appeared from the prospects of traffic development to be ample for many years to come, and as much as could be justified economically. A greater traffic capacity would have added to the cost of the project. To take full advantage of the economy of the narrow roadway, however, the designers adopted an extraordinarily small width of structure compared to its span. Their expectation that the bridge would have adequate lateral rigidity under wind pressure was unquestionably justified.

#### ASSUMED STATIC LOADS AND FORCES

Dead Load. Based on a detailed estimate made by the engineers of the Washington Toll Bridge Authority, the average weight of the suspended structure as built is 5,700 lb. p. ft. in the center span and 5,740 lb. p. ft. in the side spans. This compares with a dead load of 6,000 lb. p. ft. of bridge assumed for the design. This quite appreciable decrease of 300 lb. or 5% in the actual weight resulted principally to a modification of the design of the roadway slab and floor steel.

The weight of 5,700 lb. is made up as follows:

|                                |                |
|--------------------------------|----------------|
| Floor slabs, lighting, etc.    | 2,650 lb.      |
| Floorsteel and laterals        | 720 "          |
| Stiffening girders             | 900 "          |
| Cables with accessories        | <u>1,430 "</u> |
| Total dead load of center span | 5,700 lb.      |

Live Load. For the design of the floor system and suspenders the H-20 loading (20-ton trucks) of the specifications of the American Association of Highway Officials was assumed. This loading is now generally adopted for the design of bridges on first class highways.

For the stiffening girders, cables and towers a live load of 1,000

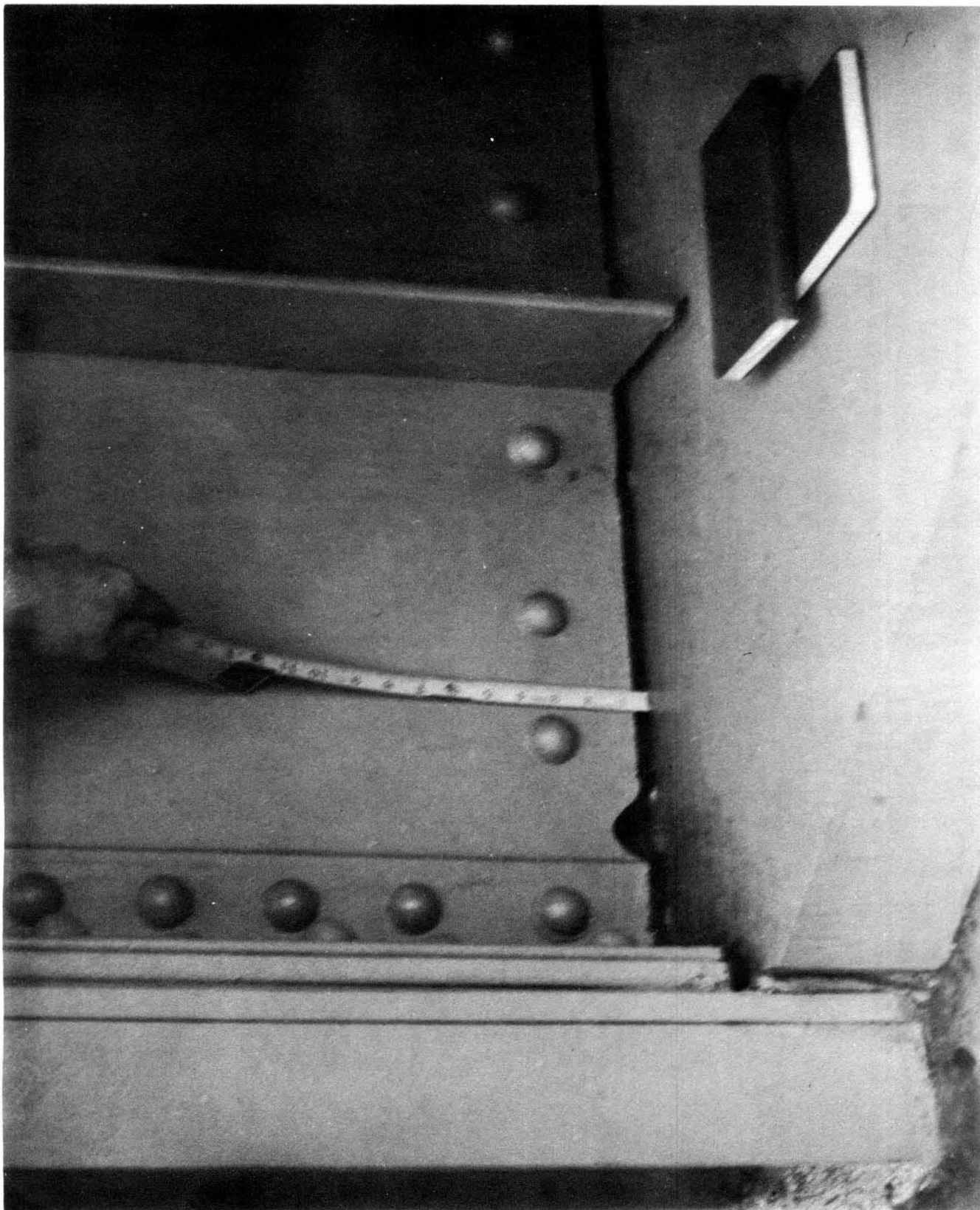


Fig. 11. Opening of 1/2 Inch between Tower Shaft and  
Base Plate on Riverward Side of Tower Shaft.

lb. p. ft. of bridge was assumed for the design. This is equivalent to an average load of about 28 lb. p. sq. ft. of area of the 26 ft. roadway and the two 5 ft. sidewalks. While this represents a load condition which is not likely ever to occur over any considerable length of the bridge, under actual traffic, it does not, in our opinion, represent the intensity of load concentration which is possible, and which should be made the basis of design of a structure such as the Tacoma Narrows Bridge. In a bridge of this span highway live load is relatively small compared to the dead load and an ample margin for possible future increase in loading is warranted.

This review is therefore based upon a live load of 1,500 lb. p. lin. ft. of bridge (42 lb. p. sq. ft. of floor area) which is more nearly comparable to the loads assumed for other, much wider, recent suspension bridges with long spans designed for highway traffic only (Golden Gate Bridge, 4000 lb. p. ft., or 50 lb. p. sq. ft.; Bronx-Whitestone Bridge, 3000 lb. p. ft. or 46 lb. p. sq. ft.; Triborough Bridge, 4000 lb. p. ft. or 41 lb. p. sq. ft. This difference in assumed live load has no bearing on the failure of the bridge, nor does it affect materially the maximum stresses in the cables, towers and anchorages, because it is largely offset by the aforementioned reduction in dead load. Both reduction in dead load, indirectly, and higher live load affect unfavorably the live load stresses in the stiffening girders, but since the sections of the latter are largely governed by the wind stresses the effect is not material.

For maximum live load stresses the load is placed in the most unfavorable position. For torsion or transverse tilting of the floor it

is assumed distributed over part of the floor and in the most unfavorable position.

Wind Forces. The following static wind forces were assumed for the design.

On suspended structure, per lin. ft. of bridge:

|  |         |
|--|---------|
| Wind on moving load                              | 200 lb. |
| Wind on floor structure (1-1/2 x 8 ft. x 30 lb.) | 360 "   |
| Total wind on floor                              | 560 "   |
| Wind on two cables                               | 60 "    |
| Total wind on loaded suspended structure         | 620 "   |

On towers: 50 lb. p. sq. ft. of exposed area.

This horizontal wind force is equivalent to over 50 lb. p. sq. ft. of exposed area of the unloaded floor, the two cables, and the towers, as seen in elevation, is adequate as a static force for the proportioning of the wind system and has been adopted for the design review.

For the investigation of torsional effects the following assumptions have been made: (a) 200 lb. p. ft. of bridge acting on the moving load applied 6 ft. above the top of the roadway, combined with a vertical load of 400 lb. p. ft. of bridge located on the leeward lane, and (b) a uniform live load of 750 lb. p. ft. of bridge extending from one tower to center of span, applied 39 x 1/4 feet from the near cable and the same load extending from the center of span to the other tower applied 39 x 1/4 feet from the far cable. Assumption (b) causes the most severe torsion near the quarter points of the center span.

Temperature. A maximum change in temperature of plus or minus 40° F. from the normal was assumed for the design. This is entirely sufficient for the locality of the Tacoma Narrows Bridge.

## THE SUSPENDED FLOOR STRUCTURE

The Floor System. The 26 ft. roadway is carried on a reinforced concrete slab 5-1/4 in. thick. It is flanked on each side by a concrete curb 15 in. high and by a 5 ft. sidewalk on a 2-1/2 in. reinforced concrete slab (Drawing 2). The roadway slab rests on five 21 in. I-beam stringers spaced 5 ft. 9 in. on centers. The stringers are framed into the floor beams which are spaced 25 ft. apart on centers and are plate girders 4 ft. 4-1/2 in. deep. The floor beams in turn are framed into the two main longitudinal girders which carry the loads to the suspenders. The general arrangement and makeup of the floor system is exceedingly simple and, except for some details, meets the specifications of the American Association of Highway Officials. All material is structural carbon steel.

The curves of vertical deflections under maximum assumed live load at normal temperature are shown in Fig. 15. The maximum deflections from live load and temperature are 13.5 ft. near the quarter points and 13.4 ft. at the center of the center span. The greatest deflection in the side spans is 10.0 ft.

The floor slab and stringers are located near the neutral axis of the stiffening girders, and therefore are practically free from bending stresses from the vertical deflections imposed by the cables. To avoid stresses in the floor system from horizontal deflections and temperature changes the stringers were provided with sliding bearings and the concrete floor with expansion joints 150 ft. apart. In addition the concrete slab had asphaltic joints at every floorbeam. The main expansion joints of the floor structure and of the stiffening girders are located at the



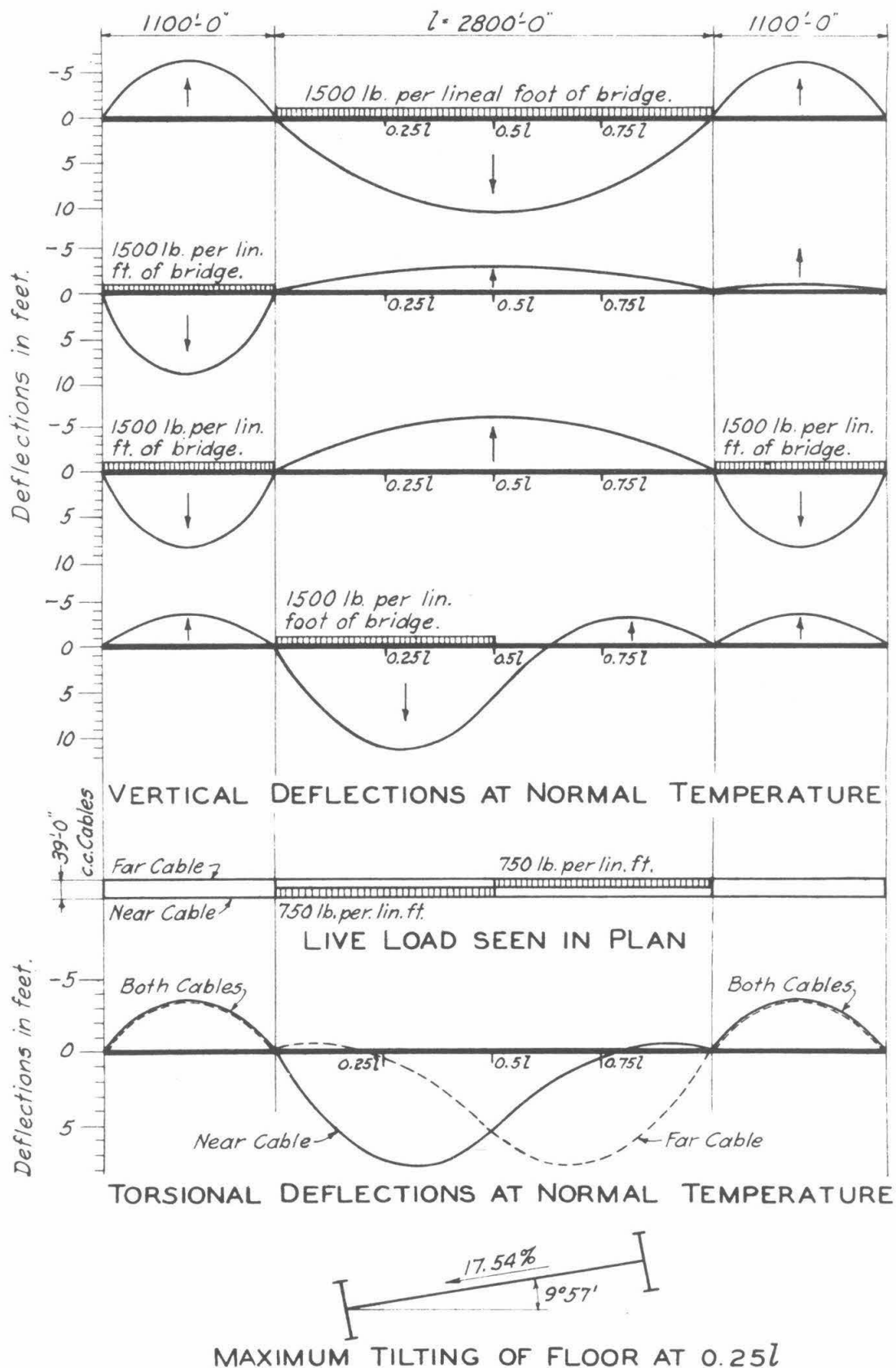


FIG.15 - VERTICAL AND TORSIONAL LIVE LOAD DEFLECTIONS OF THE TACOMA NARROWS BRIDGE.

towers. Thus the floor system had great flexibility and adaptability to vertical deformations and probably at no time sustained severe stresses as long as the cables moved in phase.

The vertical deflections from live load and temperature cause an increase in grade in the side span near the towers of 3.93%, thus bringing the maximum grade of the roadway to 6.52%. On a highway bridge and under the extremely improbable load assumptions this grade cannot be considered excessive. It is not greater than on some of the other large suspension bridges.

In a wide and relatively rigid suspension bridge the transverse tilting of the floor and the resulting torsional stresses in the floor structure caused by non-uniformly distributed live load and eccentrically applied wind pressure are negligible, but in a narrow flexible bridge, such as the Tacoma Narrows Bridge, they may be appreciable and must be investigated. For maximum torsion the load condition hereinbefore mentioned was assumed (Fig. 15). Under this very improbable condition the floor at the quarter point of the center span would tilt sideways 17.5% or 6.84 ft. in the width of 39 ft. While such a torsion would probably never be produced under actual load conditions it is considerably in excess of similar tilting in other large suspension bridges (Table 4) and indicates the relatively great torsional flexibility of the bridge. The stresses resulting from this highly improbable but possible static load condition in the floor structure are, however, well within permissible limits.

Stiffening Girders and Wind Truss. The two main girders are located in the planes of the suspenders and cables 39 feet apart on centers.

They are hung from the rope suspenders spaced 50 ft. apart. They are solid plate girders 8 ft.  $1/2$  in. deep b. to b. of flange angles. Besides carrying the floor they act vertically as stiffening girders and horizontally as chords of the wind truss.

Each girder is made up of a web plate 96 in. x  $1/2$  in., 4 angles 8 x 6 x  $1/2$  in. and 2 cover plates 20 x  $1/2$  in. It has a sectional area of main material of 95.0 sq. in. and for stiffening an effective moment of inertia about the horizontal axis of  $1283 \text{ in}^2 \text{ ft.}^2$ . Both web and flanges are of structural silicon steel.

The general arrangement of the stiffening system and the make-up of the stiffening girders are not unusual. Plate girders have been used in a number of suspension bridges. They are being adopted to a growing extent, and with increasing proportions, in bridges of all types because of economy and structural simplicity. When the required depth exceeds the practicable or economical depth of plate girders, open trusses are the proper solution.

The 8 ft. girders are so shallow and flexible that they exerted almost no stiffening effect upon the cables, except locally. Evidently the designers had relied on the weight of the long span of the bridge and on the relatively small cable sag to provide the necessary vertical rigidity. There is every evidence that insofar as rigidity under moving loads is concerned their judgment was justified. They had not expected that aerodynamic action could become so powerful as to put the great suspended mass of the structure into dangerous or objectionable motion.

Under the static live load, temperature changes and lateral wind forces assumed for this review the stiffening girders are subject to

the following maximum stresses in lbs. p. sq. in.

|   | Near 1/4 Point of<br>Center Span |             | Near Center of<br>Side Span |             |
|---|----------------------------------|-------------|-----------------------------|-------------|
|   | Tension                          | Compression | Tension                     | Compression |
| Dead load + live load<br>+ temperature            | 9,780                            | 9,780       | 13,200                      | 13,200      |
| Wind only   | 23,350                           | 19,840      | 16,960                      | 19,960      |
| Dead load + 1/2 live load<br>+ temperature + wind | 26,890                           | 23,730      | 23,760                      | 21,060      |

These stresses are within permissible limits for structural silicon steel, the minimum yield strength of which is 45,000 and the ultimate strength 80,000 lbs. p. sq. in. The fact that the top flange of the girder is elastically braced at the floorbeams has been considered. The critical buckling strength of the flange is calculated at 40,000 lb. p. sq. in.

The K system of laterals between the two stiffening girders are located in a plane below the bottom flanges of the roadway stringers to which they are fastened at the points of intersection. All laterals are 8 in. WF beams. The stresses in the laterals have been determined by assuming the wind as a moving load placed in a position to cause maximum shear in the respective panels of the wind truss.

The largest stresses occur in the laterals at the towers, where they are 21,170 lb. p. sq. in. in tension and 17,150 lb. p. sq. in. in compression ( $1/r = 77$ ). These laterals are of silicon steel. The largest stresses in carbon steel laterals are 15,540 and 12,570 lb. p. sq. in. respectively. These stresses are within permissible limits.

The width of the wind truss between centers of stiffening girders

is 39 ft. or only  $1/72$  of the span length. It is thus the most slender and flexible wind truss of any of the large suspension bridges, the nearest being the Golden Gate Bridge with a width to span ratio of  $1/47$ . The lateral deflection of the truss at mid-span under full wind force on the center span is 20.0 ft. or  $1/140$  of the span length. This deflection is controlled largely by the cables to which the major part of the floor wind load is transmitted (Fig. 16).

The lateral deflection does not produce excessive stresses in the floor structure. In a suspension bridge the cables and, with them the wind truss, remain always in stable equilibrium and there is no danger of the wind truss buckling as a whole laterally in its plane, such as might be the case with a flexible wind truss in a simple span. cantilever or arch bridge in which compression chords of the main trusses form the chords of the wind truss. The wind truss of the Tacoma Narrows Bridge is discontinuous at the towers so there are no sharp curvatures in its elastic line and therefore no unusual secondary stresses in its members from the deformations.

Furthermore, it has been observed generally that the actual lateral deflections and therefore stresses from wind forces remain below those considered in the design. Wind pressure never acts over such great lengths with its maximum intensity. In the Tacoma Narrows Bridge the lateral deflection of the floor was observed as about two feet under the wind velocity of 42 mi. p. hr. on the day of failure. The calculated lateral deflection of 20 feet in the Tacoma Narrows Bridge would cause an horizontal rotation of the floor at the towers of about 2.8%, which would be noticeable. Again under actual conditions the angle would remain

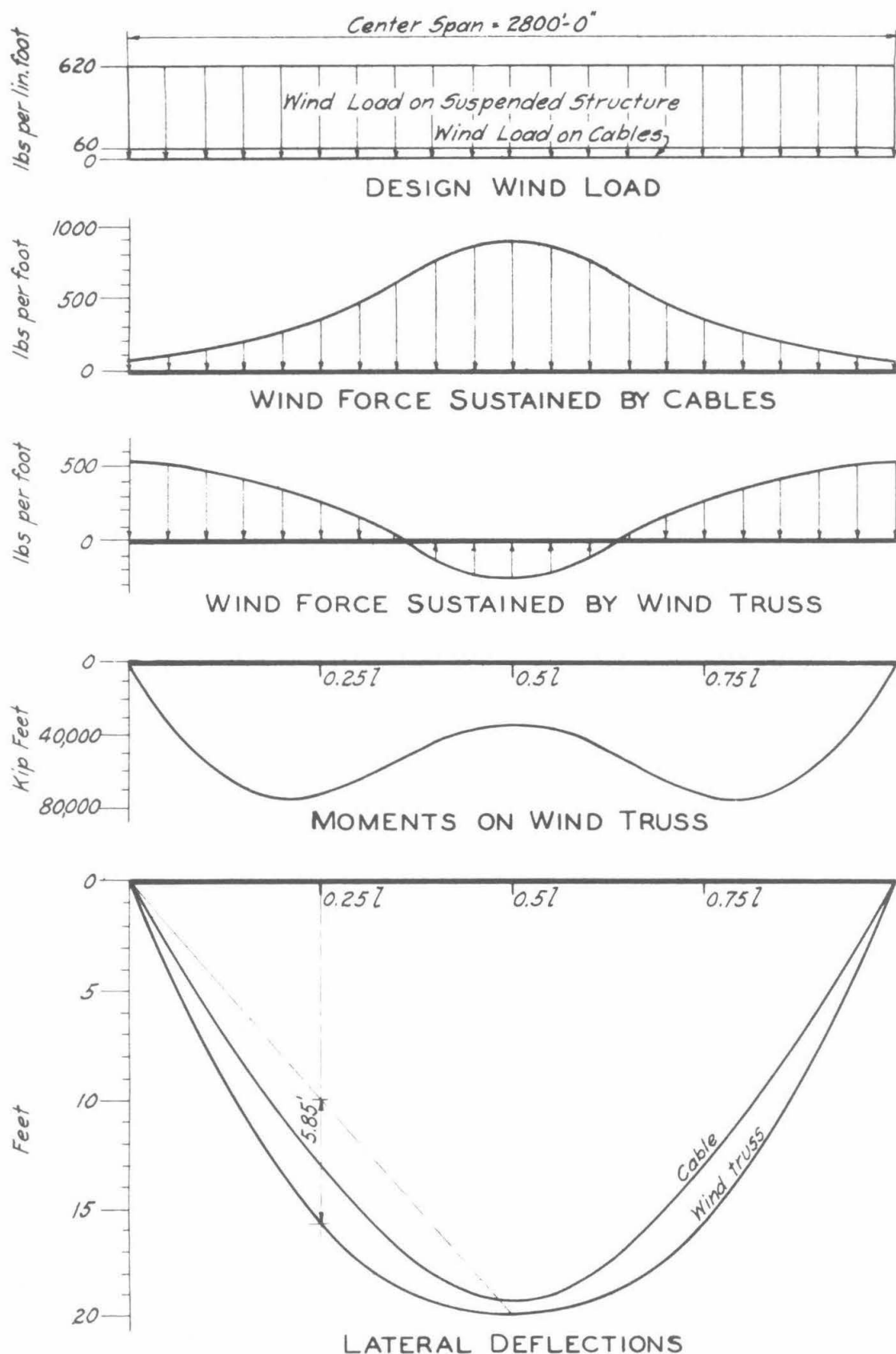


FIG.16- DISTRIBUTION OF 620 LBS PER LINEAL FOOT DESIGN WIND LOAD AND RESULTING LATERAL DEFLECTIONS OF THE TACOMA NARROWS BRIDGE.



below this value and it has therefore no significance in its effect upon traffic.

Neither the horizontal deflections resulting from wind nor the angular change at the towers are a measure of the allowable width to length ratio or of the required rigidity of suspension bridges.

#### THE CABLES AND SUSPENDERS

The Suspenders. The suspenders from which the floor is hung at points 50 ft. apart consist at each point of four parts of  $1\frac{1}{4}$  in. diameter galvanized steel ropes with an average ultimate strength of a two part rope tested over a sheave of 298,300 lb. The average for a single rope was 165,400 lbs. The maximum axial stresses in a four rope suspender are: Dead load 106,000 lb.; live load 45,900 lb.; impact 10,200 lb.; total stress 162,100 lb. This is 27.2% of the ultimate strength of the four ropes combined and provides an ample margin under the assumed loads.

The Cables. The floor structure is suspended from two cables, one on each side of the floor in the plane of the stiffening girder. The cable sag of 232 feet at normal temperature, or  $\frac{1}{12}$  of the span length, is smaller than that of other modern suspension bridges in which it ranges generally between  $\frac{1}{9}$  and  $\frac{1}{11}$  of the span (Table 3). It is well known that a small cable sag produces greater rigidity against both vertical and lateral deformations under static loads, and it was for that reason and for the express purpose of producing greater rigidity that the designers adopted relatively flat cables, undoubtedly at some extra expense.

The analysis which is contained herein, corroborated by the behavior

of the structure, indicates that the effect of a flat cable on the vertical rigidity of the suspended system was overestimated, and was by far not sufficient to offset the flexibility permitted by other elements of the structure.

Each of the two cables consists of 6308 No. 6 galvanized cold drawn steel wires laid parallel in 19 strands of 332 wires each. The theoretical net area of wire in each cable is 190.32 (actual 191.296) sq. in. The diameter of the cable after wrapping is 17-1/4 in. The wire has a specified average strength for each heat of at least 225,000 lbs. per sq. in. The average of actual tests was 235,000 lbs.

The maximum axial stresses in the cable based on 190.32 sq. in. wire area are:

From dead load and live load (1500 lb. p. ft.) and temperature 40°F. below normal, 84,500 lbs. p. sq. in.

From dead load only at normal temperature, 67, 700 lbs. per sq. in.

These axial stresses are not excessive and reflect a recent tendency toward higher unit stresses in wire cables which is justified by the greater uniformity of material and superior practice in splicing and laying the wires in the cable. The axial stresses in a suspension bridge cable include an allowance for secondary or bending stresses which are produced at the saddles, principally those at the tops of the towers, when the cables deflect under load. In the Tacoma Narrows Bridge these bending stresses are in the order of 5% of the primary stresses under maximum deflections from live load and temperature.

The Towers. The two towers as seen in side elevation are flexible shafts 425 ft. high from top of pier to center of cables, 19 ft. wide

at the bottom and 13 ft. at the top. As seen in transverse elevation they form simple rigid frames. The two shafts which have a uniform width of 13 feet from top to bottom, and on which the cables are centered, are connected by four deep horizontal struts, two above and two below the floor (Drawing 3). The two shafts are slightly battered so that the width between shaft centers increases from 39 ft. at the top to 50 ft. at the base.

Each shaft has a cellular cross-shaped section built of solid web plates, connected by angles. The sectional area varies between 898 sq. in. at the top to 1199 sq. in. at the bottoms. The bottom section is increased to 1879 sq. in. at the base. The maximum moment of inertia of each shaft is 2,061,000 in<sup>4</sup> longitudinally and 3,250,000 in<sup>4</sup> transversely. The struts are box shaped sections 3 ft. 11 in. wide and of depths ranging between 27 and 35 ft.

All materials in shafts and struts are structural carbon steel with a minimum yield strength of 33,000 lbs. per sq. in. and ultimate strength of 60,000 lbs. per sq. in.

The tower shafts are firmly anchored to the piers by a steel anchorage frame which extends to 16 ft. below the base into the concrete.

The general design of the towers and major details are in accordance with best recent practice.

The maximum unit stresses in lb. p. sq. in. in the towers from a combination of dead load, one half live load, temperature and wind are as follows:

|                          | At Point of<br>Maximum Axial<br>Stress - 16 ft.<br>above base | At Point of<br>Maximum Bending<br>Stress - 163 ft.<br>above base | At Point of<br>Maximum Combined<br>Stress - 392 ft.<br>above base |
|--------------------------|---|--|---|
| Axial Compression Stress | 17,600  | 13,100   | 14,700  |
| Bending Stress           | <u>2,200</u>  | <u>9,500</u>   | <u>8,800</u>  |
| Combined Stress          | 19,800  | 22,600   | 23,500  |

With the towers rigidly fixed at the base the value of  $l/r$  can be taken at 60 and the permissible axial stress at 18,000 lb. p. sq. in. The permissible value for combined axial and bending can be assumed at 22,000 lb. p. sq. in. The maximum calculated stresses are therefore somewhat high, but in view of the excellent behavior of the towers under far greater stresses they cannot be considered unduly excessive.

The maximum longitudinal deflection of the towers at the top from live load (1500 lb. p. ft. of bridge) and temperature is 2.34 ft. riverward.

#### DAMPING INSTALLATIONS

The installations erected in the bridge prior to its opening to traffic, in order to dampen the oscillations from wind which had developed during erection of the deck structure, consisted of hydraulic jacks at each tower, acting longitudinally against the end of the floor structure of the center span, and diagonal stay ropes connecting the cables at the center of the span to the stiffening girders. These devices followed in general similar installations which had at that time been placed in the Bronx-Whitestone Bridge and, after various improvements, has proven effective. The amplitudes of the oscillations on the Bronx-Whitestone Bridge before damping devices were installed were, however, much smaller.

On the Tacoma Narrows Bridge these devices evidently reduced longitudinal motions of the floor, but had little effect in reducing the vertical oscillations, partly because the design of the devices was not entirely adequate and partly because the amplitudes were too great.

Hydraulic Jacks. The devices at the towers were designed primarily to steady the floor against dynamic longitudinal motions from wind action,

while permitting the slow but much larger motions from temperature changes. Hydraulic jacks acting between the ends of the floor and the towers (Drawing 3) were adopted for the Tacoma Narrows Bridge to act as bumpers or shock absorbers. Acting in this way they prevented longitudinal motion only partially because the structural details transmitting the force from the floor to the hydraulic jacks permitted considerable play with the result that the installation proved largely ineffective in checking vertical oscillations.

Experience at the Bronx-Whitestone Bridge proved that such damping devices at the towers were not fully effective until they had been made sufficiently strong and rigid to prevent play and elastic yield. This was possible only by fixing the ends of the floor to the towers by direct friction which permits motion only when, under the relatively much larger temperature forces, the frictional resistance is overcome. The effectiveness of this device in damping vertical oscillations was observed, not only on the bridge proper, but was subsequently demonstrated on the model of the Whitestone Bridge.

It is doubtful that even similarly effective friction devices in the Tacoma Narrows Bridge would have damped the structure sufficiently to prevent, or materially reduce, the much larger oscillations.

Stay Ropes at Center of Span. The stays installed at the center of the center span of the Tacoma Narrows Bridge consisted of a pair of 1-1/2" wire ropes at each cable, extending from the central cable band about 14 ft. on each side to points on the stiffening girder (Drawing 3). They were provided with adjustable turnbuckles and were erected with a nominal initial tension. Each rope had a specified minimum strength of 270,000 lb.

These stays were intended primarily to assist in checking longitudinal motions of the floor by tying the latter to the cables. Incidentally in combination with fixed ends of the floor they tend to dampen the vertical oscillations of the cables by restraining the longitudinal movement of the center of the cables. They could not, and were not intended to, prevent longitudinal motions of the cables entirely, because this would involve very large stresses in the stays and would materially alter the deformations and the stress condition in the cables. These center stays, because of their short length and therefore limited elastic stretch, can therefore be designed only for very moderate deformations of the cables. Within such deformations they exert a noticeable, but limited, damping effect. This was clearly observed on the Bronx-Whitestone Bridge. On the Tacoma Narrows Bridge it was apparently less marked. When the motions become as large as they did in the Tacoma Narrows Bridge, and the stays do not have a high initial tension, the cables impose alternately slackening of one and excessive strains with impact on the other of the two stays. The result is that under large distortions of the cables the stays, or their connections, may break or, as happened in the Tacoma Narrows Bridge, the cable band to which they were attached may slip when the stay rope stress becomes sufficient to overcome the frictional resistance of the cable band.

Similar experience on the Bronx-Whitestone Bridge in respect to alternate slackening of the central stay ropes, as at first installed, led to their replacement by heavier ropes, without turnbuckles, with more rigid connections, and with a large initial tension which prevented slackening.

From these experiences the conclusion may be drawn that central



stay ropes, when properly designed, exercise a limited damping effect on vertical oscillations, but they are largely ineffective when loose and may be a source of danger when the motions of the cables become large. Observations on the model of the Bronx-Whitestone Bridge showed that the damping effect of the central stays becomes overshadowed by the effect of stay ropes from the tops of the towers to points on the floor.

The Slipping of the Cable Band and its effect on the motions. It can be estimated that under certain possible force conditions, particularly torsional forces which tend to create out-of-step oscillations of the two cables, stresses approaching the strength of the center stay ropes or the frictional resistance of the cable band can be produced.

The stay ropes had a strength of at least 270,000 lb. The cable band was tightened by six bolts with a stress which may reasonably be estimated at 225,000 lb. The cable band friction was then probably at most 60% or 135,000 lb. It is possible that, under the rapidly alternating and eccentrically applied dynamic stresses from the stay ropes, the bolts, and therefore the cable band, became looser and the frictional resistance decreased.

After the north cable band slipped, the cable moved forward and backward through the band with a measured longitudinal amplitude of 40 in. The resistance of the band practically disappeared, thus bringing about, perhaps suddenly, a materially different dynamic condition of the suspended structure.

There can be little doubt that, as long as both cable bands were tight, and the stay ropes tied the cables to the floor, they were effective

in preventing out-of-step motions of the cables with a node at the center. When one cable band became ineffective it rendered the structure more sensitive to torsional motions. Its sudden slipping induced a large torsional moment in the horizontal plane which in turn caused the cables to oscillate out of phase. This action was possibly aggravated by dynamic effects created by the sudden slipping of the cable band. Observations indicated that the change from the moderate parallel motions of the cables to the more violent out-of-phase motions was sudden.

This explanation of what happened appears to be strengthened by the results of torsional tests made by Prof. F. B. Farquharson on the model of the Tacoma Narrows Bridge. These tests indicate that without center stays the bridge was considerably more sensitive to torsion than with stays attached to both cables. Therefore the stays must have exerted considerable restraint on the cables to get out of phase and this accounts for the fact that no torsional motions were observed as long as the stays held both cables. The model tests show also that with stays removed at one cable the bridge became more sensitive to torsion although not as much as without stays.

Stay Ropes at Towers. Mention of the studies made by the engineers of the Washington Toll Bridge Authority with respect to the installation on the Tacoma Narrows Bridge of stay ropes attached to the tops of the towers, as an additional damping device, and comments on the effect such stays might have had on the behavior of the bridge, are contained elsewhere in this report. The study of these stays, although undertaken promptly, together with their proper design and installation was bound to consume months. The failure occurred before such installation could be made.

Tie-Down Ropes in Side Spans. Some time prior to the failure wire ropes attached to the floor in the side spans and firmly anchored were installed with an initial stress of 100,000 lbs. in an effort to dampen the motions. These ropes evidently had an effect in damping the side spans, but, inasmuch as the center span motions were to a large extent independent of the side spans, the effect of the tie-down ropes on the amplitudes of the center span oscillations was not appreciable. Their presence could have had no influence upon the failure of the floor structure.

#### STRESS CONDITION OF SUPERSTRUCTURE AT TIME OF FAILURE

The calculations of stresses made on the basis of the observed dynamic deformations of the structure show that where failure or permanent local damage in various members occurred, the stresses had reached at least the yield strength of the composite member. This indicates that neither faulty design, nor defective material or workmanship, contributed to such failure or to permanent local damage.

Under the alternating  $45^{\circ}$  torsion in the final one-node motion the floor system, more particularly the concrete floor slab, received its most severe stresses near the center of the center span. The steel floor had great flexibility to adjust itself to the large distortions, but the torsional shearing stresses in the concrete slab exceeded the ultimate strength and this explains the breaking down of the slab in that vicinity as one of the first failures. Severe shearing stresses were also caused between the floor slab and the stiffening girders, accounting for the longitudinal motions observed between these parts. Examination of remaining parts of the floor slab confirms its excellent qualities.

During the vertical parallel motions of the cables, with double

amplitudes up to 5 ft., and the mode of oscillation corresponding to a frequency of 38 p. min., the stiffening girders were subjected to vertical bending which caused stresses approaching the buckling strength of the flanges. Under the more severe torsional motions, the girders were subjected to breaking stresses near the center of the center span, where the first buckling was observed. The buckling of the girders near the quarter points of the center span can be explained by a combination of stresses from bending in the plane of the web, alternate  $45^{\circ}$  torsion and bending stresses in the flanges from the lateral component of the suspender pull.

The side span girders were most severely stressed when a portion of the floor structure of the center span tore away, thus causing the side spans to deflect at their center as much as 60 ft., six times the maximum live load and temperature deflections. This vertical deflection alone produced stresses which caused the buckling of the girders in that vicinity. The failure occurred at web splices where the horizontal Z stiffeners were discontinuous (Fig. 7). The buckling stress of the compression flange was reached at a deflection of about 40 ft. With increasing deflection the buckling progressed rapidly through the depth of the girder.

The type of failure clearly indicates that the floorbeams and the stiffener angles which connect them to the girder were rigid enough to limit the buckling length of the top flange, and of the whole girder, to the panel length of 25 feet. The top flange apparently had ample lateral support.

Under the severe tilting of the floor, each of the two parts of a rope was subjected alternately to severe increases in the axial dead

load stress, on account of the frictional resistance of the rope over the cable. This is evidenced by observations to the effect that at some suspenders one part of a rope was slack. In addition the ropes were subjected to sharp alternate local bending of  $45^{\circ}$  at their connections to the girders. The dynamic effect of the falling mass of the structure hung from the suspenders greatly exceeded the corresponding static stresses. Under these dynamic stresses the ropes failed. After failure of one suspender, that of the others progressed rapidly, resulting in the successive dropping of large portions of the floor structure of the center span.

During the large deflections of the side span the bending stresses in the cables at the top of the towers, together with the axial stresses, were much higher than the safe design stresses. There is not enough information available that would permit an even approximate appraisal of the stresses actually produced. While surface examination does not reveal breaking of wires, except where caused by the cable band slipping, it is not certain that incipient failure was not produced at other points.

The stresses in the towers reached their highest values when, as a result of the dropping of part of the center span suspended structure, the side spans were subjected to a vertical deflection of about 60 ft. The corresponding shoreward deflection of the towers at their tops was about 27 ft. or 12 times the maximum deflection from full live load and temperature. Throughout the height of the tower shafts the bending stresses reached the critical buckling stress in the shoreward cover plates, accounting for the buckles that are plainly visible in these plates. The stresses on the tension side also reached the yield

strength of the material. The splices failed probably by shearing deformation of the rivet shank. We do not have sufficient information to permit a definite statement as to the damage to the main material on the tension side. This severe bending also overstressed the upper part of the tower anchorage frame, lifting the tower base on the riverward side by about 1/2 in. above the pier surface without, however, damaging the pier except locally. During the torsional motions of the center span and the sagging of the side span the towers were subjected to torsion about their vertical axes. This caused severe stresses and consequent permanent deformation in the top strut between the two tower shafts.

#### DESIGN OF MAIN PIERS AND ANCHORAGES

Main Piers. The two main piers which support the steel towers are of rectangular section with beveled corners. The pier shafts are of cellular concrete construction 64.5 ft. wide and 117.5 ft. long. They extend from an elevation of 23.09 ft. above to depths of 163 and 217 ft. below mean low water, where they are founded on a firm bed of sand and gravel. At the bottom the caissons are sealed with a 25 ft. layer of concrete and the top is formed by a solid concrete slab 20 ft. thick.

In the pier design forces and dynamic action from tidal currents, waves and ship impact were considered. The resulting stresses in the piers and the soil pressures are found to be conservative. Under combinations of weight, live load, temperature and one-half the assumed wind force, or weight and full wind force, the extreme edge pressures range between 9.2 and 9.7 tons per sq. ft., and the stresses in the concrete are within 400 lb. p. sq. in. A combination of tidal currents,



waves and ship impact is estimated to increase the extreme edge pressure between 3 and 5 tons. Of the above pressures only about 1.9 tons are due to the weight and forces acting on the superstructure, and about 5.5 tons to the great mass of the piers themselves. Even under the severe deflections of the towers at the time of the failure the additional stresses imposed upon these massive piers were comparatively small and the piers could therefore not have suffered in any way from the failure except locally near the tower base. The margin of carrying capacity of the piers is such that they may be utilized for a new structure, with certain alterations near the top, even if the weight and live load of the superstructure should be doubled.

Anchorage. The cable anchorages are concrete blocks which rest on a sand and gravel formation about 100 ft. above mean low water and about 500 ft. back of the river banks. The layer of sand and gravel is about 80 ft. deep and overlies a bed of clay approximately 100 ft. thick. The maximum cable pull combined with the weight of the anchorage blocks, neglecting passive earth resistance in front of the anchorages, causes maximum bearing pressures of 3.8 tons per sq. ft. and horizontal forces equal to 44% of the resultant vertical reaction. This provides sufficient resistance against sliding of the anchorages. As would be expected, the anchorage suffered no damage as a result of the failure and they may be safely utilized in a new structure. It would not be advisable, however, to increase the bearing pressure nor to permit an increase in the ratio of horizontal pull on the anchorage to the resultant vertical reaction.

CHAPTER III  
COMPARISON OF TACOMA NARROWS BRIDGE  
WITH OTHER SUSPENSION BRIDGES

The foregoing review of the design of the Tacoma Narrows Bridge with respect to its behavior and safety under the static forces usually assumed for design reveals no explanation of its behavior and final failure under wind action, except that the relatively large deformations under the design forces indicates the great flexibility of the bridge. The behavior of the bridge can only be explained by its response to aerodynamic forces. The present status of knowledge on aerodynamic forces acting on suspension bridges, and those produced indirectly by the oscillating motion of such a structure, gives a partial explanation of the excessive motions and ultimate failure of this bridge, but a complete quantitative analysis requires further experiments and theoretical studies.

A semi-empirical approach, based on the characteristics and behavior of comparably large modern suspension bridges may, therefore, partially explain the behavior of the Tacoma Narrows Bridge. Furthermore, only a comparative analysis of the elastic and dynamic characteristics will explain partially why similar aerodynamic forces do not cause motions of similar magnitude in these other bridges. There are in existence today a number of suspension bridges of a magnitude comparable to that of the Tacoma Narrows and among them two with relatively flexible stiffening girders, the Golden Gate Bridge and the Bronx-Whitestone Bridge, which have experienced to a mild degree aero-

dynamic effects of a nature similar to those of the Tacoma Narrows Bridge. The five longest suspension bridges have been selected for a comparison of their elastic characteristics and actual behavior. This comparison has involved extensive calculations and the study of results from model tests.

Developments in Suspension Bridge Design. Oscillating motions in insufficiently stiffened suspension bridges under dynamic forces from both wind and moving loads have long been known. Practically all early suspension bridges built before the middle of the 19th Century were comparatively flexible. Oscillations to a marked degree, without ill effects on traffic or structure, were not uncommon. The following accounts are given by the late William Hildenbrand, expert on suspension bridges (Trans. Am. Soc. C. E. 1902, p. 436):

"A highway bridge, of about 400 ft. span, over the Elk River, near Charleston, W. Va., is suspended from wire cables without any kind of stiffening construction. The bridge accommodates a heavy traffic from the adjoining lumber region, and the oscillations and undulations of the floor, under the moving load of a four-ox lumber wagon, are enough to make a person seasick, but without causing an apparent inconvenience to the travel of vehicles, and certainly without detriment to the strength and durability of the structure. The bridge, as far as the speaker could ascertain, was built long before the civil war; therefore, at the time it came under his observation, it had seen at least thirty years' service, and it was still in an excellent state of preservation."

Of the famous bridge over the Merrimac River near Newburyport, Mass., which was built in 1810 and lasted over a century, Mr. Hildenbrand says:

"When, some twelve to fifteen years ago, trolley cars were invented, the community owning this bridge did not hesitate to permit the crossing of electric cars, without making any change or addition to the structure, except to provide it with the necessary girder rails. The effect of a trolley car, weighing, perhaps, 12 to 14 tons, on the unstiffened floor was more appalling

to the speaker than anything he had ever experienced. Standing at one end of the bridge and seeing a car enter at the other end, it seemed suddenly to vanish from sight, giving the impression that the bridge was breaking down, but in a few moments the car emerged again, and after it had reached the opposite end of the bridge assumed its usual shape and appearance, as though it had never been disturbed. Actual measurement showed that the car caused a local deflection of about 20 ins. in one quarter of the span, and a corresponding rise in the opposite quarter. The combination of the depression and rise projected on the car produced the impression that the car was gradually vanishing from sight. The speaker made a careful examination of all parts of the bridge, and found them in perfect condition and of ample strength for sustaining the loads."

There are also on record a number of failures of early flexible suspension bridges. Some failed under the action of wind, others under moving loads. Troops marching in step and thus producing synchronized oscillations of the structure constituted a source of danger for such bridges, and in at least one case led to serious disaster.

When these early suspension bridges were built very little was known about the intensity of wind forces acting on structures. In many cases failure was attributable to faulty structural design, material defects or poor maintenance. The early suspension bridges were of comparatively short span and of very light floor construction, usually of timber. The cables were not stiffened other than by the floor itself and the hand railings. In some bridges a certain degree of rigidity was secured by various kinds of stayropes.

The Brighton Bridge in England, built in 1823 was twice destroyed by windstorms, in 1833 and in 1836. (17) It had spans of only 255 ft. length. Repeated serious damage by wind gales was done to the famous Menai Bridge in Wales, built in 1826 with the then unprecedented span of 570 ft., until by 1840 the floor structure had by successive improvements been sufficiently strengthened and stiffened. The suspended

structure was again damaged by a severe gale in 1937, and was modernized in 1939. The most notable suspension bridge failure by hurricane in the U. S. A. was that of the Ohio River Bridge at Wheeling, W. Va., in 1854. This bridge, built in 1847, had a span of 1010 ft. which held the record in length for 20 years. More complete historical data are contained in a recent article, "Wind Failures of Suspension Bridges", by J. K. Finch in Engineering News-Record, March 13, 1941.

Although these early very flexible suspension bridges showed the characteristic wave motions under wind action, and pointed to the danger of extreme flexibility combined with insufficient weight, they furnish no explanation of the behavior of the Tacoma Narrows Bridge because of their much shorter spans and exceedingly small weights. The floor of the Wheeling Bridge, 24 ft. wide, consisted of timber weighing only 23 lb. per sq. ft., or barely enough to keep it from being lifted up by static wind pressure. It was carried by 12 light separate iron cables. The total weight of the center span was only 460 tons compared to the 8,000 tons of the Tacoma Narrows Bridge. It was practically without stiffening system.

As the weight and speed of moving loads increased, and in particular with the advent of trolley cars and trains with their heavy load concentrations, in the second half of the 19th Century, the extreme flexibility of the practically unstiffened type of suspension bridge became objectionable from the traffic point of view, and led to more effective means of stiffening such bridges. The effectiveness of stiffening girders, first in the form of rigid longitudinal floor timbers, or as braced hand railings, had long been recognized and towards the latter part of the 19th Century became the most common

method of stiffening suspension bridges. One of the outstanding early examples of the application of deep rigid stiffening girders or trusses was the Niagara suspension bridge built by John A. Roebling in 1855 which carried, for those days, heavy railroad trains in addition to highway traffic.

In the latter part of the 19th and early 20th Century the aim in suspension bridge design was solely to provide ample rigidity against excessive deformations and oscillations under traffic. The suspension bridges then built, according to such design, with moderately long spans of up to 1600 ft., proved to have ample rigidity against dynamic wind action. Consequently no attention was paid or required to such action, other than proportioning the structures safely under accepted requirements for equivalent static wind pressure.

With the development of the theories of elastic structures, the function of the stiffening girders became more clearly understood and their proportioning on a more scientific and rational basis became possible. The early, so-called elastic (Rankine) theory, was, however, largely responsible for the tendency to provide excessively deep and rigid stiffening trusses, as exemplified notably by the Williamsburg Bridge across the East River in New York, completed in 1904. Its trusses have a depth of 40 ft. or  $1/40$  of the span length. This theory ignored the fact that weight has an important influence on rigidity and that, with increasing span length and suspended mass of the structure, it becomes an increasingly potent factor in stabilizing a bridge against dynamic motions. The trend in recent years towards greater weight has resulted largely from the development of heavy concrete floor slabs in place of the older wooden flooring.

With the construction of the Manhattan Bridge in 1910 and as a result of the increased application of the so-called "deflection theory" the trend has been towards more flexible stiffening girders. This is indicated in Table 3 which contains the principal dimensions of the major suspension bridges built since 1900. It illustrates the marked increase in span lengths accompanied by a general trend towards smaller depth ratio of stiffening truss. In some bridges new low ratios of width to span length were established.

The importance of weight as a stiffening factor in the design of the modern suspension bridge was first fully realized in the George Washington Bridge. Its great span (3500 ft.) and its large traffic capacity resulted in a suspended weight of center span of 68,000 tons (56,000 tons in the initial single deck condition). Other stabilizing factors are the relatively flat catenary, short side spans, and rigid towers. An analysis of these influences induced the designers to adopt, for the ultimate double deck condition, the then unusually shallow stiffening trusses of 29 ft. depth or  $1/120$  of the span length, and to omit all vertical stiffening in the first stage of a single deck designed to carry highway traffic only. As had been expected the rigidity of this bridge proved to be sufficient under all dynamic influences, traffic as well as wind.

Encouraged by this example engineers were led to the adoption of a progressively greater degree of flexibility of stiffening girders, accompanied by progressively decreasing suspended weight in other long span highway suspension bridges, the Golden Gate Bridge 4200 ft. span (45,000 tons weight of center span) the Bronx-Whitestone Bridge (2300 ft. span, weight 13,000 tons) and finally the Tacoma Narrows Bridge



**TABLE 3 — DIMENSIONS OF SUSPENSION BRIDGES BUILT SINCE THE YEAR 1900  
WITH SPANS OF 1200 FEET OR OVER.**

| Name and Location                    | Year Completed | Center Span ft. | Side Spans ft.                   | Cable Sag ft. | Width c.c.Cables ft. | Depth of Stiff'g Girder ft. | Dead Load Center-Span lbs. per ft. of Bridge | Design Live Load lbs. per ft. of Bridge | Width of Roadway & Sidewalks                 | Nº of Elec. R.R. Tracks | Ratio Cable Sag to c.c.Cables Span | Ratio Width to Span | Ratio Depth of Stiff'g Girder to Span | Nº & Dia. of Cables  |
|--------------------------------------|----------------|-----------------|----------------------------------|---------------|----------------------|-----------------------------|--|---|--|-------------------------|------------------------------------|---------------------|---------------------------------------|----------------------|
| Golden Gate.<br>San Francisco. Cal.  | 1937           | 4200            | 1125                             | 475           | 90                   | 25                          | 21,600                                       | 4,000                                   | 1-60' Rdy.<br>2-10' S.W.                     | None                    | $\frac{1}{9}$                      | $\frac{1}{47}$      | $\frac{1}{168}$                       | 2-36 $\frac{3}{8}$ " |
| George Washington.<br>New York. N.Y. | 1931           | 3500            | 610-650                          | 325           | 106                  | Ultimate<br>29              | Double Deck<br>39,000                        | Condition<br>8,000                      | 1-30-6' Rdy.<br>2-28-9' Rdy.<br>2-10-0' S.W. | 4                       | $\frac{1}{11}$                     | $\frac{1}{33}$      | Double Deck<br>$\frac{1}{120}$        | 4-36"                |
|                                      |                |                 |                                  |               |                      | Present<br>—                | Single Deck<br>31,500                        | Condition                               | do.  | None                    |                                    |                     | Single Deck                           |                      |
| Tacoma Narrows.<br>Tacoma. Wash.     | 1940           | 2800            | 1100                             | 232           | 39                   | 8                           | 5,700  | 1,000                                   | 1-26' Rdy.<br>2-5' S.W.                      | None                    | $\frac{1}{12}$                     | $\frac{1}{42}$      | $\frac{1}{350}$                       | 2-17 $\frac{1}{4}$ " |
| Transbay.<br>San Francisco. Cal.     | 1936           | 2310            | 1160                             | 231           | 66                   | 30                          | 18,800                                       | 8,000                                   | 1-58' Rdy.<br>1-31' Rdy.                     | 2                       | $\frac{1}{10}$                     | $\frac{1}{35}$      | $\frac{1}{47}$                        | 2-28 $\frac{3}{4}$ " |
| Bronx-Whitestone.<br>New York. N.Y.  | 1939           | 2300            | 735                              | 200           | 74                   | 11                          | 11,000                                       | 3,000                                   | 2-27-6' Rdy.<br>2-5' S.W.                    | None                    | $\frac{1}{1.5}$                    | $\frac{1}{31}$      | $\frac{1}{209}$                       | 2-22"                |
| Ambassador.<br>Detroit. Mich.        | 1929           | 1850            | 817-973<br>Unloaded<br>Backstays | 209           | 67                   | 22                          | 12,400                                       | 3,300                                   | 1-47' Rdy.<br>1-8' S.W.                      | None                    | $\frac{1}{8.9}$                    | $\frac{1}{27.6}$    | $\frac{1}{84}$                        | 2-19 $\frac{1}{4}$ " |
| Delaware River.<br>Philadelphia. Pa. | 1926           | 1750            | 716.7                            | 197           | 89                   | 28                          | 26,000                                       | 6,000                                   | 1-57' Rdy.<br>2-10' S.W.                     | 4                       | $\frac{1}{8.9}$                    | $\frac{1}{9.7}$     | $\frac{1}{62.5}$                      | 2-30"                |
| Bear Mountain.<br>West Point. N.Y.   | 1924           | 1632            | Unloaded<br>Backstays            | 200           | 61.3                 | 30                          | 11,540                                       | 3,160                                   | 1-38' Rdy.<br>2-5' S.W.                      | None                    | $\frac{1}{8.2}$                    | $\frac{1}{26.6}$    | $\frac{1}{54}$                        | 2-18 $\frac{1}{4}$ " |
| Williamsburg.<br>New York. N.Y.      | 1903           | 1600            | 596<br>Unloaded<br>Backstays     | 178           | 67                   | 40                          | 17,200                                       | 5,700                                   | 2-20' Rdy.<br>2-17' S.W.                     | 6                       | $\frac{1}{9}$                      | $\frac{1}{23.7}$    | $\frac{1}{40}$                        | 4-18 $\frac{3}{4}$ " |
| Lions Gate.<br>Vancouver. B.C.       | 1938           | 1550            | 615                              | 150           | 40                   | 15                          | 4,600  | 1,230                                   | 1-29' Rdy.<br>2-5' S.W.                      | None                    | $\frac{1}{10.3}$                   | $\frac{1}{38.8}$    | $\frac{1}{104}$                       | 2-13 $\frac{1}{4}$ " |
| Mid Hudson.<br>Poughkeepsie. N.Y.    | 1930           | 1500            | 750                              | 150           | 42                   | 20                          | 8,800  | 3,000                                   | 1-30' Rdy.<br>2-4' S.W.                      | None                    | $\frac{1}{10}$                     | $\frac{1}{35}$      | $\frac{1}{75}$                        | 2-16 $\frac{3}{4}$ " |
| Manhattan.<br>New York. N.Y.         | 1909           | 1470            | 725                              | 148.5         | 96                   | 24                          | 23,280                                       | 8,000                                   | 1-35' Rdy.<br>2-13' S.W.                     | 8                       | $\frac{1}{9.9}$                    | $\frac{1}{15.3}$    | $\frac{1}{61}$                        | 4-21 $\frac{1}{4}$ " |
| Triborough.<br>New York. N.Y.        | 1936           | 1380            | 705                              | 138           | 98                   | 20                          | 20,000                                       | 4,000                                   | 2-43-6' Rdy.<br>2-5-6' S.W.                  | None                    | $\frac{1}{10}$                     | $\frac{1}{14.1}$    | $\frac{1}{68}$                        | 2-20 $\frac{3}{4}$ " |
| St. Johns.<br>Portland. Ore.         | 1930           | 1207            | 430                              | 121           | 52                   | 18                          |  |   | 1-40' Rdy.<br>2-5' S.W.                      | None                    | $\frac{1}{10}$                     | $\frac{1}{23.2}$    | $\frac{1}{67}$                        | 2-16 $\frac{3}{4}$ " |
| Mount Hope.<br>Providence. R.I.      | 1928           | 1200            | 504                              | 120           | 34                   | 18                          | 5,300  | 1,500                                   | 1-27' Rdy.<br>1-2-6' S.W.                    | None                    | $\frac{1}{10}$                     | $\frac{1}{35}$      | $\frac{1}{67}$                        | 2-11"                |

(2800 ft., weight 8,000 tons). Even in three recent bridges of much shorter span and smaller weight, the Deer Isle Bridge in Maine (1080 ft. span, weight 1,300 tons) and the Thousand Island Bridges (spans 800 and 750 ft., weight 1,280 and 1,200 tons) stiffening girders only 6-1/2 and 6 feet deep were adopted.

All of these lighter and relatively flexible bridges have experienced oscillations under wind of various degrees. In the Bronx-Whitestone Bridge and in the three small bridges the oscillations have been reduced by stay ropes and other damping installations.

Observed Motions and Damping Appliances on other Modern Suspension Bridges. Small motions under dynamic wind action are caused in all suspension bridges. As far as is known no unusual motions, vertically or laterally, have been observed in any of the suspension bridges of the span range of 1200 to 1850 ft. listed in Table 3, nor in the San Francisco-Transbay Bridge of 2310 ft. span, all of which have relatively rigid stiffening trusses, although most of them probably have experienced windstorms more severe than that which wrecked the Tacoma Narrows Bridge. Their freedom from objectionable motions under wind action can be ascribed largely to the rigidity of their stiffening trusses and to their ample width.

The George Washington Bridge, likewise, has not experienced in its 10 years of existence any objectionable motions. A few times, and for very short duration, mild vertical and very slight torsional oscillations have been observed. It is not certain whether these resulted from wind or from frictional resistances set up, especially under freezing conditions, at the expansion joints of the floor. No

motions, vertically or laterally, have been observed under heavy winds. Since in its present stage of single deck serving highway traffic only, it has no stiffening girders whatsoever, its ample rigidity must be ascribed mainly to its great weight.

Since its completion in 1937, the Golden Gate Bridge has shown sufficient rigidity under all usual wind conditions. Frequently, under moderate winds, it has a slight torsional motion which is imperceptible except when special efforts are made to observe it. On two occasions, February 9, 1938, and February 11, 1941, with wind velocities respectively 75 and 62 m.p.h., vertical oscillations with amplitudes of approximately 2 ft. were observed. On the same occasions, lateral deflections of about 8 and 5 ft. were observed. A more complete record of these observations by Mr. Russell G. Cone, Chief Engineer, Golden Gate Bridge and Highway District, are given in Appendix IX.

The oscillations observed in the Bronx-Whitestone Bridge shortly before and after its completion in April, 1939, and the corrective measures taken to check them are described in Engineering News-Record of December 5, 1940. The motions of this bridge may be summarized as follows: Before any damping installations were erected, the bridge exhibited occasional oscillations, generally under moderate wind velocities and wind directions of less than  $45^{\circ}$  to the axis of the bridge, of maximum double amplitudes of approximately 24 ins. The prevailing and most severe motion was one with a single node near the center, although change into no-node and two-node conditions were observed. Under higher wind velocities the bridge exhibited invariably greater steadiness.

After installation of efficient stay ropes at the center and friction devices at the ends of the floor of the center span in the summer of 1940, the observed oscillations became much less frequent and remained within considerably smaller amplitudes. Studies made on a scale model of the bridge led to the further installation of stay ropes from the tops of the towers to points on the stiffening girders in the center and side spans. This installation was completed very recently.

The oscillations of the Thousand Island and Deer Isle bridges and the corrective installations are described in an article in Engineering News-Record of December 5, 1940, entitled "Two Recent Bridges Stabilized by Cable Stays." According to this article oscillations were first noticed in June, 1938, on the 800 ft. span of the Thousand Island Bridge, and attained double amplitudes, estimated at more than 24 ins. After various trials the installation of diagonal stayropes attached to the cable bands was finally completed in January, 1939. These bridges are more nearly comparable in size to some of the early flexible suspension bridges. They give no clue to the possible behavior of a suspension bridge 3-1/2 times longer and 6 times heavier.

These experiences with modern relatively flexible suspension bridges point clearly to a limit of flexibility, even in very long spans, beyond which oscillations become objectionable not only from the traffic point of view, but in their effect upon the stress condition and safety of the structure.

Comparison of the Dimensions of Tacoma Narrows Bridge with Those of Other Long Span Suspension Bridges. A comparison of the modern suspension bridges with the moderately long span range from 1200 ft. to 1850 ft., Table 3, built within the last 30 years, reveals a trend towards greater flexibility of stiffening trusses, as expressed by their depth to span ratio, especially those carrying highway traffic only, and of greater flexibility laterally, as expressed by the width to span ratio. In all these bridges the depth ratio remains above  $1/84$  and the width ratio above  $1/36$ . There is no record that any of them have revealed objectionable motions under wind pressure. All were designed to secure ample vertical rigidity under traffic and ample strength under lateral wind pressure. Being of a span range far below that of the Tacoma Narrows Bridge, they furnish no evidence for the behavior of the latter. Even within this group of bridges, comparison of rigidities on the basis of depth and width ratios is crude and misleading. It disregards such influences as weight, cable sag, rigidity of towers and side spans, damping effects, and the important effect of vertical rigidity and width of floor on torsional deformations.

With a depth ratio of stiffening girders of  $1/350$ , the Tacoma Narrows Bridge with a much smaller weight went far beyond precedents, notably the Golden Gate Bridge with a depth ratio of  $1/168$ , and the Bronx-Whitestone Bridge with one of  $1/209$ . This extraordinary small depth explains to some extent why the amplitudes of oscillations in the Tacoma Bridge were so much larger than in other bridges and why it was more susceptible to motions from small dynamic forces.

In respect to width also, the Tacoma Narrows Bridge, with a ratio of  $1/72$  surpassed in slenderness all others, which show a range of ratios between  $1/14$  (Triborough) and  $1/47$  (Golden Gate).

Since neither the Tacoma Narrows Bridge, nor any of the other bridges have shown lateral deformations or oscillations that have proven at all objectionable in their effect, either on the structure or on traffic, it becomes evident that limitation of width is primarily a question of sufficient torsional resistance. It was the combination of an exceedingly small vertical rigidity and its narrow width that made the Tacoma Narrows Bridge susceptible to excessive torsional flexibility.

Comparison of Deflections of the Five Longest Suspension Bridges. The elastic deformations of suspension bridges under static forces form a better approach to a comparison of susceptibility to oscillations than their dimensional proportions. Static deformations reflect the important effect of weight, as well as the stiffness of the suspended structure, the towers and side spans. They do not, however, furnish a direct measure for the dynamic rigidity since they do not reflect the damping effects produced by elastic deformation nor other dynamic characteristics of the compound structure.

It has been a general practice to use the ratio of live load deflections to span length as a criterion and limitation for the rigidity of bridges of ordinary types. Table 4 lists for the five longest suspension bridges the principal properties which affect the static deformations, and also the maximum deflections, vertically, laterally and in torsion, under the design loads. This criterion evidently does not apply to the rigidity of long span suspension bridges under wind action.

It is now evident that limitation of flexibility in long span

highway suspension bridges is not primarily a question of safety of the structure under maximum loads, nor of sufficient rigidity under impact effects of moving loads, but one of keeping oscillations produced by wind within such small limits so as not to produce the feeling of danger or discomfort on the part of patrons of the bridge. Such limiting oscillations are far below those which endanger the safety of the structure.

It is now well established by observations on suspension bridges and on models that such oscillations may be produced by comparatively small wind forces acting longitudinally on towers, suspenders, cables, and floor structure, as well as by wind acting at a right angle to the structure.

Characteristics of the Suspension System. The regularity, frequency and other characteristics of the oscillations are evidently not so much the result of intensity, frequency, direction and distribution of the wind impulses, but lies largely in the behavior of the suspension system when acted upon by dynamic forces. Freely suspended cables are very sensitive to displacements under forces acting in their vertical planes. Very small forces may create comparatively large displacements of the cable catenary, and since little of the energy input by dynamic forces is absorbed by the elastic deformations, successive energy impulses may easily create large oscillations. Model tests show that, after cessation of dynamic impulses, the relatively flexible suspension systems continue to oscillate for a long time before they come to rest. Comparatively small energy impulses applied at very irregular and infrequent intervals are sufficient to keep unstiffened cables oscillating



**TABLE 4 — ELASTIC PROPERTIES AND PRINCIPAL DEFLECTIONS FROM DESIGN LOADS OF FIVE LONGEST SUSPENSION BRIDGES.**

| Name of Bridge   | Golden Gate      | George Washington | Tacoma Narrows   | Transbay         | Bronx-Whitestone |
|--|------------------|-------------------|------------------|------------------|------------------|
| <b>GENERAL DIMENSIONS AND WEIGHT.</b>                                  |                  |                   |                  |                  |                  |
| Length of center span, feet.   | 4,200            | 3,500             | 2,800            | 2,310            | 2,300            |
| Length of side spans, feet.  | 1,125            | 650               | 1,100            | 1,160            | 735              |
| Width between cables, feet.  | 90               | 106               | 39               | 66               | 74               |
| Average weight of center span, lb. per. lineal foot.                   | 21,035           | 31,590*           | 5,700            | 18,740           | 11,000           |
| <b>PROPERTIES OF CABLES.</b>   |                  |                   |                  |                  |                  |
| Number of cables.  | 2                | 4                 | 2                | 2                | 2                |
| Cable Sag, feet.   | 475              | 319.2*            | 232              | 231              | 200              |
| Diameter, inches.  | 36 $\frac{3}{8}$ | 36                | 17 $\frac{1}{4}$ | 20 $\frac{3}{8}$ | 22               |
| Net section, all cables, sq. in.                                       | 1664             | 3195              | 383              | 1050             | 594              |
| <b>PROPERTIES OF STIFFENING GIRDERS.</b>                               |                  |                   |                  |                  |                  |
| Number of girders and type.  | 2 Trusses        | 2 Chords          | 2 Pl. Girders    | 2 Trusses        | 2 Pl. Girders    |
| Depth (c.c. chords for trusses), feet.                                 | 25               | —                 | 8                |                  | 11               |
| Mom. of inertia, all girders, in. <sup>2</sup> ft. <sup>2</sup>        | 88,000           | 168               | 2,567            | 156,000          | 5,860            |
| <b>PROPERTIES OF LATERAL SYSTEM.</b>                                   |                  |                   |                  |                  |                  |
| Horiz. wind force acting on floor and vehicles, lb. per. lin. foot.    | 1130             | 1200              | 560              | 1400             | 800              |
| Horiz. wind force acting on cables, lb. per. lineal foot.              | 200              | 300               | 60               | 145              | 120              |
| Total wind force on floor and cables, lb. per. lineal foot.            | 1330             | 1500              | 620              | 1545             | 920              |
| Width of wind truss, feet.   | 90               | 106               | 39               | 66               | 74               |
| Mom. of inertia, wind truss, in. <sup>2</sup> ft. <sup>2</sup>         | 1,236,000        | 481,000*          | 95,000           | 743,000          | 410,000          |
| <b>PROPERTIES OF TOWERS.</b>   |                  |                   |                  |                  |                  |
| Height of towers, (top of pier to center of cable).                    | 702              | 582               | 425              | 462.41           | 353              |
| Width at top (parallel to bridge axis), feet.                          | 24.6             | 37.5              | 13               | 15               | 12               |
| Width at bottom (parallel to bridge axis), feet.                       | 52.6             | 56                | 19               | 32               | 18               |
| Area at top—both tower legs.   | 7340             | 5978              | 1524             | 3712             | 3300             |
| Area at bottom—both tower legs.  | 14,384           | 14,624            | 2287             | 8,000            | 4,200            |
| Aver. M. of I, both tower legs, in. <sup>2</sup> ft. <sup>2</sup>      | 1,500,000        | 3,130,000         | 40,500           | 290,000          | 60,000           |
| <b>VERT. DEFLECTIONS FROM LIVE LOAD AT NORMAL TEMPERATURE.</b>         |                  |                   |                  |                  |                  |
| Live Load, lb. per. lin. ft. bridge.                                   | 4,000            | 4,000             | 1,500            | 6,000            | 3,000            |
| Down at $\frac{1}{4}$ pt. of center span.                              | 12.41            | 5.88              | 10.95            | 10.70            | 8.41             |
| Load $\frac{1}{2}$ center span, tower to center.                       |                  |                   |                  |                  |                  |
| Up at $\frac{3}{4}$ pt. of center span, same loading.                  | 6.72             | 3.54              | 2.74             | 7.50             | 3.30             |
| Down at center—load center span.                                       | 7.34             | 2.92              | 10.50            | 11.85            | 6.50             |
| Up at center—load side spans.  | 2.34             | 0.48              | 5.93             | 8.18             | 2.35             |
| Maximum change in grade.   | 2.6              | 1.2               | 3.6              | 3.3              | 2.8              |
| Maximum grade.   | 5.7              | 3.4               | 6.2              | 6.3              | 6.8              |
| Tilting from live load at $\frac{1}{4}$ pt. of center span.            | 10.88            | 4.44              | 17.54            | 14.27            | 7.88             |
| Lateral deflection of truss from design wind at center of center span. | 21.5             | 10.7              | 20.03            | 11.42            | 8.93             |
| Maximum longitudinal deflection of tower top from live load.           | 1.36             | 0.34              | 1.98             | 2.53             | 0.83             |

\* Single Deck Condition.

Final Condition :-39,500 lb. per lin. ft. of bridge. Sag 325 ft.

indefinitely or to cause progressively increasing amplitudes.

The more flexible the stiffening system, that is, the nearer the compound system approaches the unstiffened cables, the more sensitive it is and the longer it persists in the state of oscillation after energy impulses are removed. Moreover, the more flexible the system the more easily it oscillates in increasing numbers of waves or nodes at higher frequencies. This is clearly illustrated by the relative behavior of those bridges which have experienced oscillations and also by the models.

When the stiffening girders are very flexible the motions of the compound suspension system are almost completely controlled by the motions of the cables. In the case of the Tacoma Narrows Bridge the stiffening girders restrain the static cable deformations by only 1.3% (Table 5 and Fig. 20). In the Bronx-Whitestone Bridge their effect is about 2.5% and in the Golden Gate Bridge 7.5%. In the George Washington Bridge the articulated floor, neglecting the elastic resistance of the large cables against bending, exerts practically no restraining influence on the deformations of the cables. In the San Francisco-Transbay Bridge the more rigid stiffening trusses influence the deflections to the extent of 29%. The effect of damping by the suspended structure tends to further restrain the motions of the cables.

The comparative deflections as an indication of the susceptibility of different suspension bridges to dynamic motions have been calculated for the Tacoma Narrows Bridge and the four other suspension bridges of comparable magnitude. These bridges show a wide variation in the elements which influence the deformations of the compound structure, vertically, laterally and in torsion.

Comparative Vertical Deflections. For a comparison of the deformations in a vertical plane a uniform load of 200 lb. p. ft. of bridge has been assumed and placed in a position to produce approximately maximum downward deflections at the quarter points and at the center of the center span at which points the greatest deflections occur. In the Tacoma Narrows Bridge the greatest amplitudes occurred at the quarter points when the cables oscillated out of phase.

Table 5 shows the comparative vertical deflections at the center and the quarter points for the five bridges. As expressed by the following ratios, the vertical flexibility of the Tacoma Narrows Bridge is far in excess of that of any of the other bridges:

|  | For 1/4 Point | For Center |
|--|---------------|------------|
| Golden Gate Bridge                     | 2.3 to 1      | 3.9 to 1   |
| Bronx-Whitestone Bridge, without stays | 2.6 to 1      | 3.3 to 1   |
| Bronx-Whitestone Bridge, with stays    | 3.0 to 1      | 3.7 to 1   |
| San Francisco-Transbay Bridge          | 4.0 to 1      | 3.5 to 1   |
| George Washington Bridge               | 5.1 to 1      | 10.4 to 1  |

Fig. 17 illustrates graphically the comparative vertical deflections of these five bridges.

Lateral Deflections and Their Relationship to Width. The comparison of lateral rigidities of long span suspension bridges and their bearing on the behavior of the Tacoma Narrows Bridge presents a different and much simpler problem than the vertical rigidity. None of the modern suspension bridges listed in Table 3, inclusive of the Tacoma Narrows Bridge, have shown objectionable lateral movements under wind action comparable to the vertical oscillations. Such lateral deflections as have been ob-

served under high wind velocity, notably a deflection of 8 feet on the Golden Gate Bridge during a wind storm of 75 miles per hour, have been slow and sluggish without any objectionable effect on people on the bridge, and they were well within the safe deflections for which the bridges are designed. The Tacoma Narrows Bridge during its most violent vertical torsional motions under a wind velocity of 42 m.p.h. exhibited lateral movements to the extent of only a few feet. No lateral motions of more than a few inches have ever been observed on the George Washington Bridge and on the Bronx-Whitestone Bridge.

It is easy to explain the difference between vertical and lateral behavior. Lateral deflections result from the average pressure of the wind. While small oscillations may be superimposed on this deflection, even a comparatively flexible wind truss, as the one of the Tacoma Narrows Bridge, because of its much greater rigidity and damping effect, offers far greater resistance to dynamic oscillations laterally than the very flexible stiffening girders do vertically. In the Tacoma Narrows Bridge the lateral moment of inertia of the wind truss is over 35 times the vertical moment of inertia of the two stiffening girders and the dynamic resistance is in even greater ratio.

Table 4 gives the maximum lateral deflections of the five longest suspension bridges based upon the design wind pressures. Based upon the ratio of deflection to span, the Tacoma Narrows Bridge is about 40% more flexible than the Golden Gate and Transbay Bridges, 80% more than the Whitestone Bridge and 125% more than the George Washington Bridge.

In long span suspension bridges with a relatively narrow flexible wind truss, a large part of the wind force acting on the suspended

structure is transmitted to the cables and through them to the towers. It follows that the lateral stability and deflection of the suspended structure is largely independent of the rigidity and width of the wind truss.

If the wind truss is very flexible, the center of the center span acts as an elastic central support for the wind truss. For the Tacoma Narrows Bridge the distribution of wind forces to floor and cables is shown on Fig. 16, which illustrates the supporting effect of the cables on the wind truss.

The above considerations lead to the conclusion that the relatively narrow and flexible wind truss of the Tacoma Narrows Bridge supplied sufficient lateral rigidity. It is evident from observations of the behavior of the bridge that this rigidity was sufficient to prevent dangerous or even objectionable lateral oscillations.

The suspended structure of a suspension bridge is always stable against lateral wind pressure, provided it has sufficient weight and vertical rigidity. The width of bridge between cables, or depth of wind truss, has a far more important effect on the torsional deformations resulting from insufficient vertical rigidity combined with narrow width.

Comparative Torsional Deformations. All bridges are subject to forces which cause torsion or lateral tilting of the floor. In most ordinary bridges and in suspension bridges of ample width and vertical rigidity such torsional forces are of small consequence, but in a narrow and vertically very flexible suspension bridge such as the Tacoma Narrows Bridge, they may cause excessive distortions of the floor structure.

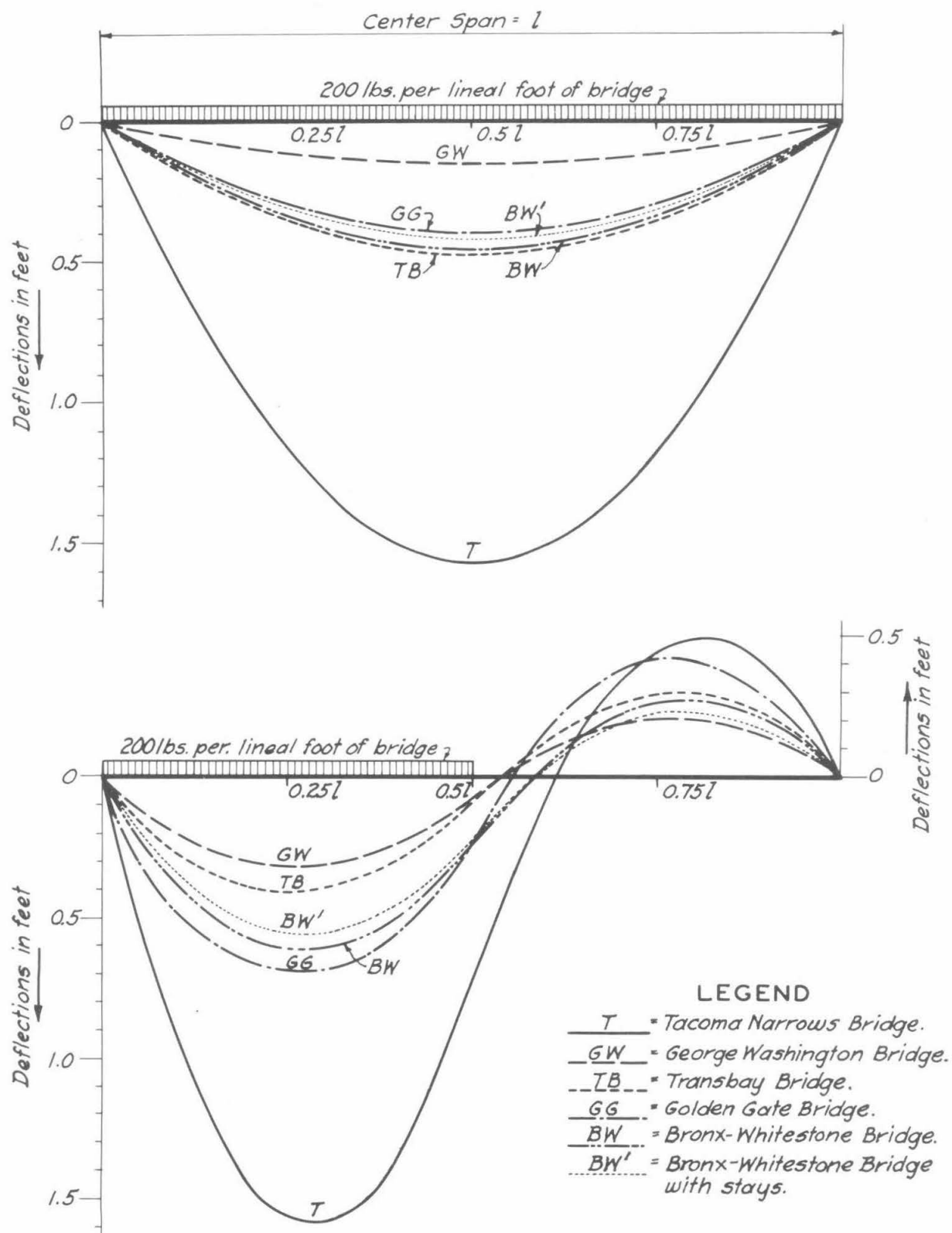


FIG.17- COMPARATIVE VERTICAL DEFLECTIONS OF THE FIVE LONGEST SUSPENSION BRIDGES DUE TO A LOAD OF 200 LBS. PER LIN. FOOT OF BRIDGE.

Torsion may be caused by moving loads or vertical wind forces distributed unequally across the floor of the bridge, or it may result from horizontal wind forces which act eccentrically above or below the points of suspension of the rigid floor structure, thus creating a torsional moment resulting in vertical reactions of opposite direction on two cables. Such moments, as well as vertical forces, are also caused by turbulence of the wind as it passes the floor structure.

A comparison of torsional deflections of different suspension bridges as an indication of their susceptibility to torsion by wind action may therefore be based either on an assumed torsional moment or on assumed small vertical forces which create torsion.

In the suspension bridges selected for comparison the torsional resistance of the floor structure is small compared to the resistance of the cables and stiffening system when forced to deflect vertically in opposite directions. The torsion of the floor under given forces is therefore, almost entirely a function of the rigidity of the vertical suspension system and only to a negligible extent of the torsional resistance of the floor structure. The greater the differential vertical deflections or oscillations of the two cables the greater the angular distortion of the floor and the torsional stresses caused in the floor.

The angle of distortion is also a function of the width of the floor between cables. For any given torsional moment the floor reactions on the cables and therefore, the deflections are approximately inversely proportional to the width, and the angle of torsion, or the lateral inclination of the floor, is, for small motions, inversely proportional to the square of the width. For given small forces acting



vertically in the planes of the two cables the deflections are approximately proportional to the forces and the resulting torsion is inversely proportional to the width.

Either of these two force conditions furnish a qualitative comparison of torsional rigidities. The second conditions, assuming the vertical forces acting in the planes of the cables, would appear to furnish a more rational quantitative comparison.

In Table 6 the comparative torsional rigidities have been calculated on the basis of a torsional moment of 10,000 ft. lb. p. ft. of bridge acting in such position as to cause maximum differential vertical deflections at the opposite quarter points of the two cables, that is, if "d" represents the vertical deflections at the quarter point from a load of 100 lb. p. ft. per cable, as given in Table 5, and "b" the width between cables, the relative angles or torsion, expressed in percent, are  $t = \frac{200 d}{b^2}$ .

TABLE 6 - COMPARATIVE TORSIONAL DEFORMATIONS  
FOR THE FIVE LONGEST SUSPENSION BRIDGES  
Based on a Torsional Moment of 10,000 lb. per lin. ft. of Bridge.

| Name of Bridge                    | d        | b       | l        | b/l   | t      | r      |
|-----------------------------------|----------|---------|----------|-------|--------|--------|
| George Washington                 | 0.31 ft. | 106 ft. | 3500 ft. | 1/33% | 0.0055 | 38.0:1 |
| Golden Gate                       | 0.68 "   | 90 "    | 4200 "   | 1/47% | 0.0168 | 12.4:1 |
| Transbay                          | 0.40 "   | 66 "    | 2310 "   | 1/35% | 0.0184 | 11.4:1 |
| Bronx-Whitestone<br>with stays    | 0.54 "   | 74 "    | 2300 "   | 1/31% | 0.0196 | 10.7:1 |
| Bronx-Whitestone<br>without stays | 0.61 "   | 74 "    | 2300 "   | 1/31% | 0.0223 | 9.4:1  |
| Tacoma Narrows                    | 1.59 "   | 39 "    | 2800 "   | 1/72% | 0.2091 | 1.0:1  |

d = Comparative Vertical Deflection near 1/4 pt. from 200 lb. per lin. ft. of bridge.

b = Width c.c. Cables.

l = Span.

b/l = Width to Span Ratio.

t = Comparative Torsion in %.  $200 d/b^2$ .

r = Ratio of Torsion Tacoma Narrows Bridge to others.

In Fig. 18 the comparative torsion or tilting of the floor is based on a load of 100 lb. p. ft. along the near cable from one tower to the center and the same load along the far cable from the center to the other tower.

The ratios of torsional flexibility of the Tacoma Narrows Bridge to that of the other four large suspension bridges, on the basis of the above two force conditions is as follows:

|  | Based on<br>Torsional<br>Moment<br>(Table 6) | Based on<br>Vertical<br>Forces<br>(Fig. 18) |
|--|--|---|
| Golden Gate Bridge                     | 12.4 to 1                                    | 4.2 to 1                                    |
| Bronx-Whitestone Bridge, without stays | 9.4 to 1                                     | 4.4 to 1                                    |
| Bronx-Whitestone Bridge, with stays    | 10.7 to 1                                    | 5.9 to 1                                    |
| San Francisco Transbay Bridge          | 11.4 to 1                                    | 4.9 to 1                                    |
| George Washington Bridge               | 38.0 to 1                                    | 14.1 to 1                                   |

This comparison shows that the Tacoma Narrows Bridge was far more susceptible to torsion than any of the other bridges. This comparatively great torsional flexibility is due to a combination of exceptionally great vertical flexibility and unusually narrow width, and not to lack of torsional resistance of the floor structure, nor to the narrowness of the bridge alone. The Golden Gate Bridge, which is by a wide margin, next in torsional flexibility to the Tacoma Bridge, has experienced small unobjectionable torsional motions. It is questionable as to whether it represents a limit for torsional flexibility, since these motions might be greater and yet not objectionable. Furthermore, we do not yet have sufficient information as to the effect of proportions of the floor structure on the torsional moments and vertical forces from

wind to permit a quantitative comparison.

The above qualitative comparison would indicate, however, that the Tacoma Narrows Bridge, with its relatively narrow width and small weight would have insufficient torsional rigidity even if its vertical rigidity were equal at least to that of the Golden Gate Bridge. Either its vertical rigidity would have to be made still greater, or else its width or weight, or both, would have to be increased to secure a degree of torsional rigidity at least equivalent to that of the Golden Gate Bridge.

#### Effect of Various Elements of Structure on Vertical and Torsional

Deformations of Tacoma Narrows Bridge. In order to be able to visualize more clearly the reasons for the unusually large vertical and torsional deformations of the Tacoma Narrows Bridge, and to determine how and to what extent they may be controlled to obtain greater rigidity, it is essential to analyze the extent to which the various elements of the structure contribute to the deformations of the compound system. The principal elements which influence vertical deflections are weight, cable sag, stiffening girders, towers and side spans. Torsional deformations are influenced by the vertical deflections, hence by all the above-mentioned elements, and by the width between cables.

Effect of Weight. Weight or dead load is one of the most potent elements affecting the static rigidity of a long span suspension bridge. If applied as weight of floor structure it requires additional sectional area and, therefore, weight of the cables and thus indirectly decreases deflections. The static deflections from loads or wind forces are thereby decreased, roughly in inverse proportion to the total dead load.

It is predominantly its great weight which gives the George Washington

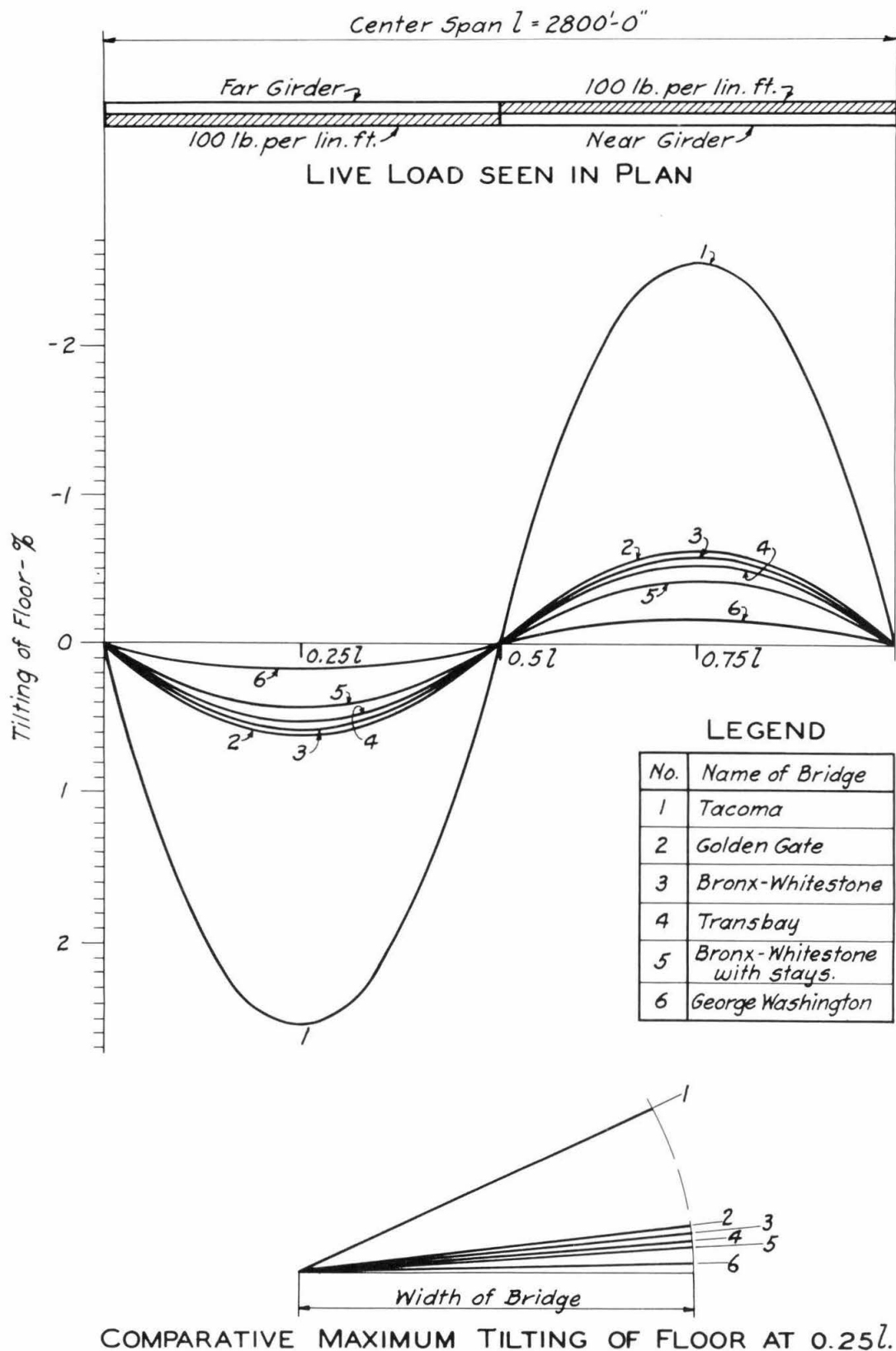


FIG.18 - COMPARATIVE TILTING OF FLOOR OF THE FIVE LONGEST SUSPENSION BRIDGES

Bridge its great rigidity. The relatively short side spans and rigid towers are other, but less effective factors. Weight is an important factor also in the Golden Gate and Whitestone Bridges. In the Tacoma Narrows Bridge weight alone was far from sufficient to supply the necessary rigidity.

Weight not needed to meet the design requirements is, however, an expensive way to supply additional rigidity in long span suspension bridges, because even when added cheaply to the floor structure, weight involves considerable additional expense for the heavier suspenders, cables, towers and anchorages which is not compensated by any possible saving in the stiffening girders.

In the Tacoma Narrows Bridge an increase in weight of 10% (to a total of 6270 lb. p. ft. of bridge), which would result from an increase in the thickness of the roadway slab by two inches, would decrease the quarter point deflection by only about 5%. An increase in weight of 100% (to 11,400 lb. p. ft. of bridge), as would result approximately from doubling the traffic capacity, would decrease the comparative static deflection near the quarter point from 1.59 to 0.83 ft. or by 48%. To bring its comparative vertical rigidity within the range of the Golden Gate and Whitestone Bridges would require that the Tacoma Narrows Bridge be made  $2\frac{1}{2}$  times heavier than it was, other proportions remaining the same. This would manifestly be wasteful. The effect of weight is shown graphically in Fig. 19.

Effect of Cable Sag. When the weight to meet design requirements is not sufficient to supply adequate rigidity, it must be supplemented by other means. This is possible only to a limited extent by a decrease

in the cable sag, because any appreciable decrease involves a material increase in cost of cables and anchorages. In the Tacoma Narrows Bridge the cable sag was made as flat as economy permitted in order to secure added rigidity. The sag ratio of  $1/12$  is less than in all other modern long span bridges, in which it varies generally between  $1/9$  and  $1/11$  (Table 3).

A decrease of 10% or from 232 to 209 ft. (ratio to span  $1:13.4$ ) in the cable sag of the Tacoma Narrows Bridge decreases the maximum deflection at the  $1/4$  point from 1.59 to 1.49 or by only 6.3%. This would evidently go only a short way towards supplying the deficiency in rigidity in the Tacoma Narrows Bridge and indicates that the effect of the relatively flat cable sag of  $1/12$  was greatly overestimated. Within the usual economically justifiable sag ratios of  $1/9$  to  $1/12$  any variation in cable sag has no marked effect upon the vertical rigidity of a long span suspension bridge. The effect of sag on vertical deflections of the Tacoma Narrows Bridge is illustrated in Fig. 22. In a suspension bridge with relatively narrow and flexible wind truss the cable sag has a material effect upon the lateral rigidity, the lateral deflection from the wind pressure being roughly proportional to the cable sag. Inasmuch, however, as in the Tacoma Narrows Bridge the lateral rigidity was sufficient, the stiffening effect of the relatively low cable sag of  $1/12$  had no practical significance in this respect.

Effect of Stiffening Girders or Trusses. Stiffening girders or trusses are the most effective and most economical means to supply rigidity as far as it is not provided by weight. In the Tacoma Narrows Bridge the girders were evidently far too flexible for that purpose. As built,

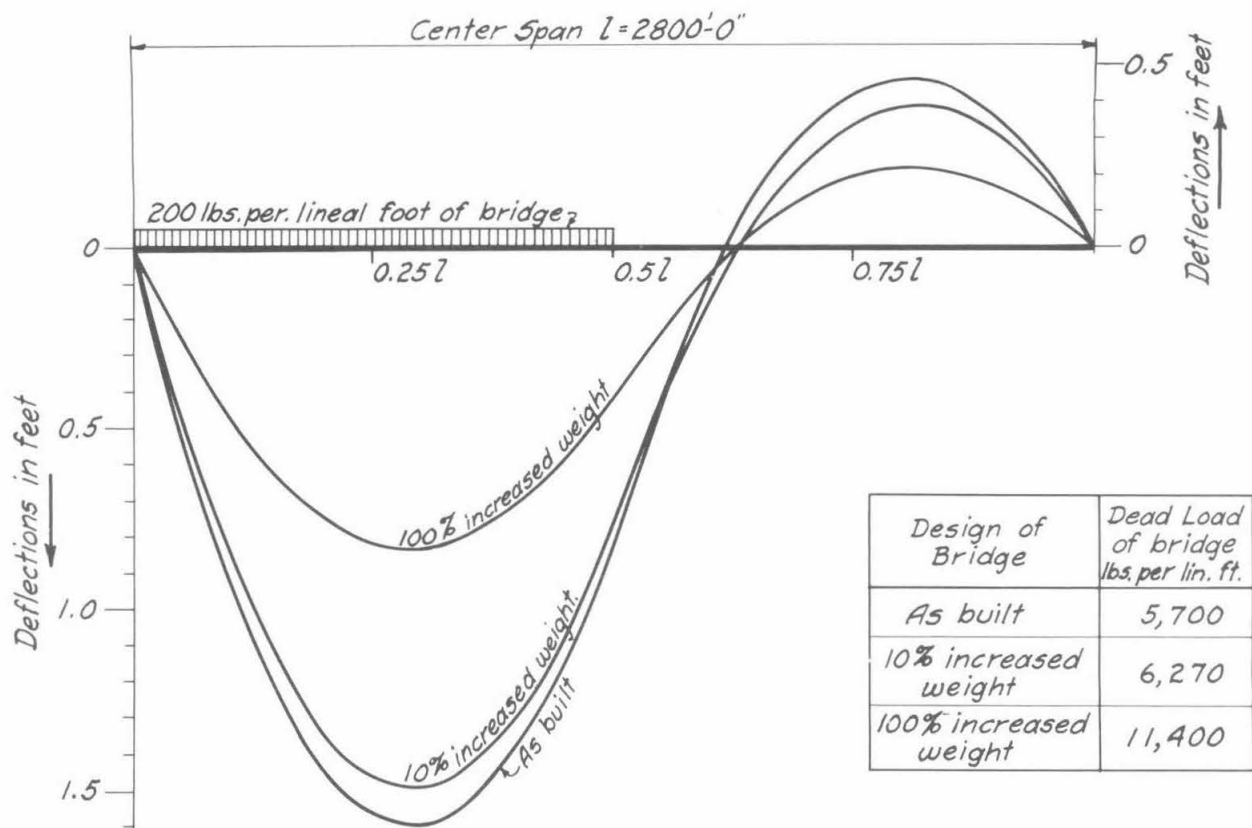


FIG.19 - EFFECT OF WEIGHT ON COMPARATIVE VERTICAL DEFLECTIONS OF THE TACOMA NARROWS BRIDGE

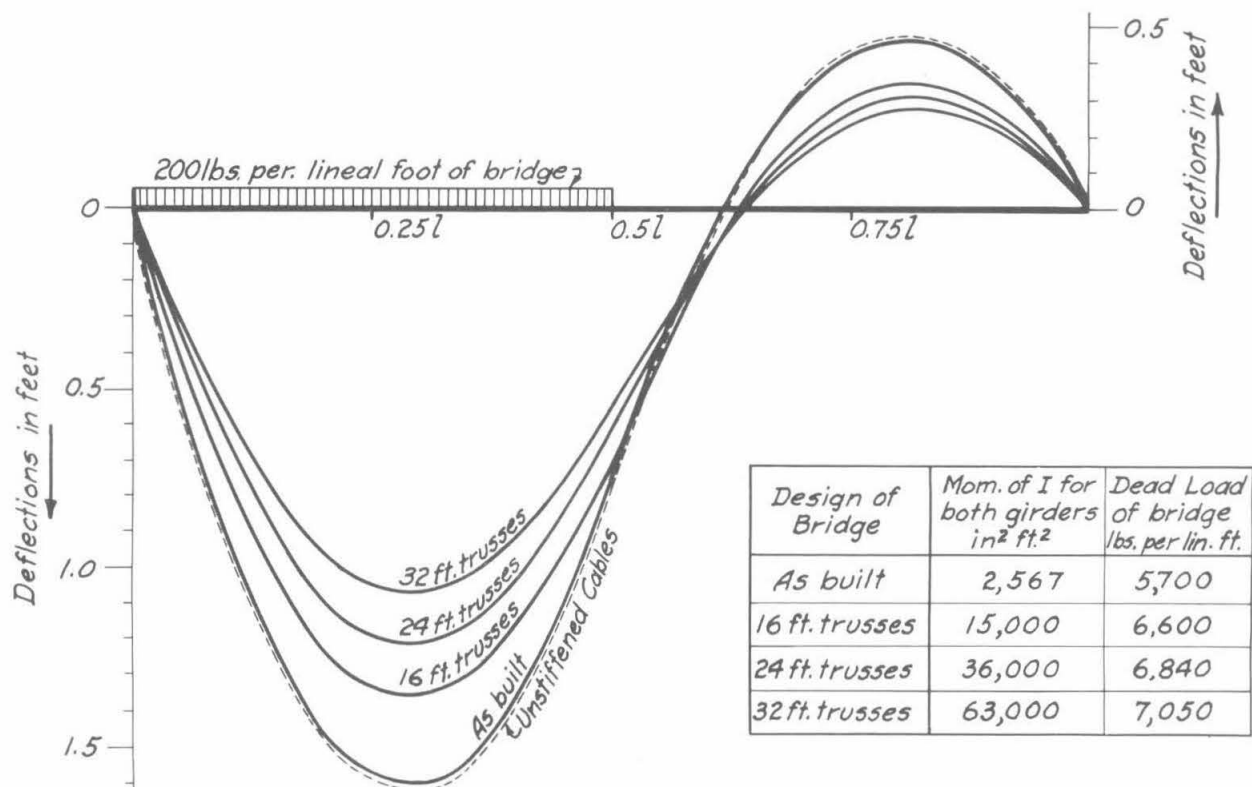


FIG.20- EFFECT OF THE RIGIDITY OF THE STIFFENING GIRDERS ON COMPARATIVE VERTICAL DEFLECTIONS OF THE TACOMA NARROWS BRIDGE.



they influenced the deflections of the unstiffened cables at the quarter points to the extent of only 1.3% (Fig. 20), in other words, they were practically ineffective in stiffening the cables except very locally.

The two stiffening girders of the Tacoma Narrows Bridge have a moment of inertia of only  $2.566 \text{ in}^2 \text{ ft}^2$ , compared to  $5,860 \text{ in}^2 \text{ ft}^2$  in the Bronx-Whitestone Bridge of twice the weight and to  $88,000 \text{ in}^2 \text{ ft}^2$  in the Golden Gate Bridge of over 3.7 times the weight.

The stiffness of girders or trusses is most effectively and economically increased by increase in depth. Increase in depth of truss in a suspension bridge, however, also means increase in live load participation with resulting increase in chord area, and therefore depth must be restricted to the required minimum.

When, as in the case of the Tacoma Narrows Bridge, the unstiffened cables have a great excess of flexibility, owing to insufficient weight, relatively deep rigid trusses are necessary to supply the deficiency in rigidity. A change from 8 ft. girders ( $1/350$  of span) to trusses 16 ft. deep, ( $1/175$  of span) with the same width and weight of floor, would increase the weights of the suspended structure to about 6,600 lb. p. ft. of bridge, the moment of inertia of two trusses to about  $15,000 \text{ in}^2 \text{ ft}^2$  and decrease the quarter point deflection from 1.59 to 1.36 ft. or by 14.5%, which would evidently be far from sufficient.

A change to 24 ft. trusses ( $1/117$  of span) would affect the respective values approximately as follows: Increase in weight to 6,840 lb. p. ft., increase in moment of inertia to  $36,000 \text{ in}^2 \text{ ft}^2$  and decrease in quarter point deflection to 1.20 ft. For 32 ft. trusses ( $1/88$  of span) the corresponding values would become: weight 7,050 lb.

p. ft., moment of inertia of two trusses  $64,000 \text{ in}^2 \text{ ft}^2$  and relative deflection 1.07 ft.

If the total suspended weight is doubled to 11,400 lb. p. ft. of bridge, as would be the case, approximately, if the traffic capacity were doubled, and if the trusses are made 24 ft. deep ( $1/117$  of span) with moment of inertia of the two trusses  $36,000 \text{ in}^2 \text{ ft}^2$ , the deflection would be 0.77 ft., approaching sufficient rigidity. The effect of the stiffening trusses of various degree of rigidity on vertical deflections is illustrated in Fig. 20.

Fig. 21 illustrates graphically the combined effect of weight and girder stiffness.

Effect of Towers and Side Spans. For the slender type of towers, fixed at the bottom, the effect of the tower rigidity on the static deformations of the suspended structure is negligible. The deflections of the tower top and the proportions of the tower are governed largely by the deformations of the side spans. If the latter are relatively short, as in the case of the George Washington Bridge, or if the side span cables do not support the floor, the motions of the tower tops may be sufficiently restricted to permit comparatively wide, rigid towers. With relatively long side spans and extended cables, as in the Tacoma Narrows Bridge, flexible towers are the only practical and economical solution.

Long side spans have a material influence upon the flexibility of the center span. In the Tacoma Narrows Bridge this influence is further increased by the elongations of the cable extensions from end supports to the anchorages. The side span contribution to the deflection at the quarter point of the center span is 0.4 ft. or 26% of the total deflection.

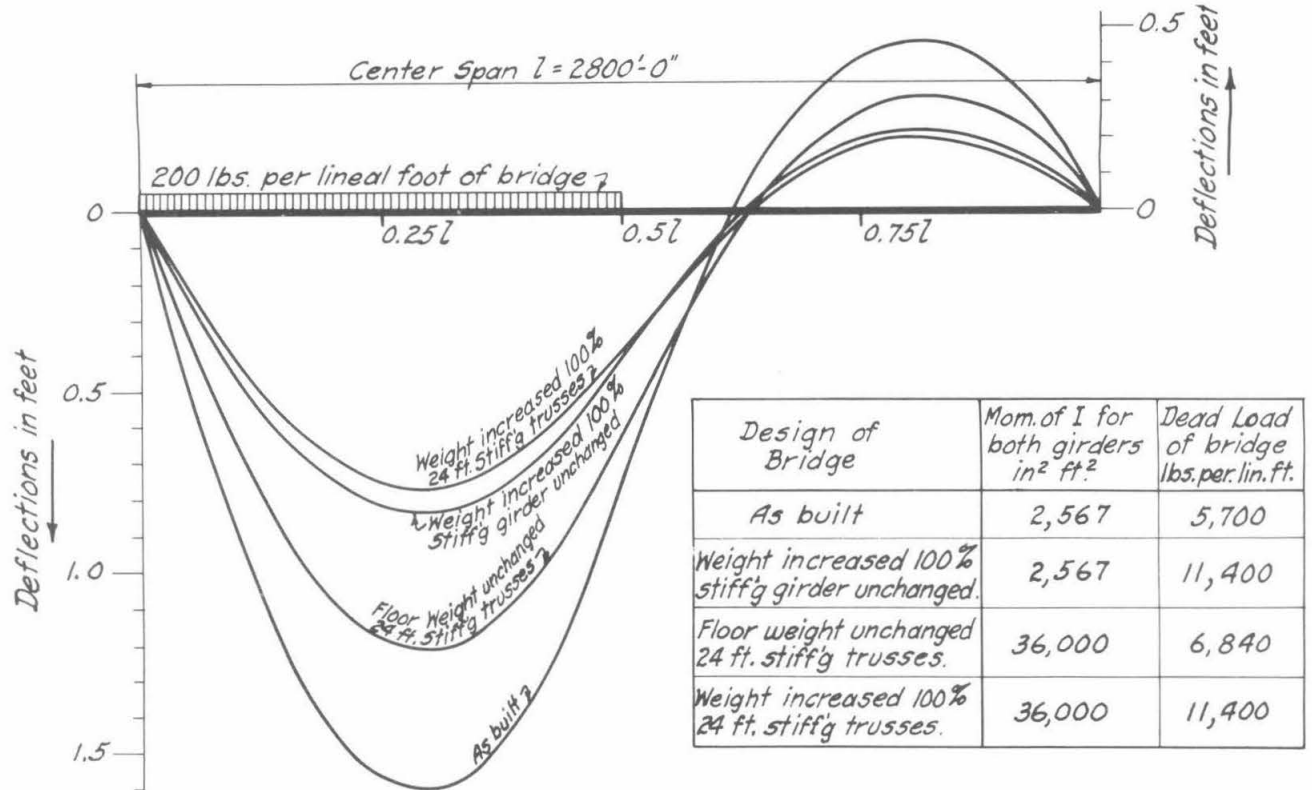


FIG.21 - COMBINED EFFECT OF INCREASED WEIGHT AND RIGIDITY OF THE STIFFENING GIRDERS ON THE COMPARATIVE VERTICAL DEFLECTIONS OF THE TACOMA NARROWS BRIDGE

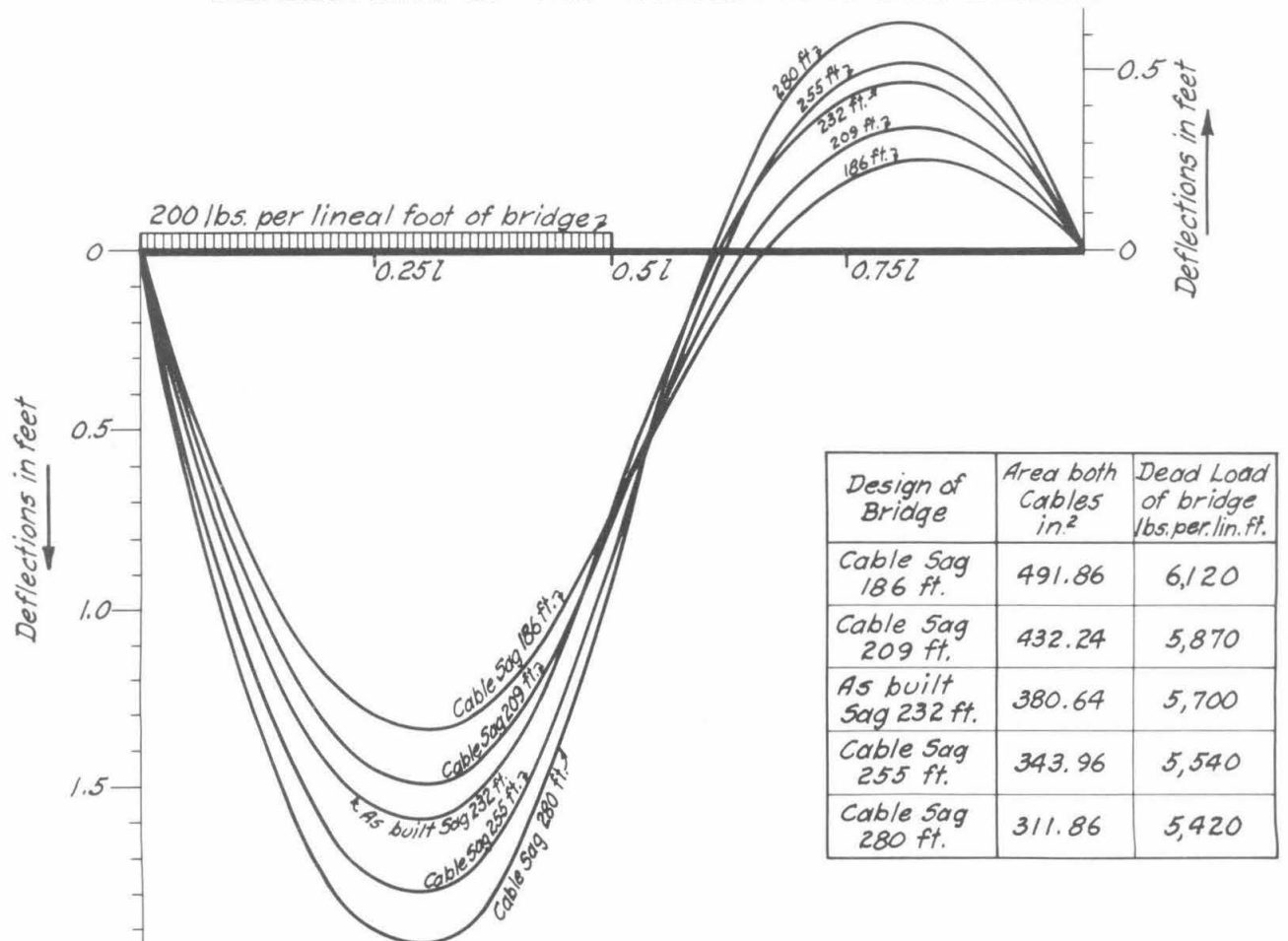


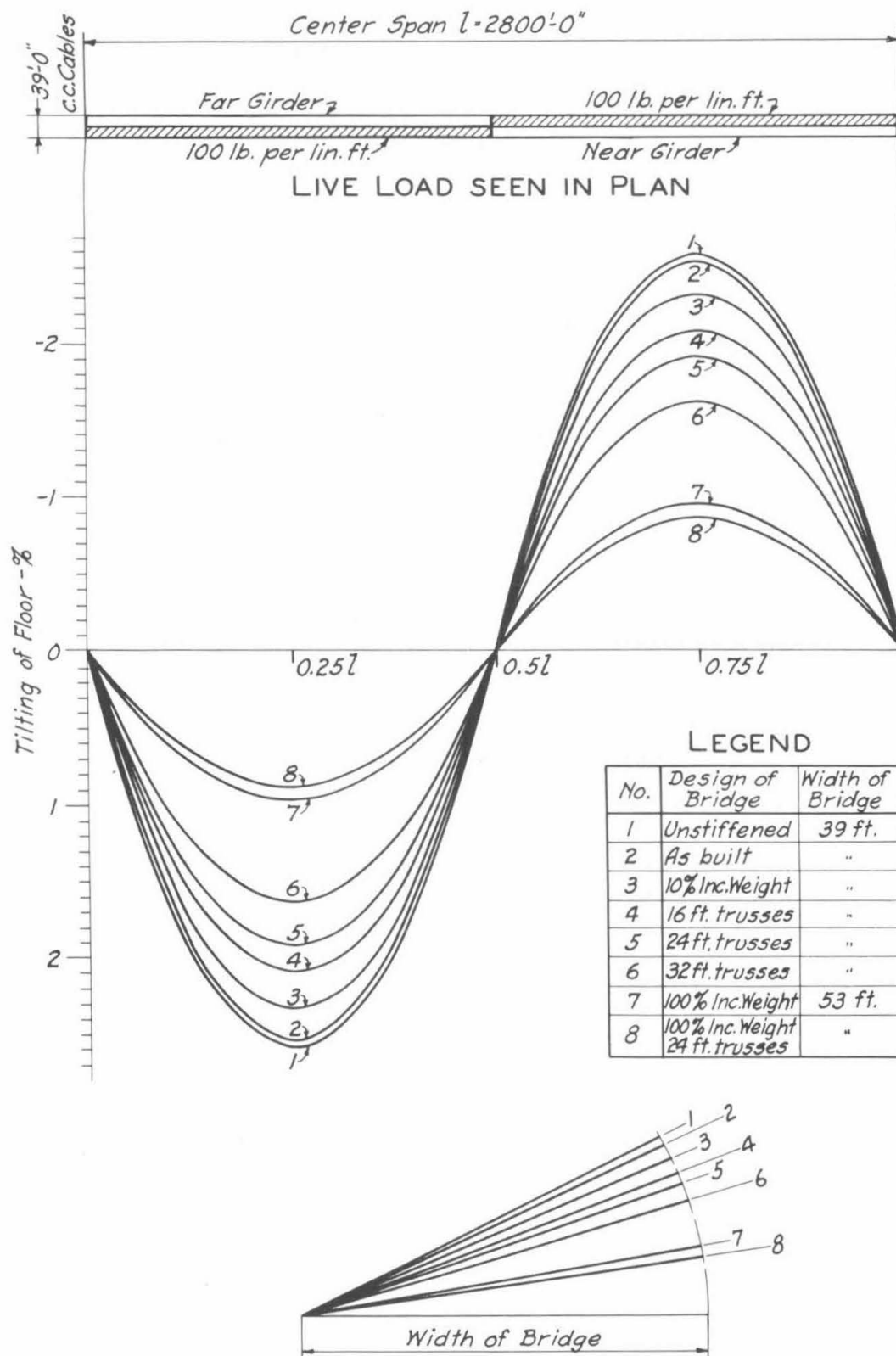
FIG.22 - EFFECT OF CABLE SAG ON COMPARATIVE VERTICAL DEFLECTIONS OF THE TACOMA NARROWS BRIDGE.

This compares with 0.11 ft. or 18% in the Bronx-Whitestone Bridge, which has side spans of only  $1/3$  of the center span, and with 0.03 ft. or 8% in the George Washington Bridge which has side spans of only  $1/6$  of the center span.

Effect of Width on Torsional Rigidity It has been explained heretofore that torsional rigidity is a function of the vertical rigidity and the width between cables. Without further aerodynamic experiments and experience on actual structures it is not possible to set a limit to torsional flexibility. Judging from the behavior of the Golden Gate Bridge, however, it would not seem advisable at this time to permit torsional flexibility exceeding materially that of the Golden Gate Bridge.

On this basis a width of 39 feet appears entirely inadequate, and even a 53 ft. width would require more rigid trusses than appears necessary for adequate vertical rigidity.

Fig. 23 illustrates the effect of width, weight and girder stiffness on torsion, and indicates that only an increase in width to 53 ft., an increase in weight by 100% and stiffening trusses 24 ft. deep, can secure a torsional rigidity approaching that of the Golden Gate Bridge.



COMPARATIVE MAXIMUM TILTING OF FLOOR AT 0.25l.

FIG. 23 — EFFECT OF WIDTH, WEIGHT AND GIRDER STIFFNESS ON THE COMPARATIVE TORSIONAL DEFORMATIONS OF THE TACOMA NARROWS BRIDGE.

CHAPTER IV  
AERODYNAMIC FORCES ACTING ON SUSPENSION BRIDGES  
AND THE OSCILLATIONS RESULTING THEREFROM

The terminology used in this chapter is based on that customarily used in aerodynamics, but is somewhat adapted to the specific character of the problem under discussion. In aerodynamic theory distinction is made between steady and unsteady forces. Steady forces are those acting on a structure at rest in a uniform wind stream or on a structure moving uniformly through calm air. Unsteady forces are those resulting from variation of the velocity and direction of the wind and from non-uniform motion, such as oscillation of the structure. In accordance with structural engineering terminology the expressions static and dynamic forces are used in this report. There is no sharp distinction between the two, for example, if the variation of the wind velocity is very slow, the corresponding forces can be considered as static forces. On the other hand, a parallel uniform wind stream produces eddies on a structure and, therefore, the force exerted by such wind stream is constant only insofar as its average value is concerned. Its instantaneous magnitude and direction is subjected to fluctuations. These variable effects will be classified as dynamic forces.

According to the aerodynamic terminology drag is the component of a force exerted on any structure, for example, an airplane wing, by a wind stream in the direction of the wind stream and lift is the component normal to the wind stream. For the present problem it is more convenient to refer to the axes of the structure and to consider horizontal and vertical components of the wind force. The vertical wind force is

identical with the lift in the case of a horizontal wind. Therefore, for practical considerations, the expression lift may be used for the vertical force normal to the floor.

The following table gives the classification of the aerodynamic forces to be considered in the case of a suspension bridge. Only the forces acting on the suspended roadway are taken into account.

A. Static forces

- (a) Horizontal force, drag.
- (b) Vertical force, lift.
- (c) Moment in the vertical plane.

B. Dynamic forces

- (a) resulting from fluctuations in velocity and direction or the wind velocity (turbulence).
- (b) resulting from periodic character of the flow around the structure (eddy formation).
- (c) produced by the vertical oscillation of the suspended structure.
- (d) produced by the torsional oscillation of the suspended structure.

The term "dynamic pressure" in lbs. per sq. ft. designates the half product of the density of the air (in slugs) and the square of the wind velocity in ft. per sec. If the air velocity is expressed in miles per hour, the dynamic pressure is equal to  $25.6 \left(\frac{V}{100}\right)^2$  in lbs. p. sq. ft.

Static Horizontal Wind Forces. The usual computation of the structural safety and rigidity of suspension bridges under action of wind is based on the estimate of the horizontal forces acting on the surfaces exposed



to the wind. It is assumed that the horizontal forces are equal to the product of the frontal area of the exposed structural parts, the dynamic pressure corresponding to the specified maximum wind velocity and an empirical numerical factor which expresses the influence of the form. In bridge engineering practice a factor equal to about 1.3 is used. Calculations for the design of the Tacoma Narrows Bridge were based on a pressure of 30 lbs. p.ft., acting on one and one-half times the frontal area. It was found that both stresses and deflections produced by this wind load are within allowable limits.

The horizontal wind pressure assumed for the design can be compared with the wind tunnel measurements given in Appendix VIII. The drag coefficients of the suspended floor, in accordance with aerodynamic practice, is referred to the horizontal projection of the roadway, and for the model of the Tacoma Narrows Bridge it was found to be equal to 0.31. Referred to the frontal area of the front girder, as customary in bridge engineering practice, this gives a coefficient equal to 1.51. The following values result for various wind velocities:

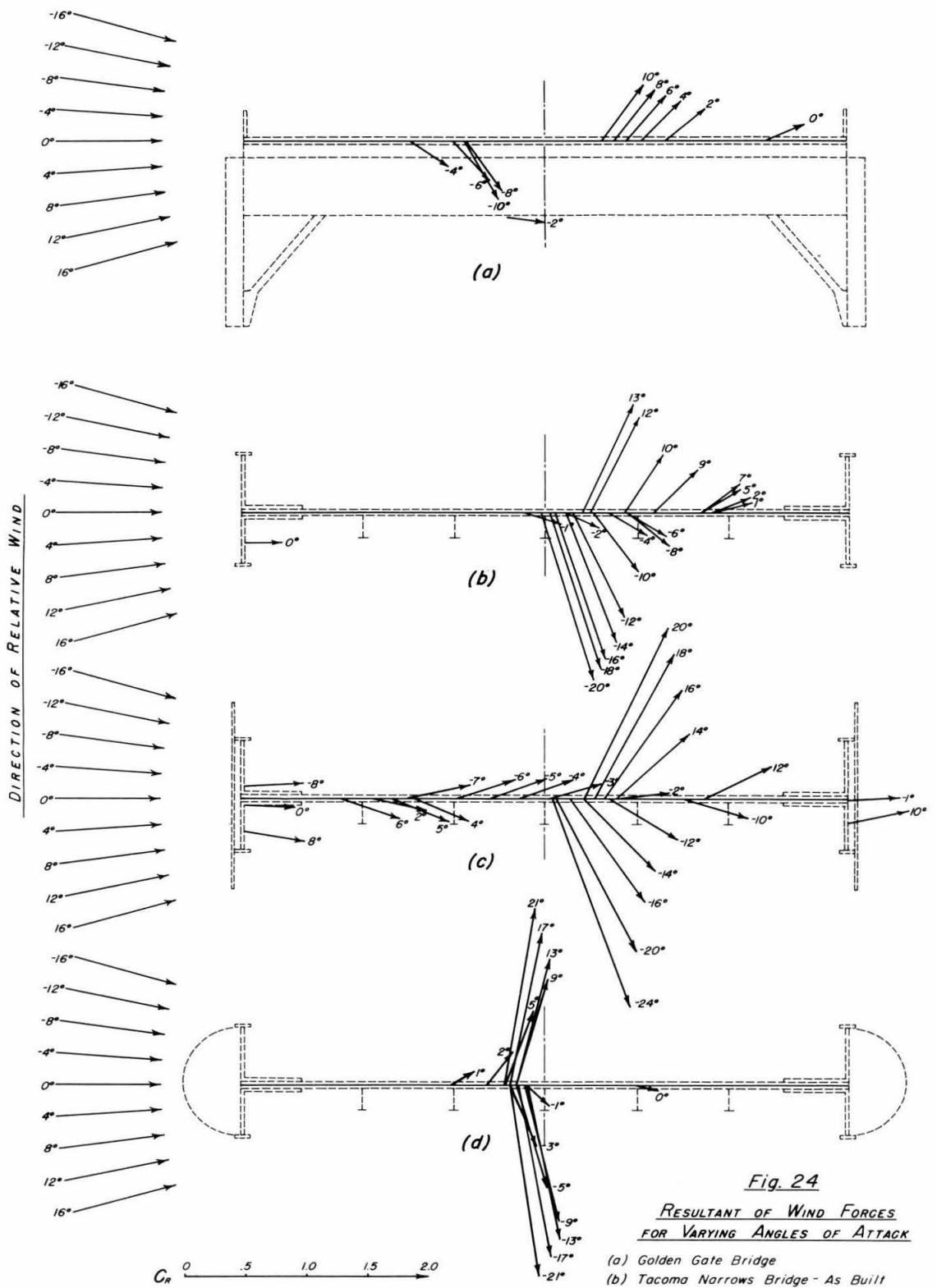
| Wind velocity<br>mi. p. hr. | Wind pressure in<br>lb. per sq. ft.<br>of front girder. | Horizontal force in<br>lb. p. lin. ft. of<br>bridge. |
|-----------------------------|---|--|
| 40                          | 6.2   | 49.6   |
| 80                          | 24.8  | 198.4  |
| 120                         | 55.7  | 445.7  |

It appears that the design assumption of a maximum wind pressure of 30 lb. p. sq. ft. on 1-1/2 times the frontal area gives a value of the drag corresponding to 108 m.p.h. wind velocity.

According to wind tunnel tests at Stanford University, the drag

coefficient referred to one square foot of the roadway, in the case of the Golden Gate Bridge, has almost exactly the same value which has been obtained for the Tacoma Narrows Bridge. Considering the ratios of the widths of the two bridges, the wind pressure on the truss structure of the Golden Gate Bridge corresponds to that of plate girders of 18.5 ft. depth. Streamlining by semicircular fairings would reduce the horizontal wind force on the girders of the Tacoma Bridge by about one-third.

Static Vertical Wind Forces (Lift forces). The vertical or lift force is the resultant of the pressures acting on the suspended floor; the lift force acting on a section of unit length of the bridge can be expressed as the product of the width, the dynamic pressure corresponding to the wind velocity and a numerical factor. This numerical factor, the lift coefficient, depends on the shape of the cross section and the angle of attack, i.e., the angle between the surface of the floor and the wind vector. The static lift forces are small in comparison with the dead and live loads and are therefore generally neglected in the strength calculations. However, they have a bearing on the dynamic behavior of the structure, especially on the magnitude of the forces produced by its vertical oscillations. The resultants of the horizontal and vertical forces for various angles of attack are shown in Fig. 24 for four different structures: (a) Golden Gate Bridge, (b) Tacoma Narrows Bridge as built, (c) Tacoma Narrows Bridge with flat plates of 13 ft. depth in front and in rear, and (d) Tacoma Narrows Bridge with semicircular fairings at the girders. In order to obtain the actual forces per unit length of the bridge the values given in the drawing must be multiplied with the width of the bridge and the



dynamic pressure corresponding to the wind velocity. For example, if the coefficient is equal to unity, the force per unit length of the roadway at 40 m.p.h. wind speed is equal to 160 lb. in the case of the Tacoma Bridge and 369 lb. in the case of the Golden Gate Bridge. It is noticed that in case (d) a small positive angle of attack (wind from below) produces a vertical force component directed downward. The tests conducted at the University of Washington (9) gave similar inverted vertical forces for the bridge as built. The reasons for this apparent discrepancy are discussed in Appendix VIII. In the Golden Gate Bridge a horizontal wind produces a slight vertical force directed upward, in the Tacoma Narrows Bridge a slight vertical force directed downward.

Moment of the Static Forces. In general the resultant of the horizontal and vertical wind forces does not intersect the plane of resistance of the horizontal forces at the vertical axis of symmetry of the suspended structure. It exerts a moment on the structure about that axis which produces different loads on the two cables and a twisting of the roadway. The moment per unit length of the bridge can be expressed by the product of the dynamic pressure, the square of the width of the bridge and a non-dimensional moment coefficient, which depends on the axis of reference and the angle of attack.

Fig. 24 shows the lines of action of the resultant forces from static wind loads as determined by the wind tunnel tests, therefore, it also indicates the direction and magnitude of the moment as defined above. The direction and magnitude of the moment for horizontal wind is of little consequence. Its rate of change, however, has some bearing on the behavior of the bridge. If the moment varies with the angle of

attack, it will work, in case of tilting of the roadway, against the restoring action of the cables. The wind tunnel measurements show that this unfavorable effect exists definitely in the case of the streamlined roadway, whereas with the roadway as built the variation of the static moment of the wind forces is favorable.

Dynamic Forces Due to Turbulence. Because of the natural turbulence of the wind, the magnitude and direction of the wind at a fixed point is subjected to more or less random variations. Consequently, the horizontal wind pressure and especially the lift forces are continually varying. The vertical forces vary not only in amount, but also in direction, and therefore are likely to produce vertical oscillations of the roadway.

The order of magnitude of the forces involved can be estimated in the following way. Records of velocity fluctuations in natural, turbulent wind show that their average magnitude is of the order of 20-25% of the mean wind velocity. The peaks of the fluctuations are considerably higher. The fluctuations of the vertical components of the wind are of about the same order as the fluctuations of the horizontal component. Hence, it can be assumed that the instantaneous angle of attack of the wind against the structure continually varies between limits of the order of  $15^{\circ}$ . Using the value of the coefficient of the vertical force for such angles measured on models of the suspended structure, one obtains, for example, for 40 m.p.h., a force of about 180 lb. p. lin. ft. of the Tacoma Narrows Bridge. It is evident that there is little probability that the wind velocity fluctuates simultaneously in the same way over the entire span; hence

the span will be subjected at any instant to positive and negative loads distributed along the span. The resulting static deflection would depend on the distribution of these loads. However, it is necessary to consider variation of the loads with time. The wind fluctuations are more or less random, i.e., there is no definite periodicity either in the change of the velocity or the direction of the wind. Nevertheless, the fluctuations exert an action on a structure similar to a great number of superimposed periodic fluctuations. As a matter of fact, there are experimental methods to obtain an harmonic analysis of the turbulent fluctuations. Therefore, if the fluctuations of the turbulent wind be assumed as composed of many periodic fluctuations, the deflections, produced by those load fluctuations the period of which approaches one of the natural frequencies of the bridge structure, will be amplified, especially if the structure is not sufficiently damped.

The distribution of the energy of turbulent fluctuations as a function of the period has been determined for artificial wind streams. As far as natural wind is concerned, it is known that, in general, higher frequencies or shorter periods prevail in winds of higher average velocity. The spacing of the eddies or gusts in the wind is governed largely by the topographical situation and the height over the ground and, therefore, the rate of the velocity fluctuations at a fixed point increases if the wind velocity increases. In addition, the weather situation, especially the stability of the atmosphere, also has a definite influence on the intensity and spacing of the gusts. Therefore, statements about the action of turbulent wind on a structure

can be only true if a statistical average of many observations is taken.

Dynamic Forces Due to Eddy Formation on the Structure. The second source of variable forces is the eddy formation which occurs with the air flow around bodies with sharp edges and of blunt cross sections. The formation of these eddies occurs, in many cases, with a periodic rhythm and, when their period of formation coincides with the natural period of the structure, a resonance oscillation often develops.

In order to produce periodically alternating lift forces on the bridge structure, it would be necessary that eddies be produced alternately at the upper and lower edge of the girders as it occurs if a single flat plate is exposed normally to a wind stream. In order to decide this question the velocity fluctuation measurements described in Appendix VIII were conducted on a 1:80 model of the roadway. A hot wire anemometer arrangement was used which permitted the simultaneous recording of the fluctuations near the upper and lower edges of the front girder. It was found that the fluctuations were of large order and of fairly periodic character, but as far as could be discovered the fluctuations were more frequently in phase than in the opposite phase (Fig. 25, (a) and (b)). This is confirmed by flow pictures taken by R. Beeuwkes at the Case School of Applied Sciences. Some of these pictures show the eddy formation about a model, similar in section to that of the Tacoma Narrows Bridge, moved in a water tank are reproduced in Fig. 25. If the model does not oscillate (c) behind the structure, the eddy formation at the front girder is fairly periodic, but is not alternating. The lift forces are chiefly produced by the pressure



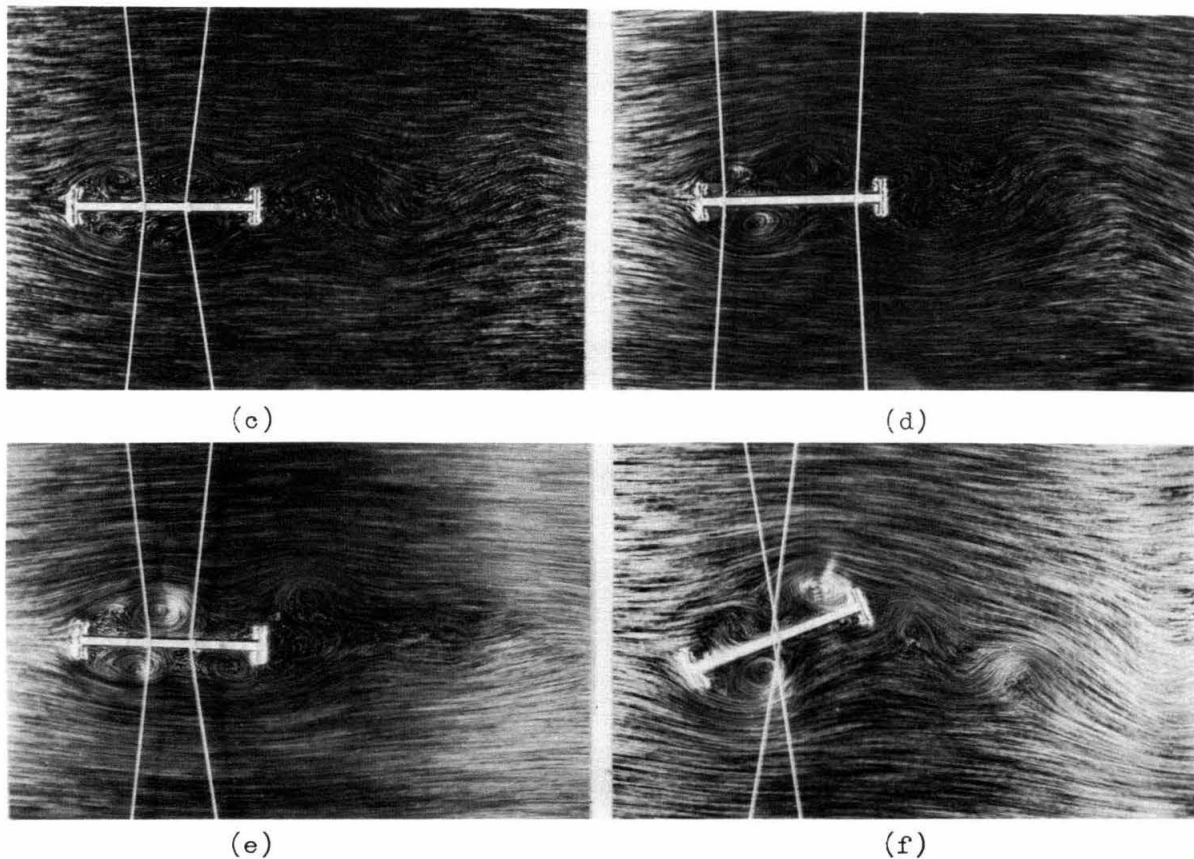
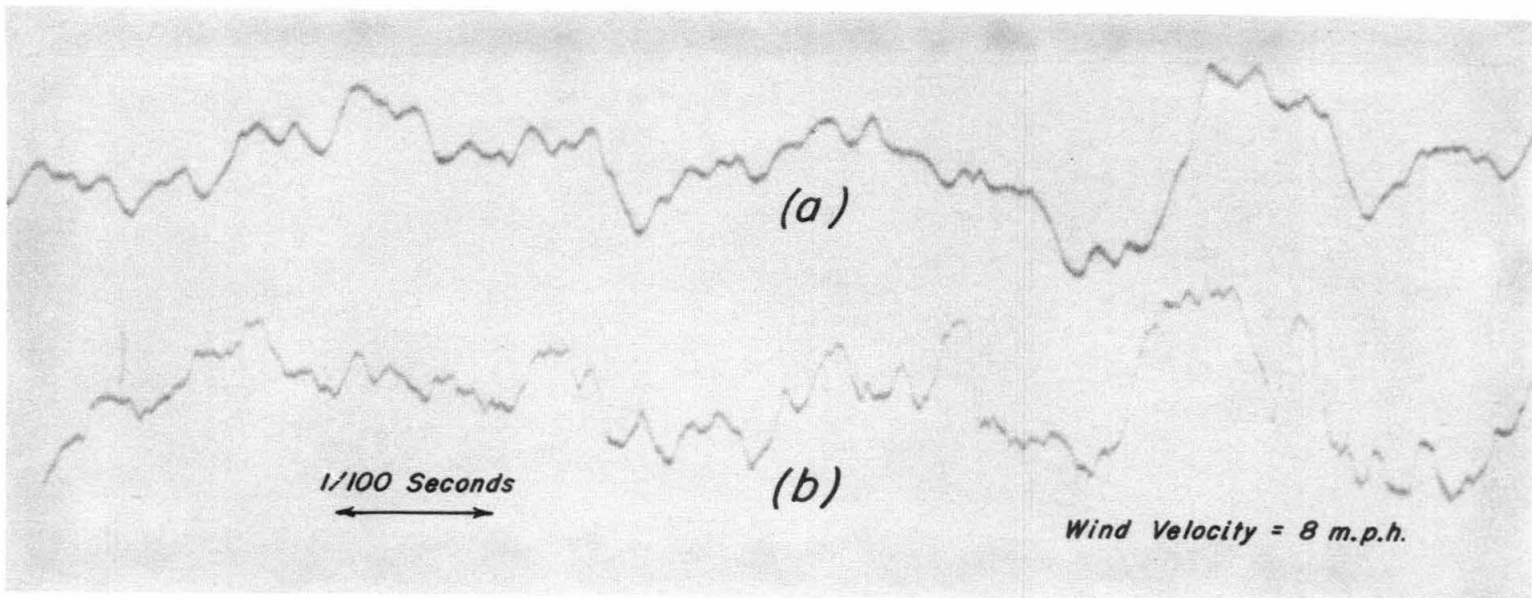


Fig. 25. Hot-Wire Records and Flow Pictures. (a), (b) - Velocity fluctuations behind the upper (a) and lower (b) edges of the front girder. (c) - Symmetric eddy formation on model at rest. (d), (e), (f) - Eddy formation at oscillating model. Note in (d) and (e) the asymmetry in eddy formation connected with incipient angular oscillation.

difference on the top and bottom of the floor. It seems reasonable to assume that this pressure difference is governed by the wake formation at the front girder. Hence, it does not appear probable that eddy formation causes periodically alternating positive and negative lift forces of any appreciable magnitude.

Dynamic Forces Produced by Oscillation of the Structure. Whereas turbulence and eddy formation are mostly responsible for oscillations of moderate amplitudes of structures exposed to wind action, the experience with aeronautical structures and power conductor lines shows that aerodynamic forces induced by oscillatory motion of such structures can cause excessive amplitudes of oscillations and complete breakdown of the structure (18). In general, self-induced forces can appear as positive as well as negative damping. If the system is set in oscillatory motion and the effect of the wind stream is such that the amplitude of the motion decreases, the induced force is denoted as positive damping, if the amplitude of the motion increases in the wind stream, we speak of negative damping. Negative damping can be considered as "dynamic instability". Another term used for oscillations whose amplitudes are increased by negative damping is "self-induced" oscillation. This latter term is in many cases more adequate, especially for large amplitudes, if the damping becomes positive so that instability is restricted to the neighborhood of the equilibrium position.

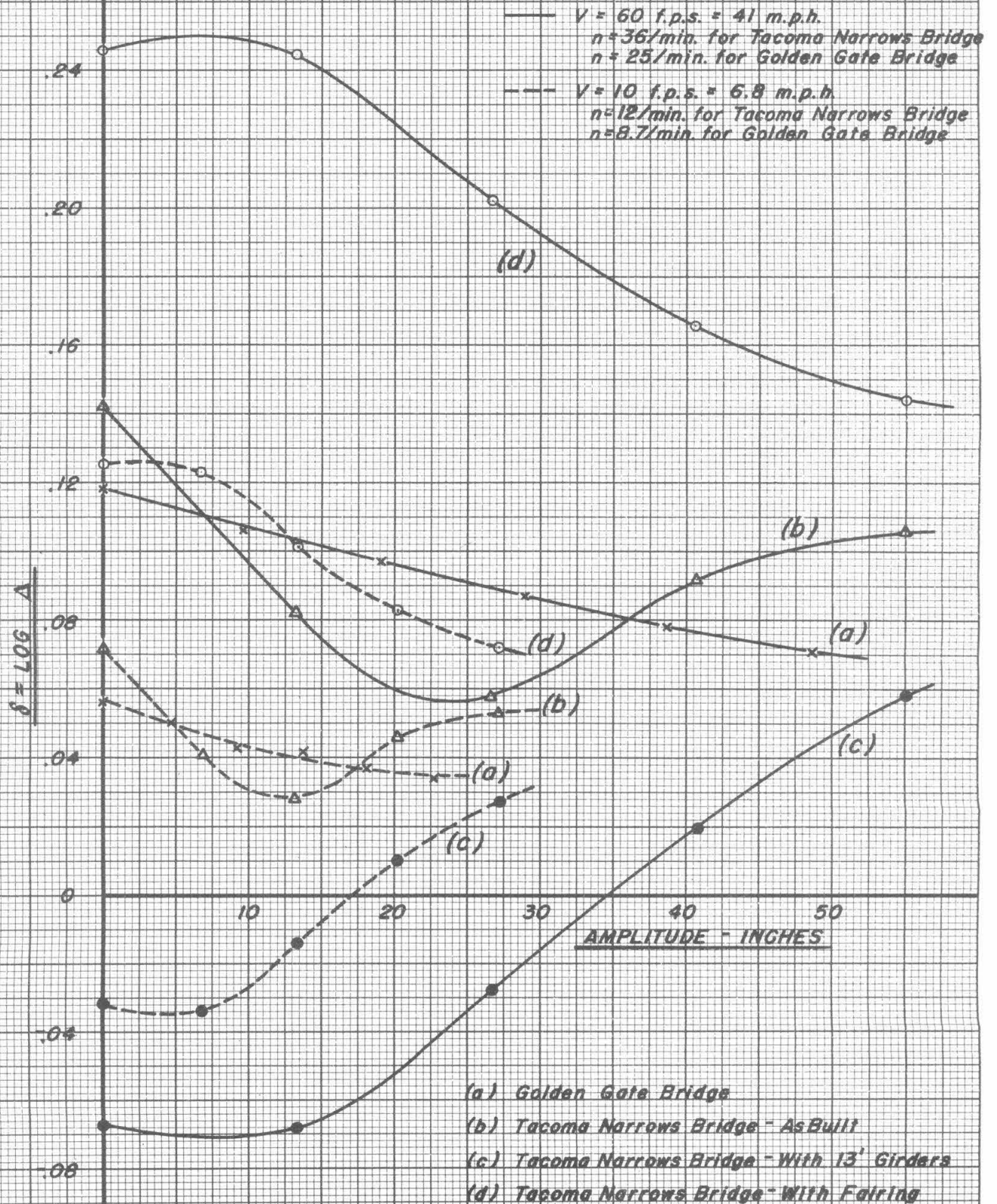
An appropriate measure of positive damping is the rate of dissipation of energy, defined by the amount of energy dissipated in one cycle of oscillation. In the case of free oscillation, the rate of dissipation is the ratio between the decrease of the energy of the

oscillating system during one full oscillation and the energy of the system at the beginning of the same. In suspension bridges we are concerned with relatively slight damping; in such cases the rate of dissipation is the double of the so-called logarithmic decrement (natural logarithm of the ratio between two successive amplitudes in the same direction). In the case of a forced oscillation a constant amplitude is reached if the rate of dissipation is equal to the energy input by the impressed forces. In the case of free oscillations with negative aerodynamic damping the self-induced forces add, in every cycle, to the energy of the oscillating system. Hence, the magnitude of negative damping can be measured by the rate of energy input per cycle. A constant amplitude will be reached, if the negative aerodynamic damping is balanced by the positive mechanical (structural) damping. The essential difference between a forced vibration induced by variable wind forces and by negative damping is that in the first case a continually varying wind load is required to establish and maintain an oscillation of certain amplitude. In the second case the oscillating system is able to extract energy from a perfectly uniform smooth wind stream.

Suspension bridges have shown several modes of oscillations. However, two classes of motions can be distinctly separated: flexural (vertical) oscillations of the roadway structure with the two cables in phase and torsional oscillations with out of phase oscillations of the cables. In the case of the Tacoma Narrows Bridge the second class of oscillations appeared only shortly before the final failure of the structure. However, torsional oscillations have been observed in other

Fig. 26

LOGARITHMIC DECREMENTS DUE TO AERODYNAMIC  
DAMPING FOR VERTICAL OSCILLATIONS



bridges, for example the Brighton (17), the Deer Isle (19) and Golden Gate bridges. The possibility of negative damping requires separate treatment for the two types of oscillations.

Aerodynamic Forces Produced by Vertical Oscillations. Negative damping of the vertical (flexural) oscillations requires that a downward motion of the roadway produces a downward directed lift force and upward motion of the roadway produces upward lift forces. This response is opposite to what one would expect. If, for example, a flat horizontal plate is moved downward, the air resistance acts upward and tends to damp the motion. However, if a vertical plate of considerable area is attached in front to the same horizontal surface, the direction of the response of the air stream becomes inverted and the structure encounters negative damping. In general the occurrence of negative damping depends on the shape of the cross section; for example, a rod with semicircular cross section oscillating in the normal direction to the air stream encounters positive damping if the round half is directed upstream and negative damping if its flat surface is exposed to the wind stream. Similarly a rod of triangular cross section with the pointed edge upstream has positive damping and with the pointed edge downstream has negative damping.

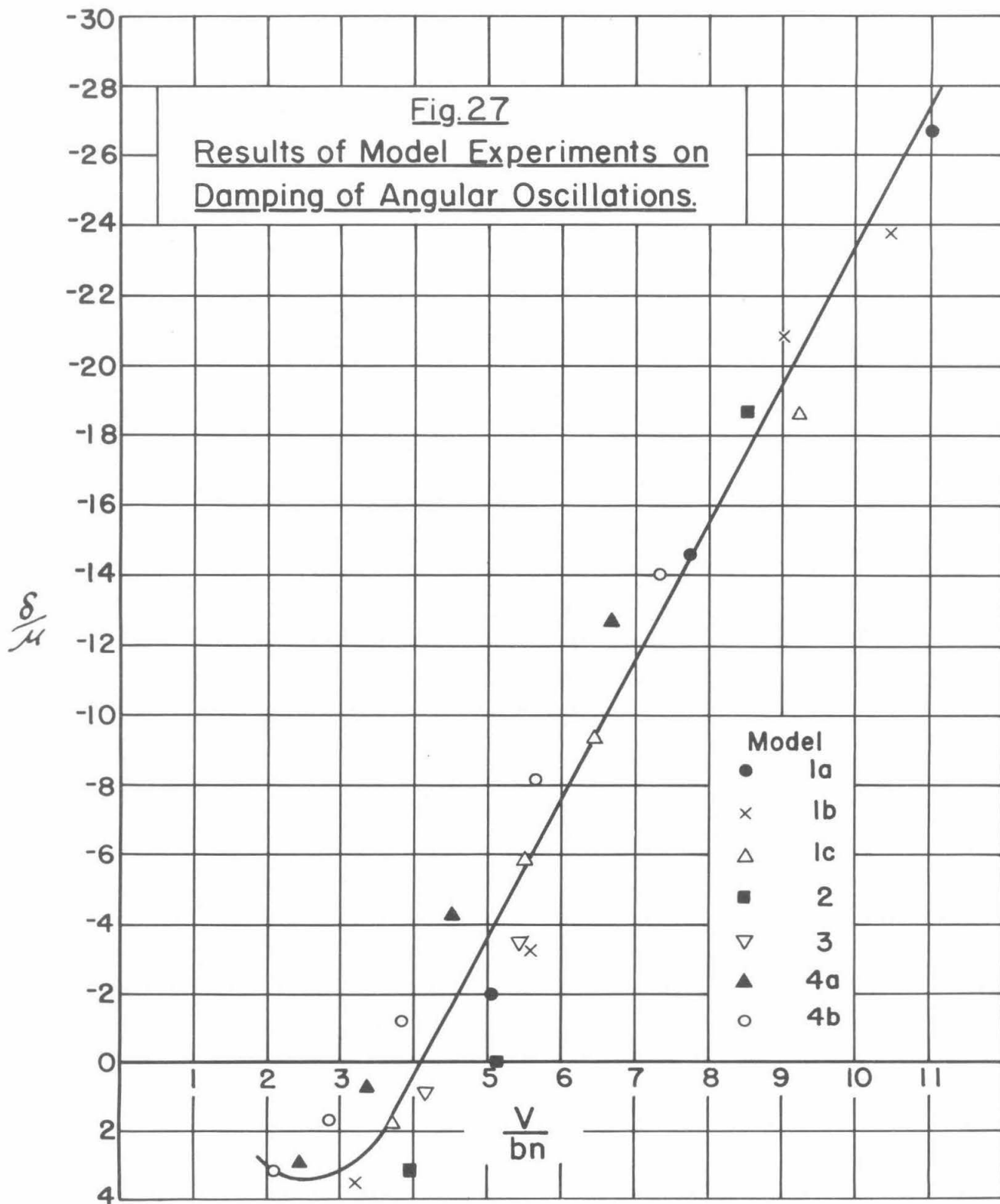
The case of negative damping of rods of different cross section oscillating perpendicular to a wind stream has been widely investigated and applied to the explanation of such phenomena as the galloping of power conductor lines (25). It has been shown that the dynamic behavior of such structures can be correlated with the static lift forces in the following way: The force produced by a downward motion of a body can be considered equivalent to the force exerted by an upward directed wind



stream and the force produced by an upward motion of the body to the force exerted by a downward directed wind stream on the body at rest. Therefore, if an upward directed wind produces upward directed static lift, positive damping results, and if, for some reason, the direction of the static lift is inverted, negative damping results.

This consideration is exact for relatively slow oscillations; as a first approximation it is applied here to the oscillation of bridge structures, using the wind tunnel data represented in Fig. 24. The results of such calculations are given in Fig. 26.

The logarithmic decrement resulting from aerodynamic damping is calculated for two cases. In the first case a wind velocity of 10 ft. p. sec. (6.8 m.p.h.) is assumed. The frequencies for the two bridges are 12 and 8.7 per minute, respectively. It appears that the aerodynamic damping is a function of the amplitude. The curve (a) refers to the Golden Gate Bridge, the curve (b) to the Tacoma Narrows Bridge as built, curve (c) to the same with 13 ft. plate girders, and curve (d) to the latter structure with fairings on the girders. It is interesting that in the case (c) oscillations with amplitudes less than 18" have negative aerodynamical damping, but oscillations with larger amplitudes are aerodynamically damped. The aerodynamic damping of the two bridges as built is of the same order of magnitude. In the second case the calculation is made for 60 ft. p. sec. (41 m.p.h.) wind velocity and the higher modes of oscillations which were found to prevail preferably at higher speeds. The calculated values give the rate of dissipation of energy by aerodynamic forces only; they do not include any structural or frictional damping. It should be noted also





that using the results of wind tunnel tests at the University of Washington a negative damping of the order of magnitude of that calculated for the case (c) would be obtained for the Tacoma Narrows Bridge as built.

Aerodynamic Forces Produced by Angular Oscillations. The most important aerodynamic effect induced by the torsional oscillation of the roadway is the moment with respect to the axis which may act as positive or negative damping. Since no information was available on this question, the aerodynamic experiments described in Appendix VIII were conducted at the California Institute of Technology with a 1:80 scale model of a section of the suspended structure of the Tacoma Narrows Bridge, in order to determine the direction and magnitude of the moment induced by the oscillatory motion of the structure.

The model used in these experiments and described in Appendix VIII was elastically suspended in a horizontal wind stream. The weight of the suspended structure and the stiffness of the elastic suspension was varied, and the decrease or increase of angular oscillations was determined for each case by motion picture records. It was found in all cases that the aerodynamic damping is positive below a certain velocity and becomes negative beyond this limit. From the experiments the rate of energy dissipation and input was calculated.

The dimensional analysis given in Appendix VIII shows that the logarithmic decrement is a function of two non-dimensional parameters. The first parameter is characteristic of the structure; it is called the density ratio and is expressed in the form  $\mu = \frac{W_a b^2}{W}$ , where  $W_a$  is the specific weight of the air,  $b$  the width and  $W$  the dead weight of the structure per unit length. The density ratio for the Tacoma

Narrows Bridge is equal to 0.0207; for the Golden Gate Bridge 0.0288. The second parameter is the reduced velocity defined by the ratio  $V_r = V/bn$ , where  $V$  is the wind velocity, and  $n$  the frequency of the angular oscillations. The logarithmic decrement  $\delta$  is proportional to the parameter  $\mu$  and is a function of  $V_r$ . Hence the ratio  $\delta/\mu$  is a function of  $V_r$  only. This result of the dimensional analysis is confirmed by the experiments with fair agreement. In the model experiments the cross-section of the Tacoma Narrows Bridge as built was simulated and both  $\mu$  and  $V/bn$  were varied over a wide range. In Fig. 27 the ratio  $\delta/\mu$  is plotted as function of  $V_r$ . The experimental points give a well defined curve. The scatter probably results from experimental difficulties especially to the yawing of the model at the higher velocities. In Fig. 28 the average curve obtained from the data plotted in Fig. 27 is applied to the prototype and the logarithmic decrement for the bridge section is plotted as function of the wind velocity in m.p.h. According to this curve the damping becomes negative above a wind velocity of 25 m.p.h. Below this limit there is a slight aerodynamic positive damping; beyond it, the negative damping increases more rapidly than the wind velocity.

The application to the prototype is based on the assumption that the flow pattern around geometrically similar bodies is geometrically similar, independent of scale. Deviations from this similarity are in aeronautical practice called scale-effect, or Reynolds number effect. In general the scale-effect is very considerable if the flow is governed by the viscosity of the air and less significant when the dynamic pressures control the flow. The probable error made in transferring the results from models to prototype both in the case of the vertical

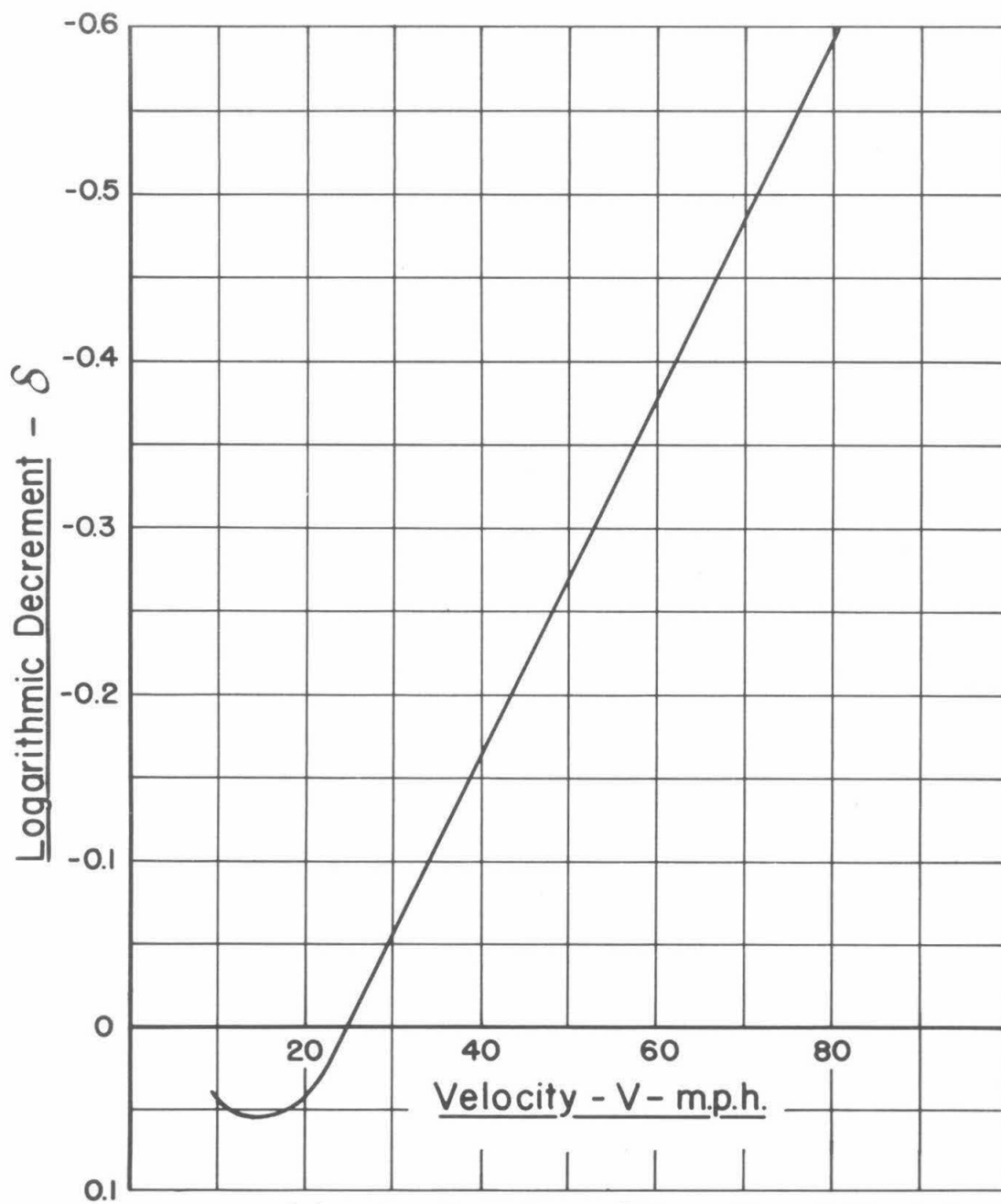


Fig.28 - Application of Results of Model Experiments to the Prototype - Tacoma Narrows Bridge.

and angular oscillations, can only be determined by further experimental investigations.

In any case the effect revealed by the model experiment has an important bearing on the development of the large torsional oscillations which preceeded the structural failure of the Tacoma Bridge. It appears that the wind velocity at which the aerodynamic damping changes from positive to negative is proportional to the width of the structure and the frequency of oscillation. This indicates that torsional oscillations are especially critical in the case of narrow bridges. The mechanism of the torsional instability is probably connected with the formation of alternating eddies created by the incipient torsional motion as shown in Fig. 25 d, e, f.

Characteristics of Suspension Bridges Which Determine Their Deflections Under Dynamic Loads.

There are essentially three characteristic properties of an elastic system which determine the amplitude of its oscillations within the elastic limit under dynamic loads; rigidity, inertia and damping capacity. The relative rigidities of various suspension bridges due to cable reaction and girder stiffness were discussed in Chapter III. The rigidity determines the deflection under dynamic loading if the latter consists of a sudden application of a certain load or force. It is known that in this case the maximum deflection is twice the static deflection produced by the same load.

Another type of dynamic loading, for which the theory gives a simple answer consists of the application of a force over a length of time which is small in comparison with the natural period of oscillation. In this case an impulse of certain magnitude is transferred

to the elastic system. The resulting maximum amplitude, when damping is neglected, is inversely proportional to the inertia of the system, expressed by mass and moment of inertia, and the frequency of the natural oscillation excited by the impulse.

The application of periodic forces is a third type of dynamic loading. In this case the maximum deflection is equal to the static deflection multiplied by an amplification factor, which depends on the ratio of the frequency of the applied periodic load to the natural frequency of the system and on its damping. In case of resonance, i.e., if these frequencies coincide, the factor of amplification is inversely proportional to the logarithmic decrement of the free oscillation of the system.

In the case of completely random loads or forces, the deflection is essentially determined by the mass and frequency. Because by the completely random character of the load application the probability of development of resonance is very small. However, if the random forces can be considered as composed of superposition of periodic forces, acting in proper phase relation over a short length of time, partial resonance effects can develop and the lack of damping has great influence on the amplitude of the oscillations. Experience shows that the aerodynamic forces due to the turbulence of natural wind are of this type. Hence, the damping characteristics of a bridge, both structural and aerodynamic, greatly influence the amplitudes under the action of aerodynamic forces.

Rate of Energy Dissipation in Suspension Bridges. The structural damping of a suspension bridge is by its very nature much smaller than that

of a bridge of the beam or arch type. In each half cycle of an oscillation of any system the kinetic energy is converted into potential energy and again into kinetic energy. In the case of a vibrating beam the potential energy consists entirely of the elastic stress-strain energy. The case of a suspension bridge resembles to some extent a pendulum, in that a very considerable part of the kinetic energy is converted into gravitational energy corresponding to lifting of the center of gravity of the compound system. The ratio of this part of the energy to the total energy depends on the design, especially on the relative contribution of the cable system to the stiffness of the whole bridge. In a flexible suspension bridge this contribution of the cables is relatively very large in comparison to the contribution of the stiffening girders and other parts of the structure.

Structural damping is provided essentially by hysteresis of the structural material subjected to variable stresses and by friction due to small relative displacements in joints, rivets, and in such composite elements as the cables and the floor. The hysteresis effect in a material is roughly proportional to the stress-strain energy which is periodically introduced in the material and converted again into kinetic energy. Hence, in a beam the hysteresis effect is proportional to the total kinetic energy of the oscillating system, whereas in a suspension bridge it is proportional only to a small portion of the total kinetic energy; namely to the portion which is equal to the stress-strain energy of the girders, or trusses, and the floor. In addition some damping results from the bending of the cables, the fluctuating cable stress, the varying stressing of the towers and

suspenders and from friction in the joints.

Few data are available for the value of the logarithmic decrement due to the structural damping of a suspension bridge. Values of the general order of 0.01 have been derived in Appendix VII from experiments of Prof. Farquharson on the dynamically similar model of the Tacoma Narrows Bridge. In the analysis, the relative contributions of various components to the structural damping have been estimated. It was found that the logarithmic decrement increases with the increasing contribution of the girders to the resulting damping of the whole system. The experiments made on the Bronx-Whitestone Bridge model lead to a value of the order 0.05. This higher value probably arises in part from friction in the recording device. While the absolute quantitative values require further confirmation, it appears that, compared with the average value of 0.3 given by Bernhard (23) for bridges of ordinary types, the values between 0.01 and 0.05 for suspension bridges of the flexible type appear reasonable.

Analysis of the Vertical Oscillations. The observations of the Tacoma Bridge oscillations recorded in Tables 1 and 2 show that in most cases one definite mode of oscillations prevailed over a certain length of time. However, the modes frequently changed. In order to decide whether there was a correlation between the wind velocity and the prevailing modes a statistical analysis of the available material was made. The result of this analysis is shown in Fig. 29. The wind velocity is plotted as abscissa, the frequencies of the prevailing modes are plotted on the axis of the ordinates. The heights of the blackened areas are proportional to the number of observations showing

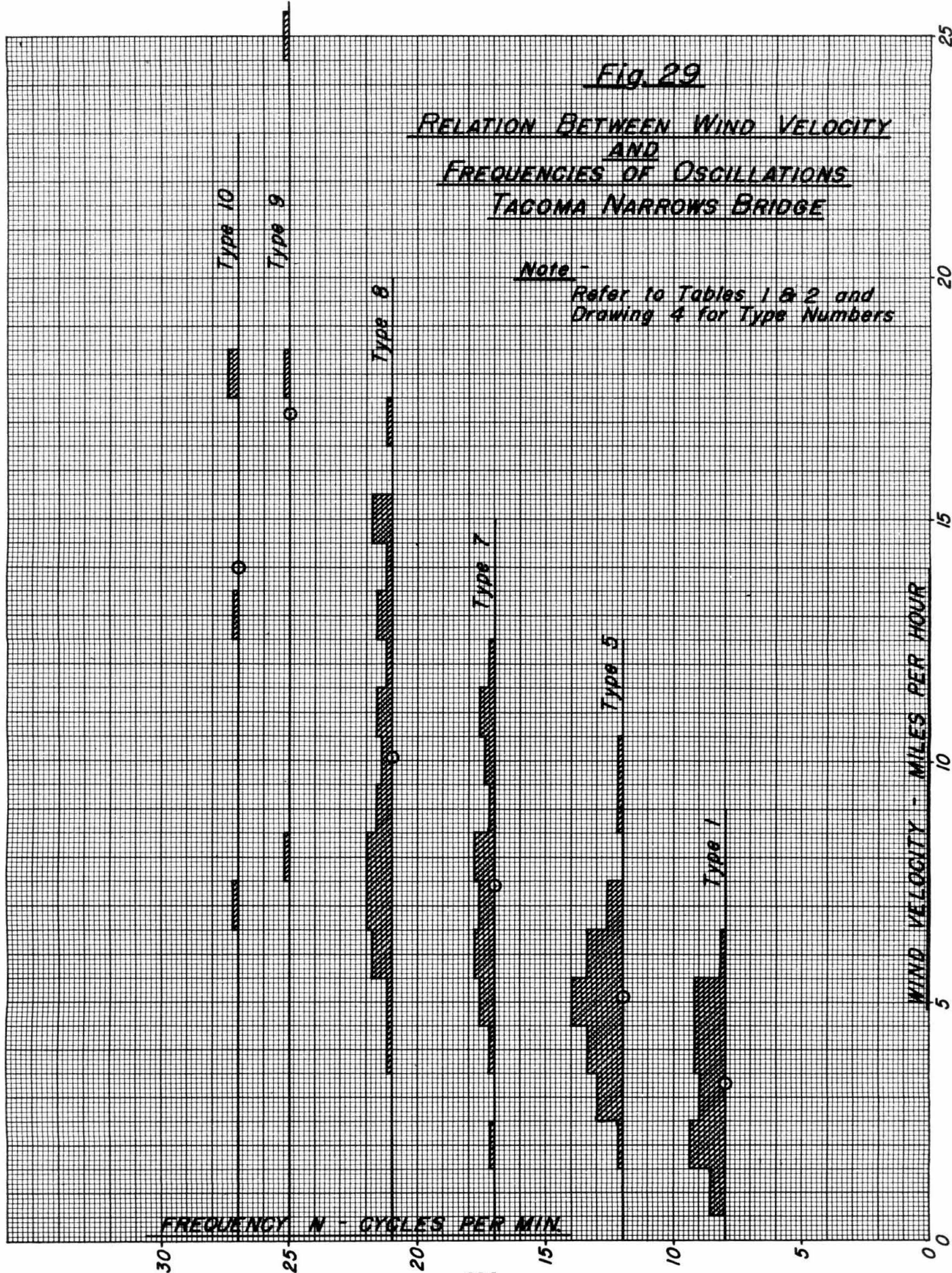


Fig. 29

RELATION BETWEEN WIND VELOCITY  
AND  
FREQUENCIES OF OSCILLATIONS  
TACOMA NARROWS BRIDGE

Note -

Refer to Tables 1 & 2 and  
Drawing 4 for Type Numbers



a particular mode of oscillation at a particular wind speed. The circles represent the abscissae of the centers of gravity of the blackened areas; the corresponding wind velocities can be considered as the most favorable ones for the particular modes of oscillation. It is seen that a definite correlation exists between frequencies and wind velocities; higher velocities favor modes with higher frequencies. The observations extend over the range between zero and twenty-five miles per hour. In this range the most favorable wind velocities are closely proportional to the frequencies of the excited modes. On the day of the failure the prevailing wind speed was of the order of 40 m.p.h., the frequency of the vertical oscillations is reported to have been between 36 and 38. This observation also confirms the general result that the frequency, with corresponding higher modes, increases with the wind speed.

The oscillations of a suspension bridge may be considered as forced vibrations of a system with a large number of degrees of freedom under action of variable forces. Mathematical calculation of the amplitudes resulting from this forced vibration is impossible because the magnitude and distribution of the acting forces is unknown. However, the typical cases of dynamic loading considered earlier in this chapter give some indication as to the relative amplitudes to be expected. It was shown that the amplitude produced by a single impulse of given magnitude or by random impulses of a given average magnitude is inversely proportional to the mass (or moment of inertia) of the system and the frequency of the induced oscillation. This explains the apparent paradox that the amplitudes of the vertical oscillations

recorded at low and high wind velocities were of about the same magnitude, so that apparently there is correlation between wind velocity and amplitude.

It was stated above that the main effect of the wind turbulence on the bridge structure consists of lift forces produced by fluctuations of the wind direction. If it is assumed that the relation of the fluctuations to the average wind is the same at different velocities, it follows that the lift forces are proportional to the square of the wind velocity. Also, the prevailing frequencies of the fluctuations increase approximately in proportion to the wind velocity, so that the time of the action of an individual impulse is inversely proportional to the wind velocity. It follows that the magnitude of the random impulses is proportional to the wind velocity. Assuming that the average amplitude of the oscillation produced by the random impulses is proportional to the average magnitude of the impulses and inversely proportional to the frequency, which itself increases with the wind speed, it is seen that the amplitude must be approximately independent of the wind speed. The energy stored in the oscillating structure is proportional to the square of the amplitude and the square of the frequency. Hence, the same result can also be expressed in the form that the energy of the oscillations is proportional to the energy of the wind fluctuations which create the oscillations.

The correlation between the oscillation frequency and wind velocity, especially the fact that higher modes are favored by higher wind velocities, is sufficiently explained by the fact that the turbulent velocity fluctuations can be considered as composed by superposition of

periodical fluctuations and the fluctuations of higher frequency are preponderant at higher wind velocities.

It is seen that the fundamental characteristics of the oscillations observed in the Tacoma Narrows Bridge can be satisfactorily explained by the action of turbulent wind forces. However, the question arises as to the cause of the relatively much larger amplitudes in this bridge in comparison with much smaller amplitudes observed in other bridges. It has been suggested (9) that the vertical oscillations of the Tacoma Narrows Bridge were amplified by negative aerodynamic damping, i.e., they were self-excited oscillations. This suggestion is derived from wind tunnel experiments made at the University of Washington, but is not confirmed by the tests described in Appendix VIII. The diagrams, Fig. 26, show that the aerodynamic damping characteristics of the Golden Gate and Tacoma Narrows Bridges, as far as the magnitude of vertical forces produced by vertical oscillations is concerned, are only slightly different. It is believed that:

a. The evidence for the occurrence of negative damping in the case of vertical oscillations or undulations of the Tacoma Bridge is at least questionable. However, there is definite evidence that deep plate girders can cause negative damping in vertical oscillations of the roadway structure. The negative damping is restricted to oscillations of moderate amplitudes, but is of about the same order as the structural damping of suspension bridges of the degree of flexibility of the Tacoma Bridge. Consequently, such bridges are very sensitive to aerodynamic effects which depend on the aerodynamic characteristics of the cross section.

b. The difference between the amplitudes observed in the Tacoma Bridge and those of other bridges can be explained by the larger deflections under static forces of the same magnitude, by the smaller weight and especially by the smaller structural damping of the Tacoma Bridge. That damping, in addition to flexibility and inertia, must be one of the deciding factors for the magnitude of the amplitudes

follows from the small order of the lift forces which produce the oscillations. For example, at 4 m.p.h. wind velocity, assuming that the lift has its maximum possible value over the whole center span, one obtains a static deflection of 0.16" at the center. The observed amplitudes of the order of 18 to 24 ins. must have resulted either from negative dynamic damping which may or may not have been present, or from resonance effect produced by the semi-periodic character of the turbulent wind indicated earlier in this chapter. In either case, the resulting average amplitudes are governed by the structural damping. The amplitudes observed in the Tacoma Narrows Bridge and the Whitestone Bridge are about in the same ratio as their deflections under the same static load. It seems that the average magnitude of the impulses resulting from turbulence does not increase materially with the width of the bridge. A narrow bridge is probably susceptible to the effect of gusts of smaller size, whereas, the actions of small gusts are, to some extent, counteracted in the case of a wider bridge. The relative steadiness of the Golden Gate Bridge, at least at moderate wind velocities, may be explained by this fact and especially by its larger structural damping.

c. The "Streamlining" of the Tacoma Narrows Bridge would increase its aerodynamic damping in the case of vertical oscillations as shown in Fig. 26. However, the wind tunnel test reproduced in Fig. 24 shows that the lift forces produced by an inclined wind, and therefore by wind turbulence, are also increased by application of the fairing. In fact, it is the increase of the lift forces that provides the increased aerodynamic damping. It cannot be decided without further investigation whether streamlining or fairing is really an advantage. It all depends whether the increased damping provides sufficient compensation for the increased energy input. It was mentioned above that the fairing of the girders causes an unfavorable distribution of the static wind forces in the floor to the effect that the stability furnished by the cable reactions in case of tilting of the floor decrease with higher wind velocity.

Analysis of the Torsional Oscillations. The aerodynamic investigations reported in Appendix VIII give definite evidence of self-induced torsional oscillations for the cross section of the Tacoma Bridge beyond a certain critical wind speed. This behavior is not a special characteristic of a cross section consisting of a floor and two plate girders; practically all cross sections used or proposed for the design of suspension bridges show the same general behavior insofar as torsional oscillations are concerned. Fig. 28 shows the magnitude of the negative damping for the



case of the Tacoma Bridge computed from the model experiments under assumption of similarity of the flow pattern for model and prototype. This assumption involves that Reynolds number (22) has no essential influence on the flow pattern and the positive or negative damping resulting from it. According to Fig. 28 the negative aerodynamic damping appears as soon as the speed exceeds 25 m.p.h. The negative damping has to be subtracted from the structural damping of the system; increasing amplitudes occur only when the negative aerodynamic damping is larger than the structural damping which, of course, is always positive. Hence, the magnitude of the "critical velocity" beyond which the amplitude actually increases depends both on the aerodynamical and structural damping of the system. In the model experiments the only structural damping was from springs and cables since the roadway oscillated as a rigid body. Under these conditions the critical velocity in many cases was only slightly above that at which the aerodynamic damping becomes negative. If it is assumed that the critical velocity at the time of failure was around 40 m.p.h., it may be calculated from the values of negative aerodynamic damping given in the Figs. 27 and 28 that the logarithmic decrement resulting from the structural damping of the prototype was about 0.10. This value is about ten times that for the vertical oscillations as determined by the experiments described in Appendix VII. While sufficient data do not exist to confirm these absolute values, it seems reasonable that higher values of the structural logarithmic decrement would occur in torsion than in vertical motion since the torsional movements involve distortions of the comparatively rigid wind truss and of the concrete floor. It is

known that the logarithmic decrement of concrete is considerably higher than that of steel especially if large distortions occur.

Since the magnitude of aerodynamic damping, positive or negative, is a function of the ratio  $V/bn$  (velocity to the product of width and frequency) it follows that for similar designs the critical velocity increases with the width of the roadway structure. The danger of self-induced torsional oscillations is most critical for narrow suspended structures.

The self-induced torsional oscillations are more dangerous than self-induced vertical oscillations. The latter are limited in their amplitudes and their amplitudes increase rapidly with increasing wind velocities. The self-induced torsional oscillations, if started, reach very large amplitudes; they cannot develop below a certain wind velocity, and become very violent within a relatively narrow speed above this limiting speed. The amount of energy input by the negative damping can be estimated from the model experiments. For  $\pm 45^\circ$  amplitude one obtains a figure of the order of 5,000 horsepower.

There appears no doubt as to the self-induced character of the torsional oscillations of the Tacoma Narrows Bridge. However, two questions arise in connection with this statement: (a) why the self-induced oscillations did not appear at earlier occasions when high wind velocity prevailed; (b) why have self-induced oscillations not been observed in other modern bridges.

(a) It appears that the fundamental torsional mode is largely prevented by the resistance at the towers against their being twisted around their vertical axes. Such a distortion is necessary for large out of phase motions of the cables in the fundamental mode. The mode with one node at the center was probably prevented by the action of

the center guys, short suspenders and other torsional resistance of the floor, which efficiently hindered the differential motion of the cables near the center of the main span. Such differential motion is unavoidable in the case of an asymmetric mode of vibration with the two cables oscillating in opposite phase. The critical velocity for more than one node is higher than the maximum wind velocity observed at any time before the failure of the bridge.

The fact that, after the beginning of a torsional motion, the angular amplitude reached the value of  $\pm 45^\circ$  in a very short time, indicates that the wind velocity was above the critical limit and previously the development of torsional oscillations was suppressed, by the internal resistance of the structure.

(b) The comparatively low critical velocity is characteristic for bridges with narrow width. Taking into account their greater width and higher structural damping it will probably be found that the other long span suspension bridges do not develop self-induced oscillations within the range of probable wind velocities. However, a theoretical and experimental investigation of this point would be highly desirable. It is worthy of mention that violent torsional motions were observed in older bridges which failed in wind.



## CHAPTER V

### GENERAL CONCLUSIONS

The preceding chapters of this report have been devoted to the description of the observed oscillations on the Tacoma Narrows and other bridges, to a review of the design, a comparison of the physical properties of several of the larger suspension bridges and to an analysis of the action of the Tacoma Narrows Bridge under wind forces, as far as that is possible with present knowledge. Based on this investigation we have presented general conclusions on the causes underlying the failure and certain deductions bearing upon the design and safety of structures of this type.

In some respects the evidence is incomplete and not conclusive. The failure of the Tacoma Narrows Bridge has inspired a large amount of thought among engineers who are interested in structures and among those who are studying problems of aerodynamics and vibrations. Research on these matters is being carried on at several places and it is almost certain that it will produce additional information and evidence that has escaped our attention. To a large extent, therefore, our conclusions are subject to the development of further data.

While we have thoroughly studied and analyzed the design of the Tacoma Narrows Bridge and compared it with similar structures, and, as a result thereof, we have been able to indicate certain lines along which an adequate design may be rationally developed, we have made no attempt to setup any definite rules for the design of such structures. In this chapter some of the conclusions reached in the previous chapters

will be summarized, also with regard to the bearing of our findings on future research and development of the art.

Aerodynamic Stability. Although aerodynamic forces (wind forces) always have been considered in the design of suspension bridges the failure of the Tacoma Bridge introduced new aspects of the problem of wind action in suspension bridge design, namely, the possible influence of form of the cross section on the oscillation characteristics of the bridge and the possibility of aerodynamic instability.

In the discussions on the subject two sources of aerodynamic instability were suggested. One suggestion refers to the stability of the bridge during vertical oscillation, the second to the torsional stability of the bridge. In addition, resonance with alternating periodic eddies (sometimes called Kármán vortices) has been mentioned as a possible inducement to large amplitude oscillations.

1. Concerning instability of vertical oscillations it has been suggested that it was caused by the form of the cross section used in the Tacoma Bridge. Our investigation shows that in fact such instability, or negative damping, is correlated to the form of the cross section, but for the specific cross section used in the Tacoma Bridge the evidence for instability is not conclusive. However, similar narrow cross sections with deeper plate girders definitely can have negative aerodynamic damping. On the other side our investigation shows that:

(a) The aerodynamic instability of this type does not cause the amplitude of the oscillation to increase indefinitely and a slight aerodynamic instability can be overbalanced by ample structural damping.

(b) In the specific case of the Tacoma Narrows Bridge the oscillation characteristics of the bridge can be explained satisfactorily as forced vibration excited by random action of turbulent wind. This assumption explains the correlation between modes of oscillation and wind velocity, namely, the preference for higher frequencies at higher wind velocities and the fact that amplitudes of the same order occurred both at low and high velocities.

(c) The difference in the unusually large amplitudes of oscillations of the Tacoma Bridge, as compared to those observed in other large suspension bridges, might be caused by some degree of negative damping, but the flexibility of this bridge and its exceedingly small amount of structural damping furnishes sufficient explanation.

(d) It is at least doubtful that streamlining is the desirable solution for elimination of large oscillations or aerodynamic instability.

Nevertheless, it is highly desirable to determine for various structures the influence of their cross section form on their aerodynamic characteristics. It is especially desirable to have more information on the comparison between deep plate girders and truss girders, concerning their effect on aerodynamic stability and damping. The wind tunnel tests made at the University of Washington have indicated that the aerodynamic characteristics of the cross section are sensitive to the ratio of open and solid portions of the frontal area.

2. As to torsional instability, it has been suggested that the static moment of the wind pressure on an inclined surface, such as the floor of a bridge when tilted about its axis, acts against the restoring

moment of the cables, and at sufficiently high wind velocity it can cause an overturning of the floor. The wind tunnel tests indicate that this effect did not exist in the case of the Tacoma Bridge since the static moment acts in a favorable direction. However, it may occur in bridge structures of certain shape; it seems to exist for a streamlined section.

However, convincing evidence has been found by oscillatory tests that beyond a certain wind velocity negative aerodynamic damping is to be expected in almost any suspended bridge structure when it oscillates torsionally. The limiting velocity is relatively low for narrow bridges. For similar geometrical forms it increases with the width. The determination of the critical velocity as function of the form and dimensions of the cross section is highly desirable, although up to the present no signs of instability have been discovered in the wider and more rigid structures. It is also important to determine means and devices which prevent torsional oscillations. It appears that the effect of the aerodynamic instability is suppressed if the structure has ample structural damping. Its appearance and destructive effect in the Tacoma Narrows Bridge resulted from its narrow width and lack of structural damping.

3. It is very improbable that resonance with alternating vortices plays an important role in the oscillations of suspension bridges. First, it was found that there is no sharp correlation between wind velocity and oscillation frequency such as is required in case of resonance with vortices whose frequency depends on the wind velocity. Secondly, there is no evidence for the formation of alternating

vortices at a cross section similar to that used in the Tacoma Bridge, at least as long as the structure is not oscillating. It seems that it is more correct to say that the vortex formation and frequency is determined by the oscillation of the structure than that the oscillatory motion is induced by the vortex formation.

Causes of Slipping of the Center Cable Band and its Effect on the Failure of the Bridge. The evidence appears conclusive that the failure of the Tacoma Narrows Bridge was closely associated with the slipping of the cable band at the center of the north cable. After a study of all the evidence we believe that the slipping of the cable band can be explained in one of the following ways.

1. The wind had reached critical conditions under which torsional oscillations of large amplitude were imminent. The center ties resisted the formation of these oscillations and the forces were sufficient to result in the slipping of the north cable band and, perhaps, to break one of the guys of the south center tie.
2. The stresses resulting from the large vertical oscillations in the center ties were sufficient to force the slipping of the north cable band. The result was to transfer all the inertia forces of the suspended structure to the south center ties with a moment (in the general order of 5,000,000 ft. lbs.) which would have a tendency to establish torsion. At the same time one of the principal resistances to the torsional vibrations was released. With the torsional motion once started, the suspended structure being aerodynamically unstable with reference to torsional motions, the oscillations would build up very rapidly.

It has little practical significance as to which explanation is accepted since the results in either case would be the same. In substantiation of the latter explanation the following reasons may be mentioned:

- a. An analysis shows that stresses in the center ties probably existed up to the limit of resistance of the

cable band against slipping. The center ties and the short suspenders at the center resisted asymmetric torsional motions and would not have been affected by symmetric torsional motions.

- b. In the wind tunnel tests on oscillations, it has been observed that usually the structure resists a change from one form (i.e. from vertical to torsional or the opposite) of oscillations to another. While it is possible, and even probable, that the change did not occur as rapidly as indicated by the statements of the eye-witnesses, (Appendix V) it is certain that it occurred in a comparatively short time.

The progress of the failure, after the initial slipping of the band, is largely a matter of conjecture, aided to some extent by the records furnished by the motion pictures. Its effects have been discussed in previous chapters.

It must not be inferred from the above that the failure of the bridge would have been averted if the details of the center ties had been made stronger, or if center ties had not been installed. The evidence is conclusive that very high torsional forces existed and that their magnitude increased rapidly with the wind velocity. Even assuming that the bridge, without the slipping of the cable band, had weathered this particular storm, it is now apparent that it would have failed under wind velocities far below that of 100 m.p.h. for which it was designed.

Effect of Certain Elements of Structure on Dynamic Motions. This investigation has revealed clearly that the criteria for rigidity against dynamic forces differ from those usually considered for static rigidity. The results are far from sufficient to permit a complete analysis of the dynamic behavior of a suspension bridge, even in a qualitative sense, but they indicate certain characteristics the knowledge of which

is useful for a better understanding of the character of this type of structure.

It is evident that the dynamic characteristics of a suspension bridge are influenced by the weight and general proportions of the structure, as well as by the shape, proportions and even the quality of the materials of the composite parts, in a manner which is not reflected in the static deformations.

Side Spans and Towers. Observations of the Tacoma Narrows Bridge and on the models indicate that under certain vertical oscillations of the center span, particularly those in the higher modes, the side spans and towers remained practically motionless or oscillated with a different frequency. In other words, they did not always contribute to the dynamic motions of the center span. This is corroborated by the calculations in Appendix VI which show that in the higher modes no more energy is required for such oscillations with the tower tops fixed than with side span participation.

It appears that the side spans have greater influence on the torsional oscillations of the center span. The towers, however, offer much greater resistance to twisting than to bending parallel to the bridge axis and contribute therefore a material resistance to torsional oscillations of the bridge.

Girder Stiffness. The tests made on the Tacoma Narrows Bridge model (Appendix VII) indicate that girder stiffness has an appreciable damping effect on the suspension system, such effect increasing with the stiffness of the girders. The quantitative results are, however, not entirely



conclusive since neither the shape nor the materials used in the model to produce the desired stiffness represented fully the construction of actual bridges. Further investigations in this respect appear to be most important.

Weight. The same tests revealed that, while weight has great influence on the amplitude of oscillations, the latter being approximately inversely proportional to the weight under completely random impulses, the rate of damping decreases with increasing weight.

Stays. The effect of stay ropes on the dynamic rigidity of a suspension bridge of the Tacoma Narrows Bridge type has been mentioned in Appendix VI. That question also requires further investigation.

Remedial Devices. The remedial devices, the center ties, hydraulic buffers and holddown ropes installed in the Tacoma Narrows Bridge have been discussed in Chapter II. While beneficial to a limited extent they were largely ineffective because of the great flexibility of the structure.

Structural Damping. Comment has been previously made upon the gravitational action, similar to that of a pendulum, in the suspension system. By their very nature suspension bridges do not have as high a degree of structural damping as do those of a more rigid type. No exact data are available as to the amount of damping of structures as measured by the logarithmic decrement. Bernhard (23) states that a value of 0.30 is usual in ordinary bridges. This may be compared to 0.005 found for the model of the Tacoma Narrows Bridge as built.

The present investigation does not furnish definite quantitative

data, but it does give the following indications.

The University of Washington tests show:

1. The increase in the logarithmic decrement with increased moment of inertia of the stiffening girders.
2. The considerable damping effect of the oak floor in the model in spite of the fact that its contribution to stiffness is very small.
3. The marked effect of friction when applied to the stay cable rockers at the towers.

In discussing the torsional oscillations of the system, the marked increase in logarithmic decrement between vertical and torsional oscillations was noted and the suggestion made that a large part of the difference comes from the hysteresis effect of the concrete. Exact data are not available as to the necessary amount of structural damping. To at least some extent any deficiency in this respect may be compensated by weight. It is desirable in the design to provide as much structural damping as is possible without unduly increasing the cost. It is shown in Appendix VI that this can be done by increasing the rigidity of the stiffening truss or girders. While, as a result of this investigation it appears that the moment of inertia of girders of the Tacoma Narrows Bridge should have been many times that used in the design, it is advisable, for reasons of economy to keep the rigidity of the stiffening girders, or trusses, to a minimum. In past practice an effort has been made to keep the participation of the floor system, including the paving, in resisting live load stresses in the stiffening trusses, to a minimum. We believe that this practice should be further considered. It may be found that the advantages of this participation are greater than its

disadvantages. Similarly, an effort has been made in past practice to reduce friction at joints in the structure as much as possible. Modifications in this practice may be desirable.

Width of Structure. The effects of varying the proportions of the Tacoma Narrows Bridge have been thoroughly discussed in Chapter III. It seems desirable again to emphasize the importance of sufficient width. Width not only increases the resistance of the structure to torsion, but it is essential that the width of the structure be made such that self-induced torsional oscillations from wind cannot occur within the highest probable wind velocity.

General Conclusions on Location, Design and Construction. On our several visits to the site during the week of December 9th, 1940, we examined the location, the records of the soundings and the borings; the character of the materials and workmanship, as evidenced by the portions of the work still standing; and made a general inspection of the remaining steel work. As a result of these investigations, of our study of all available data and of the calculations, it is our opinion that:

1. The location selected formed the best possible connection to the highways on either side of the Sound and was as favorable in regard to length of span, foundation conditions, etc., as any other that might have been considered.
2. The suspension type of bridge was unquestionably the most suitable and most economical type for the purpose and the site. The failure of the bridge indicated no weakness inherent in such type when designed with an adequate degree of rigidity. Even with the additional expenditure which would have been required to attain this end the suspension type would have proven most economical.
3. The arrangement of spans and the location of the anchorages were evidently chosen after careful study of the geological and topo-

graphical conditions for the pier and anchorage foundations and the economics of the structure. These features are not subject to material improvement. Shorter side spans, and avoidance of the extension of the cables beyond them, would have furnished greater static rigidity. It would not have been advisable, however, to effect such a change by moving the anchorages closer to the banks of the river because of danger of sliding, and any increase in center span would have been more costly.

4. The capacity of the bridge, a two-lane roadway and two footwalks, appeared from the prospects of traffic development to be ample for many years to come, and as much as could be justified economically. A greater traffic capacity would have added to the cost. To take full advantage of the economy of the narrow roadway, however, the designers adopted an extraordinarily small width of structure as compared to its span. Their expectation that the bridge would have adequate lateral rigidity under wind pressure was unquestionably justified, but the combination of this narrow width with great vertical flexibility, proved to be responsible for the excessive torsion under aerodynamic action which led to the failure of the floor. This experience reveals the danger of excessive narrowness.
5. There can be no possible question that the quality of the materials entering the structure and the workmanship were of a high order. Considering the depth of water and the extreme currents, the construction of the piers was an outstanding feat. During the period preceding and during the failure of the bridge, various parts of the structure were subjected to heavier stresses than were anticipated in the design. The fact that failure did not occur sooner, and that the towers and cables remained standing, forms strong evidence that the materials and the workmanship were beyond possible criticism.
6. The changes from the contract design during the construction period were well considered and advisable.
7. All evidence points to thorough inspection on the part of the owners engineers and to thorough and competent general supervision of the work on the part of the Public Works Administration and the Reconstruction Finance Corporation.
8. The question of possible re-use of the remaining portions of the original structure is one, we understand, that falls within the scope of the Board of Engineers appointed by the Washington Toll Bridge Authority. Our opinions as to the amount of damage in the various parts have been given in Chapter II.

## ACKNOWLEDGMENTS

The preparation of this report was instituted by the Honorable John F. Carmody, Federal Works Administrator. Throughout its preparation, the Board has had the cooperation of the officials of the Public Works Administration, especially that of Col. M. E. Gilmore, Commissioner of Public Works, Mr. J. J. Madigan, Executive Officer, and Mr. Harry M. Brown, Director of Engineering. Mr. Brown placed all the engineering records at our disposal. Mr. L. R. Durkee, Project Engineer at Seattle, made all arrangements for our inspection at the site, and, throughout the investigation, has been most helpful in securing data.

Mr. Morton Macartney, and Mr. W. L. Drager, respectively Chief, Self-Liquidating Division and Chief, Engineering Division of the Reconstruction Finance Corporation, furnished an abstract of the engineering records of that Agency.

Throughout the investigation we have had complete cooperation from the Washington Toll Bridge Authority, their engineers and consultants. Mr. L. S. Moisseiff, Consulting Engineer, submitted his design calculations for the Tacoma Narrows Bridge.

Calculations and other investigations made in connection with the four other largest suspension bridges were placed at our disposal by the respective agencies who built and own them, the California Toll Bridge Authority, the Golden Gate Bridge and Highway District, the Port of New York Authority and the Tri-borough Bridge Authority.

Prof. F. B. Farquharson, of the University of Washington, has furnished us with data from both the dynamic model and the wind tunnel

tests. Several of the tests were made at the suggestion of this Board. This program was sponsored by the Public Roads Administration, the Washington State Highway Department and the University of Washington.

Mr. Russell Cone, Engineer of the Golden Gate Bridge, furnished the record of oscillations of the Golden Gate Bridge, Appendix IX and also a copy of the report on the wind tunnel tests made at Stanford University on a model section of the Golden Gate Bridge.

In connection with the calculations, the aerodynamic tests and other research work necessary for the report, the Board had the very able assistance of M. L. Balog who made the detail calculations for the data in Chapters II and III, of Mr. William D. Rannie, graduate assistant of the California Institute of Technology, who derived the equations for the vibrations of suspension bridges, Appendix VI and of Dr. Louis G. Dunn, instructor at the California Institute of Technology, under whose immediate direction the wind tunnel tests described in Appendix VIII, A and B, were conducted. Mr. R. Beeuwkes of the Case School of Applied Sciences made his flow pictures available for the report and Mr. Carl Thiele furnished the not wire anemometer records included in the report.

The wind tunnel tests reported in Appendix VIII, C, were conducted by the wind tunnel testing department of the Guggenheim Aeronautical Laboratory of the California Institute of Technology.

## APPENDIX I

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## APPENDIX II

REPORT OF MR. LEON S. MOISSEIFF TO MR. LACEY V. MURROW

### TACOMA NARROWS BRIDGE

Report 1

July, 1938

#### Preliminary Studies

The proposed bridge over Puget Sound at Tacoma is to have a roadway of 26 ft. width and two sidewalks of approximately 4 ft. each. The total centers of trusses or girders is 39 ft.

The preliminary design made by the Highway Department of the State of Washington shows a suspension bridge of 2600 ft. main span; two side spans of 1300 ft. each; a sag of 260 ft. and stiffening trusses 22 ft. deep. The two towers were made of unequal height to suit an approaching grade on the Tacoma side of 3-1/2% and of 4-1/2% on the west shore. The main span roadway has a vertical curve for its entire length. The difference in elevation between the two shore ends of the side spans is 19.5 ft., the Tacoma shore being the higher.

The bridge proper is reached at both sides by down grades of approximately 4%.

Unless there are very valid reasons which compel the making of the towers of unequal heights the towers should be of identical design and fabrication. Economic fabrication and good appearance demand it. The symmetry of the structure should be adhered to. Since both approaches lead to the bridge on down grades it should be easy to raise the west end of the bridge by 19.5 ft. and to make the grade of the bridge 3-1/2% with a vertical curve in the main span.

#### GEOMETRIC DIMENSIONS of SPANS and TRUSSES.

A study of the Highway Department design was made. It was found that the stiffening trusses as shown cannot effectively stiffen the bridge except at great cost. The side spans should be shortened and the versine of the main span should be reduced.

Assuming a truss of 1300 ft. span and 22 ft. depth with chord areas of 60 in.<sup>2</sup> each, a live load of 500 lb. per lin. ft. would produce a deflection of approximately 43 ft. To make the side span stiffening trusses effective the size of each chord area would have to be increased from 60 in. to 140 in.<sup>2</sup>. The trusses would then sustain approximately 40% of the live load. With details and stronger web member the increase in steel of the trusses would be 1600 lbs. per ft. of bridge which would

mean 4,000,000 lbs. for both side spans or, approximately, \$300,000. This brings out the fact that the stiffening trusses of the side spans are of little value unless greatly increased in size.

To approach the problem from another angle, the stiffening trusses may practically be omitted and the desired rigidity can be obtained by other means, shortening of side spans and reduction in sag ratio. It should be kept in mind that the \$300,000 which would be required to strengthen the trusses in the side spans permit an increase in each cable of approximately 45 in.<sup>2</sup>.

A study of all of the problems involved shows that it is best to attain rigidity by shortening the side spans and by a reduction of sag ratio. It is not only a better solution but also the cheapest.

#### COMPARATIVE DESIGNS.

Studies have been made of various tentative proportions of span and sag based on the same assumptions as to live load, wind, temperature and unit stresses. Concluding figures are given in the following:

##### Assumptions.

The Specifications of State Highway Officials for Floor Design and for Detailing Stiffening Trusses, Cables, Towers, and Anchorages have been used.

Congested Live load C = 1000 lb. per lin. ft. of bridge.  
Normal " " N = 700 lb. " " " " "

Wind 30 lb. per sq. ft. on suspended span and 50 lb. per sq. ft. on towers.

Temperature  $\pm 40^{\circ}$  F. from normal.

Loading combinations: D + C + T and D + N + W + T

##### Unit Stresses:

For Stiffening trusses silicon steel 26,000 lb. per sq. in.  
For Towers carbon steel maximum complete 24,000 lb. per sq. in.  
For Cables 84,000 lb. per sq. in.

The design shown by the Highway Department has a dead load of approximately 7,700 lb. per ft. of bridge including a 7 in. roadway slab. Of this amount 2,000 lb. is due to the weight of the stiffening trusses. The maximum horizontal cable pull is 14,150,000 lb. and the greatest stress per cable is 15,500,000 lb. The corresponding tower reaction is 12,000,000 lb. Each cable is composed of 19 strands at 328 wires = 6232 wires, or 188.03 sq. in. per cable. The diameter of each strand is 3.90 in. and that of the cable 17-5/8 in.

The greatest downward deflection of a side span is approximately 7.5 ft. for a live load of 500 lb. per ft. of truss at highest temperature. With no stiffening trusses the same deflection becomes 8.4 ft., indicating

ineffectiveness of side span stiffening trusses.

To stiffen the bridge vertically as well as transversely the main span has been increased to 2800 ft. and the side spans reduced to 1100 ft. The increase in main span was made by moving the "Tacoma" pier shoreward 200 ft. The width between tower shafts at the top of piers at elevation +30 has been reduced from 60 ft. to 50 ft. These changes have been given to Moran, Proctor and Freeman together with tentative pier reactions caused by deadload, liveload and wind. The approximate size of the base plates was also given.

Two designs are now in study to decide on the type of roadway slab to be used. The elevations of the roadway and those of the cables at center of main span and of shore ends of side spans are the same for both designs, as follows:

|                                   |              |
|-----------------------------------|--------------|
| Roadway at shore end of sidespans | Elev. 152.00 |
| Roadway at center of towers       | 190.50       |
| Roadway at center of main span    | 215.00       |
| Cables at shore ends of sidespans | 158.00       |
| Cables at center of main span     | 223.00       |

Low steel is slightly less than 5 ft. below the roadway level which gives a maximum clearance of 210 + ft. at center of main span for deadload at normal temperature. The roadway grade on the sidespans is 3-1/2%. The main span has a vertical curve.

For Design A an armored steel-concrete slab has been assumed weighing 58 lb. per sq. ft.

For Design B a 7 in. reinforced concrete slab of standard type has been assumed weighing 98 lb. per sq. ft.

A 3 in. sidewalk slab weighing 40 lb. per sq. ft. was taken for both designs.

For the present investigation a spacing of floorbeam of 30 ft. was taken with suspenders at each floorbeam. A further study will be made to determine the advantage, if any, of reducing the floorbeam spacing to 25 ft. and to attach the stiffening trusses at alternate floorbeams. This, however, will neither change the deadload nor the general conclusions deduced from this investigation.

#### FLOORSYSTEM.

The roadway slab, the steel curbs and the inside of the sidewalks are supported on transverse beams spaced 5 ft. for Design A and 6 ft. for Design B. The transverse beams rest on three longitudinal stringers which are attached to the floorbeams.

Attention is called to the steel curbs which are welded in sections

in the shop and field-riveted to the transverse beams. These serve as curbs as well as railings for the sidewalks. An opening of 2 to 3 in. is provided between the roadway and sidewalk slabs and the curb plates. This was done to prevent accumulation of dirt in the corners and for removal of snow.

All floor steel is carbon steel, as follows:

|  | Design A                       | Design B     |
|--|--------------------------------|--------------|
| Cross beams                                  | 14 WF 34 lb.                   | 14 WF 38 lb. |
| 2 outside                                    | 27 I 91 lb.                    | 27 I 98 lb.  |
| Stringers 1 center                           | 27 I 114 lb.                   | 30 I 124 lb. |
| Floorbeams 1 web 48 x 3/8, 4 Ls 6 x 6 x 1/2, | 2 pls. 1 web 48x3/8, 4Ls       |              |
|  | 14 x 1/2 6x6x5/8, 2pls. 14x1/2 |              |

#### STIFFENING TRUSSES.

The stiffening trusses are of silicon steel and the same for both designs. Each consists of a plate girder eight ft. deep, composed of 1 web. 96 x 1/2, 4 angles 8 x 6 x 1/2 and 2 cover plates 20 x 1/2 with a gross area of 95 sq. in. The web is reinforced by longitudinal as well as vertical stiffeners on the inside face only. On the outside face vertical stiffeners will be placed at floorbeams only. This will result in a neat and pleasing appearance.

In these comparative design studies it was aimed to produce equivalent designs; that is, that the deflections for identical causes are the same. This was closely accomplished by making the versine of Design B 14% greater than for Design A. For Design A a versine of 215 ft. and for Design B one of 245 ft. was taken, the sag ratio is thus approximately 1/13 and 1/11.4, respectively.

The deadload is as follows in lb. per ft. of bridge.

|                                 | Design A   | Design B   |
|---------------------------------|------------|------------|
| Sidewalk slabs                  | 340        | 340        |
| Roadway slab                    | 1508       | 2548       |
| Floorsteel                      | 910        | 970        |
| Stiffening girders and laterals | 1150       | 1150       |
| Cables, Suspenders, etc.        | 1480       | 1550       |
| Misc. and contingencies         | <u>112</u> | <u>142</u> |
| Total                           | 5500       | 6700       |

The pertinent data for the two designs are given in the following:

#### LIVE LOAD of 1000 lb. per ft. of BRIDGE

|                                 | Design A | Design B |
|---------------------------------|----------|----------|
| Max. horizontal cable pull kips | 14,693   | 15,340   |

|                                       | Design A | Design B                |
|---------------------------------------|----------|-------------------------|
| Max. cable stress kips                | 15,700   | 16,640                  |
| Make of cable 19 strands at 328 wires |          | 19 strands at 348 wires |
| Area of cable sq. in.                 | 188.03   | 199.5                   |
| Max. unit stress lb. per sq. in.      | 83,400   | 83,400                  |
| Dia. of each strand in.               | 3.90     | 4.0                     |
| Dia. of cable in.                     | 17-5/8   | 18-1/8                  |
| Max. deflection in sidespans ft.      | 6.1      | 5.7                     |

LIVELoad OF 700 lb. per ft. and 30 lb.  
TRANSVERSE WIND PRESSURE.

Transverse wind pressure:

|                   |                       |
|-------------------|-----------------------|
| On suspended span | 360 lb. per ft.       |
| On live load      | 200 lb. per ft.       |
| On cables         | <u>50 lb. per ft.</u> |
| Total             | 610 lb. per ft.       |

|   | Design A | Design B |
|---|----------|----------|
| Shoreward tower deflection in.              | 12.0     | 12.5     |
| Corresponding cable reaction per cable kips | 8810     | 10560    |
| Riverward tower deflection in.              | 15.5     | 15.6     |
| Corresponding cable reaction per cable kips | 9330     | 11090    |
| Transverse deflection center main span ft.  | 21.9     | 20.3     |
| Transverse deflection center side span ft.  | 4.2      | 4.1      |
| Wind reaction top of tower, kips            | 635      | 641      |
| Wind reaction at floor level "              | 553      | 547      |
| Wind on tower "                             | 465      | 532      |

Tower deflections include tolerance of 1 in. either way.

Probable location of tower top for dead load at normal temperature is 2 in. shoreward.

The towers will be carbon steel and of box type section with stiffener angles. To be explained later.

The tentative width of towers in longitudinal view is

|         | Design A | Design B |
|---------|----------|----------|
| At top  | 12 ft.   | 13 ft.   |
| At base | 18 ft.   | 19 ft.   |

The difference in the total amount of cable wire, wrapping and cable bands for the two designs is approximately 500,000 lb. for both cables or about \$75,000. This would be the increase in cost for the Cables of Design B using a 7 in. roadway slab. Comparative estimates of cost will be prepared and transmitted in time for discussion and adoption of final design.

Incidentally, the difference in cable wire between the Highway design and Design B is the same as the above, the latter having additional wire in the amount of \$75,000. Design B, however, has stiffening girders which are but half the weight of the original design. This is equivalent to a saving of 5,000,000 lb. or about \$350,000.

It should also be remembered that the plate girder design will command a price which will be about one cent per lb. less than for a truss.

## TACOMA NARROWS BRIDGE

### Report 2

#### Preliminary Studies

#### Proposed Design

The preliminary design studies which were discussed in the first report result in a total cost of superstructure of main bridge of \$2,998,000 for a 3-1/2 in. roadway slab and \$3,054,000 for the 7 in. slab for a spacing of suspenders of 30 ft. The corresponding steel in the main bridge is 13,050 tons and 13,630 tons, respectively. For a spacing of suspenders of 50 ft. the cost of the superstructure of the main bridge is reduced by approximately \$35,000 with a corresponding reduction in the total weight of steel. From a structural and aesthetic point of view it is more advantageous to use a 50 ft. spacing for the suspenders, apart from a saving in cost. Detailed estimates are given on Sheets Nos. 1 and 2 and general elevations, towers, cross sections and stiffening trusses are shown on Drawing No. T-1.

The cost estimates shown on Sheets Nos. 1 and 2 are based on a deadload of 5,500 lb. and 6,700 lb., respectively, as given in our first report. A re-checking of the floor system, stiffening trusses, et cetera, resulted in a saving of 300 lb. for each design which will reduce the cost at the unit prices given by approximately \$70,000. The unit prices assumed in the estimate are somewhat lower than those asked in the bids on the Whitestone Bridge which were let approximately a year ago. Steel prices have since declined and it is felt that the difference in cost between the two designs is but \$50,000 in favor of the design using a 3-1/2 in. roadway slab. This difference in cost is practically negligible and it is therefore recommended to use a standard reinforced



concrete slab. The above cost estimates do not include the approach spans between the end of the main bridge and the east anchorage. The cost of the superstructure of these spans is \$87,200.

It is our opinion that a 6 in. roadway slab will efficiently fulfill all the duties imposed on it by an H-20 loading. This slab may contain reinforcing trusses of 4 in. depth with 3/4 in. of concrete below and 1-1/4 in. above. This slab will weigh 85 lb. per sq. ft. and it is recommended in the proposed design.

Several additional studies were made to determine the most advantageous geometric proportions, taking into consideration the cables, towers, anchorages, et cetera.

It is very desirable to keep the height of the towers to a minimum due to the relatively great effect of transverse wind pressure.

The proposed design is given in detail on Sheets Nos. 3, 4 and 5 and a cross section with tentative size of stringers is given on Drawing T-2. An estimate of this design is given on Sheet No. 5. The total cost of the superstructure of the main bridge is \$2,906,000 and the total weight of steel is 13,100 tons. A change in unit price of steel of one-half cent is equivalent to approximately \$130,000.

The cost of the superstructure of the approach spans is \$85,900, giving a total cost of the superstructure of \$2,991,900, or, in round figures, \$3,000,000.

No information is available as to the foundation conditions of the anchorages. Assuming sand and gravel foundations for a tentative study of the anchorages, a co-efficient of friction of 0.3 was introduced into the calculations. This value corresponds to a factor of safety of 2 which is very ample taking into consideration the uniform soil pressure of approximately 6 tons per sq. ft. With these assumptions the two anchorages will contain approximately 46,500 cu. yds. of concrete.

LEON S. MOISSEIFF  
Consulting Engineer  
New York

July 27, 1938.

Mr. Lacey V. Murrow, Director  
Department of Highways,  
Highway Building,  
Olympia, WASHINGTON

Dear Mr. Murrow:

Re: TACOMA NARROWS BRIDGE.

We send you herewith prints of Sheets Nos. 1 to 21, giving part of the general computations for the design of the bridge.

You will note that the elevation of the center line of cable at the shore end of the side spans has been lowered by a foot and a half to Elevation 156.5. This was done to effect a better detail for the anchorage saddles at the Tacoma side.

Each cable contains 19 strands of 332 wires each and has a diameter of approximately 17 in.

The tower deflections which were sent to you previously were computed for a live load of 700 lb. per ft. of bridge. This load has since been increased to 1000 lb. and the deflections are correspondingly greater. It is proposed to have a shoreward bending of 3 in. of the tower tops for deadload and normal temperature. The extreme deflections then are:

Greatest shoreward bending 18-1/2 in.  
Greatest riverward bending 17-1/2 in.

In computing the transverse wind, the rigidity of the suspended span was computed for two assumptions, namely, the stiffening girders to act without help from the roadway and sidewalk slabs and for the assumption that the whole bridge acts as a unit. The rigidity in the latter case is but 15% greater than for the effect of the stiffening girders alone.

The greatest vertical deflections due to a live load of 1000 lb. per ft. of bridge at highest temperature are 6.0 ft. in the side spans and 10.1 ft. in the center span.

The greatest lateral deflections due to a wind pressure of 30 lb. on the structure plus 200 lb. on the live load, are 3.4 ft. in the side spans and 20 ft. in the center span.

The suspenders are spaced every 50 ft. and will have an approximate diameter of 1-1/8 in. and a cross section of about 0.80 sq. in. The specified ultimate strength of one rope over a sheaf of approximately 17 in. diameter will be 280,000 lb. for both parts of the rope. This will give a factor of safety of 3.3 for the normal loading of H-20 and a factor of safety of 2.2 in case one suspender should be out of order.

We also send you a print of Drawing No. T-3 of the towers. Further details on the towers will be finished in a short time and will then be sent to you by airmail. This applies especially to the spacing of transverse stiffeners, brackets for the stiffening girder rockers and for the anchorage steel which is embedded into the piers.

We also send you prints of sheets Nos. 22 and 23 giving the size and make of the laterals.

Additional information will be sent as soon as possible.

Very truly yours,

LEON S. MOISSEIFF

APPENDIX III

LETTER FROM MR. J. J. MADIGAN, ACTING COMMISSIONER, PUBLIC WORKS  
ADMINISTRATION, RELATIVE TO THE PROCEDURE OF THE ENGINEERING  
DIVISION IN EXAMINING ENGINEERING FEATURES OF A PROJECT.

December 4, 1940  
Docket No. 1870-F (Wash.)  
Tacoma Narrows, Wash.  
Bridge  
Eng-1-A

Mr. O. H. Ammann,  
111 Eighth Avenue,  
New York, New York.

Mr. Glenn B. Woodruff,  
2709 Dwight Way,  
Berkeley, California.

Mr. Theodore von Karman,  
California Institute of Technology,  
Pasadena, California.

My dear sirs:

You requested a statement covering the procedure of the Public  
Works Administration with respect to

1. The examination of applications for loans and/or grants.
2. The supervision of the design and construction of projects to which funds from the Administration were allotted.

In response to your query, I submit the following:

Upon receipt, applications were submitted to and examined by our Legal, Finance and Engineering Divisions. Generally speaking, the matters considered by each division were such as might be indicated by the name of the division. In the present instance, I assume you gentlemen are interested primarily in the engineering aspects of our procedure.

The following items were reviewed by the Engineering Division:

1. Present facilities. The extent and character of any existing facilities similar to the project were studied to develop the necessity, proper relationship and practical coordination of the proposed facility.
2. The project. Outline plans and specifications were reviewed to determine the degree of development of the project, and its indicated

soundness from a technical point of view. In this connection, considerable weight was placed upon the past record of the designated architects or engineers. No attempt was made to analyze thoroughly the general or detailed design. Dependent upon the magnitude, type, location and other pertinent factors, the Engineering Division scrutinized and considered the personnel engaged by the applicant rather than the project itself. In the event it was disclosed that the project had not received adequate engineering study by recognized engineering or architectural personnel, the applicant was required to engage such technical service as would insure, beyond a reasonable doubt, the proper and sound development of the contemplated facility. I should state at this point, that, although in many instances, applicants were required to retain architectural or engineering services to supplant or supplement those earlier engaged, nevertheless in no instance did this Administration nominate, or express any preference for any particular individual, group or firm.

The cost of the project was reviewed to determine the adequacy of the requested funds. The progress schedule was studied to reveal any conflict with legislation governing starting and completing the work and to disclose any lack of proper planning. These features were also factors given consideration in securing a general picture of the soundness of the entire enterprise.

3. General plan. To assure proper coordination, to protect the interests of other bodies and parties, and to disclose any related weakness in the projected work the relationship of the proposed project to any general, state, or local plan was considered. Further, any objections, of knowledge or record were weighed and evaluated.

4. Economic soundness. In the event the application requested a loan which was to be paid from revenues to be derived, solely or in part, from the operation of the project, the economic features were studied and reported upon. Probable revenues, probable operating and maintenance costs, and other items of expense were forecast. The effect of competition and general policy matters were considered in this connection.

5. Employment. The classification and volume of employment were analyzed in order to have any expenditure conform with the broad plan of affording relief to unemployment.

In the event an allotment was made to a project, supervision by this Administration was in such degree as to assure the successful, harmonious construction, in accord with our rules and regulations, of a sound structure.

Construction plans and specifications were submitted by the applicant for review by this Administration. Again, much reliance was placed upon the architects and engineers who had been engaged by the sponsor. The review of such plans, specifications, and other contract documents was more to assure compliance with the Administration's requirements as

to hours of labor, rates of pay, open, competitive bidding, insurance, and similar administrative details than to review matters of technical design. In the event that unorthodox construction was disclosed, fundamental design principles clearly violated, poor choice of materials definitely evident, or other discrepancies were openly disclosed, the attention of the Owner was directed to such shortcomings and release of the plans for receiving bids withheld until such shortcomings were eliminated. A reasonable consideration of the wide scope of the Public Works program, involving the construction of over 16,000 projects at a cost in excess of \$4,000,000,000. will indicate quite clearly that to go beyond the review indicated above was definitely impractical, would involve the expenditure of several additional millions of dollars, and afford no commensurate protection.

I am attaching hereto a copy of our Form 230, Terms and Conditions, on pages 8 to 14 of which you will find our Construction Terms and Conditions. I also refer you to item (K) on page 2 of Form 230. As in the case of design personnel, this Administration required that the supervising and inspecting staff of the Owner be efficient and adequate. Again, this Administration refrained from designating any individual or firm as agent or agency to supervise or inspect the work or materials on behalf of the Owner. The activities of our field representatives, such as Project Engineers, Resident Engineer Inspectors and the like, were devoted to securing compliance with the above cited regulations, and not with the specific inspection, as such, of materials and workmanship.

With respect to the subject project, it must be remembered that the allotment of the Public Works Administration grant was made to the Washington Toll Bridge Authority, a body created by the State of Washington. It was indicated that the project would have the benefit of design and supervision of construction by the following:

The Director of Highways of the State of Washington and his staff.

Moran, Proctor and Freeman, Consulting Engineers.

Leon Moisseiff, Consulting Engineer.

Subsequently the Washington Toll Bridge Authority selected a Board of Consulting Engineers, comprising the following:

Mr. L. V. Murrow

Mr. C. E. Andrews

Mr. R. B. McMinn

Mr. R. H. Thompson

Admiral L. E. Gregory

It was, and still remains, the opinion of this Administration that the above named individuals, bodies and firms were eminently qualified by training, experience, judgment and integrity to direct and supervise closely the design and construction of the bridge over the Tacoma Narrows.

If you require fuller explanation or more complete detail with respect to any of the features discussed herein, I will be glad to answer your further request.

Sincerely yours,

/s/ J. J. Madigan  
Acting Commissioner of Public Works

## APPENDIX IV

### EXCERPTS FROM

#### REPORT OF T. L. CONDRON, SUPERVISORY ENGINEER

Docket No. PA 174      Eng. 466  
Application Received Sept.      1938  
Date of Report Sept. 21, 1938

1. NAME AND ADDRESS OF APPLICANT

Washington Toll Bridge Authority of the State of Washington,  
State Highway Department Olympia, Washington.

2. CHARACTER OF PROJECT

Toll Bridge over Tacoma Narrows at Tacoma, Washington.

6. PURPOSE OF LOAN AND SUMMARY OF APPLICANT'S STATEMENT

#### CONSULTING ENGINEERS TO WASHINGTON TOLL BRIDGE AUTHORITY

The Washington Toll Bridge Authority engaged on July 1938, as Consulting Engineers on sub-structure, the firm of Proctor, Freeman and Mueser, (Operating as Moran, Proctor and Freeman), 420 Lexington Avenue, New York, and as Consulting Engineer on Super-structure, Leon S. Moisseiff, C. E. (his associate, being Frederick Lienhard, C. E.) 99 Wall Street, New York. There is also a Board of Consulting Engineers for the Tacoma Narrows Bridge project appointed by the Washington Toll Bridge Authority, consisting of Charles E. Andrews, C. E. of San Francisco, Calif., Chairman; Luther E. Gregory, Rear Admiral U. S. Navy (Retired) of Olympia, Wash.; and R. B. McMinn, C. E. of Portland, Oregon.

The New York engineers were identified as designers of sub-structures and super-structures of the San Francisco-Oakland Bay Bridge and the Golden Gate Bridge. Mr. Andrews was "Bridge Engineer" during the designing and construction of the San Francisco-Oakland Bay bridge. Admiral Gregory, was formerly Chief of the bureau of Yards and docks, U. S. N. and also Commandant of Bremerton (Wash.) Navy Yard. Mr. McMinn is Bridge Engineer, U. S. Bureau of Roads.

The Report of this Board of Consulting Engineers to Mr. Lacey V. Murrow, Director of Highways, regarding the designs of the sub-structure and super-structure of the proposed Tacoma Narrows Bridge dated August 31, 1938 is appended hereto as Exhibit F.

## REPORT OF BOARD OF CONSULTING ENGINEERS

### Quotations & Comments

#### SUPERSTRUCTURE DESIGN

The report of this Board of Consulting Engineers comments at length on the Super-structure design as to its general features, but owing to lack of time had not attempted to check any of the calculated stresses and deflections. In this connection the report states:

"We have full confidence in Mr. Moisseiff, and consider him to be among the highest authorities in suspension bridge design."

In regard to the ratio of width of bridge to span, which is 1:72, they state:

"In our opinion this feature of the design should give no concern. -----In a long narrow span the force of the wind is to a large extent transferred to the cables and through them to the towers ----- . The greater the total dead load stress in the cables the more difficult it becomes to deflect them laterally and the less the side deflection of the bridge." -----In the proposed bridge, the dead load stress in the cables is approximately  $\frac{6}{7}$  of the total stress. ----- We believe that the present span could be materially increased if it were necessary, ----- . In consequence we have no concern as to the general features of the proposed design of the super-structure."

#### GENERAL COMMENTS ON DESIGN OF SUPER-STRUCTURE

The super-structure of this bridge has been designed by, or under the immediate direction of Mr. Leon S. Moisseiff, C. E. and his associate Mr. Fred Lienhard, C. E., of New York City. These two engineers have a very enviable reputation as experts in the calculation of stresses and deflections of suspension bridge structures. Mr. Moisseiff has been associated as a designer with the most noteworthy suspension bridges erected in the United States, and these structures are of greater magnitude than any suspension bridges erected elsewhere in the world.



He was engineer of design in the Department of Bridges, New York City, from 1897 to 1914, and in the office of the Delaware River Bridge at Philadelphia, for a period of six years. He was also Consulting Engineer for the Port of New York Authority from 1927 to 1930, and more recently Consulting Engineer for the Detroit River Bridge; the Golden Gate Bridge; the San Francisco-Oakland Bay Bridge and the Whitestone Bridge in New York.

In view of Mr. Moisseiff's ability and reputation, I hesitate to make any criticism of the structural design, but from a practical standpoint, I would feel that the width of this Bridge relative to the length of spans was open to criticism, particularly, since it was without precedent. The Golden Gate Bridge is the longest span bridge in the world, and the width of the structure is  $1/57$ th of the span length. That is the highest ratio of any large bridge that has been built up to date, so far as I can learn. The proposed Tacoma Narrows Bridge has a ratio of  $1/72$ nd. I learned that certain tests had been made on models of suspension bridge spans at the University of California, and as I could find no published report of this test, I went to Berkeley and conferred with Prof. R. E. Davis particularly with reference to horizontal and vertical deflections. Prof. Davis felt reasonably confident that the lateral deflections of the Tacoma Narrows Bridge as designed and as determined by Mr. Moisseiff, would be in no way objectionable to users of the bridge. He seemed satisfied that the theoretical determination of these lateral deflections by Mr. Moisseiff could be depended upon as representing very closely what would be experienced in the actual structure.

In the report of these tests on models published by the University of California in 1933, the statement is made

"The deflection theory permits the calculation of the main cable stress without appreciable error and the calculation of vertical bending moments of the stiffening truss with a maximum error of approximately 10 per cent, occurring in the vicinity of the quarter points of the main span. -----

"Moisseiff and Lienhard have presented a method which is closely accurate for determining lateral deflections of truss and cable stresses in the truss due to lateral forces."

In view of Mr. Moisseiff's recognized ability and reputation, and the many expressions of approval and comment of his methods of analyses of stresses and deflections in the designs of long span suspension bridges, particularly as expressed by the engineers who participated in the discussion of the paper presented before the American Society of Civil Engineers by Messrs. Moisseiff and Lienhard entitled "Expansion Bridges under the Action of Lateral Force", I feel we may rely upon his own determination of stresses and deflections.

Mr. Moisseiff in his letter to me of September 14th (Exhibit A2) says:

"The width of the Tacoma Narrows Bridge is 39 ft. center to center of girders. It is very narrow for the length of its center span, 2800 ft.,; the ratio of width to length is 1 to 72. To obtain a reasonable degree of lateral stiffness for the bridge flatter cable curves than usual have been adopted. The versine of the cables has been made 232 ft. This is in the ratio of about 1:12.1. The increased horizontal stiffness of the cables is obtained by increasing the tension in them and thus increasing their resistance to distortion.

"The maximum horizontal deflection of the center span has been determined at 19.9 ft. which is 1:141 compared to the Golden Gate Bridge with a horizontal deflection of 27.7 ft. or 1:153 for a span of 4200 ft. For a bridge of such horizontal slenderness the obtained stiffness is rather satisfactory. I see no structural objection to the magnitude of the horizontal deflection as long as all necessary clearances have been properly provided for.

"The change in grade due to liveload is limited to 2.5%. The greatest change in grade of the Whitestone Bridge now under construction, is 2.8%.

"As the stiffening girder in a suspension bridge also acts as a horizontal wind chord it becomes of determining importance in the case of narrow bridges.

"At the middle of the center span the stress due to wind attains in the Tacoma Narrows Bridge 72% of the total stress and 62% in the Whitestone Bridge. The stiffening girder to that extent will act as a column in compression and must be proportioned and designed as such. To insure the girder acting as a column against buckling an unstiffened web plate of 2 in. thickness would be required. The necessary resistance to buckling of the web was obtained in the adopted design by stiffening the web on one side by three zeos longitudinally and by two vertical stiffeners in each panel on the other side of the web. The web was then made  $1\frac{1}{2}$  in. thick."

It will be noted in the above quotation that in Mr. Moisseiff's own opinion, "For a bridge of such horizontal slenderness the obtained stiffness is rather satisfactory."

There seems to be some question even in his mind as to whether the obtained stiffness is other than rather satisfactory. The ratio of width to span is  $1/72$  which greatly exceeds the corres-

ponding ratios of other long suspension bridges, which are as follows:

| <u>Bridge</u>     | <u>Span (ft.)</u> | <u>Width (ft.)</u> | <u>Ratio</u> |
|-------------------|-------------------|--------------------|--------------|
| Delaware River    | 1,750             | 89                 | 1:19.7       |
| Ambassador        | 1,850             | 59.5               | 1:31.1       |
| Whitestone        | 2,300             | 74                 | 1:31         |
| San Francisco Bay | 2,300             | 66                 | 1:35         |
| George Washington | 3,500             | 106                | 1:33         |
| Golden Gate       | 4,200             | 90                 | 1:46.7       |
| Tacoma Narrows    | 2,800             | 39                 | 1:72         |

It therefore seems to me that it would be advisable to widen the super-structure to 52 ft. so as to provide for a forty-two foot roadway and two 3 ft. 3 in. sidewalks with protection curbs. This width would give a ratio of 1:54 approx. and would provide greater convenience and capacity for highway traffic. The cost would be increased considerably, but the additional cost would certainly be justified.

### CONCLUSIONS

#### TECHNICAL SOUNDNESS

With regard to the super-structure, I do not pretend to be qualified to analyze and check the design of the long span suspension bridge, but I have studied this design in connection with the designs of other bridges, which have been successfully erected, and are in successful operation. I also have great confidence in the ability and integrity of the Consulting Engineers under whose direction the computations and design drawings for the super-structure have been made. Moreover, these engineers have earned a very enviable reputation as experts in this field, as evidenced by the commendation from other suspension bridge experts which they have received in technical publications.

I therefore, feel that with the exception of the unusual narrowness of this bridge with reference to its span length, the super-structure design is technically sound. It is probably technically sound notwithstanding its narrowness, but there are several reasons why it would be of material advantage if the bridge could be widened at a reasonable increase in the cost, and therefore, I recommend that serious consideration be given to the possible increase in the width of this structure, before the contract is let or work begun. This would undoubtedly increase the width of the anchorage blocks and the smaller piers, but it would seem reasonable to assume that the widths of the main piers would not have to be increased.

September 28, 1938

Increasing Roadway Width

Assuming it is expedient to consider changing the width of the bridge as now designed, I would suggest increasing the width of the roadway from 26 ft. to 30 ft. and making the two sidewalks each 2 ft. 9 in. clear, instead of 4 ft. 9 in. clear.

Respectfully submitted,

(Sig.) T. L. Condrón

Advisory Engineer, RFC

REPORT  
of  
BOARD OF CONSULTING ENGINEERS

THE TACOMA NARROWS BRIDGE

to

Mr. Lacey V. Murrow  
Director of Highways

Olympia, Washington

August 31, 1938

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In accordance with your desire, we have reviewed the plans for The Narrows Bridge to ascertain whether or not, in our opinion, they were in satisfactory shape for receipt of bids. It should be stated here that the present purpose was not to go into the Project in detail, but only in respect to major features, the intention being to make a more detailed analysis later.

We have examined the superstructure design as to its general features. Time has not permitted the checking of stresses in the cables and stiffening trusses. In this regard we have full confidence in Mr. Moisseiff, and consider him to be among the highest authorities in suspension bridge design.

It might seem to those who are not experienced in suspension bridge design that the proposed 2800-foot span with a distance between stiffening trusses of 39' and a corresponding width of span ratio of 72, being without precedent, is somewhat excessive. In our opinion this feature of the design should give no concern.

The development of the deflection theory of suspension bridge design in recent years for both vertical and lateral deflections has proven beyond doubt that the matter of width ratio is limited not by structural stress but only by the amount of lateral deflection in wind which can be realized without discomfort or fear to the driver of an automobile over the bridge.

In a long narrow span the force of the wind is to a large extent transferred to the cables and through them to the towers rather than through the stiffening trusses. The greater the total dead load stress in the cables the more difficult it becomes to deflect them laterally and the less the side deflection of the bridge. In a long narrow bridge the matter of side deflection thus becomes a function of not width only but of both width between stiffening trusses and dead load cable stress with the dead load cable stress playing more and more a part as the width and sag ratios increase. In other words a suspension with a lesser distance between stiffening trusses and a low sag ratio may be just as stiff laterally as one with a greater width between stiffening trusses and a greater sag ratio.

In the proposed design the dead load stress in the cables is approximately  $\frac{6}{7}$  of the total stress. This large dead load stress is accomplished by decreasing the sag ratio of the cable. A sag ratio of  $\frac{1}{12}$  has been used while the general practice in wider bridges is to use between  $\frac{1}{7}$  and  $\frac{1}{10}$ .

In the proposed design approximately 80% of the wind stress will be taken by the cables and the result is that the equivalent span of the stiffening truss insofar as resisting wind is concerned is reduced to less than  $\frac{1}{2}$  the cable span. Side deflection in a 100-mile wind will be approximately 20 ft. In a 100-mile wind, however, autos attempting to travel would likely be overturned on the highway. In a 60-mile wind which is the velocity at which practically all traffic stops, this deflection would be reduced to approximately 9 ft. which is not noticeable in a distance of 2,800 ft. The same reasoning applies to stiffening truss depth. Here again the low sag ratio of the cables with the greater total dead load stress makes the cable more difficult to distort and in consequence reduces the bending moments and shears in the stiffening truss. This feature of stiffening truss design is strikingly demonstrated in the George Washington Bridge where no stiffening truss is used.

It may be said that the necessity of a stiffening truss and its depth and moments of inertia depend largely upon the ratio of dead to live load and cable sag. The greater the ratio of dead to live becomes and the lower the sag ratio, the less the necessity of the stiffening truss.

We believe that the present span could be materially increased if it were necessary, keeping the same width without any detrimental effect. In consequence we have no concern as to the general features of the proposed design of the superstructure.

Respectfully submitted,

Chas. E. Andrew, Chairman

Luther E. Gregory, Member

R. B. McMinn, Member-Secretary

APPENDIX V

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REPORT OF L. R. DURKEE, PROJECT ENGINEER, PUBLIC WORKS ADMINISTRATION, ON THE FAILURE OF THE TACOMA NARROWS BRIDGE AND STATEMENTS OF WITNESSES

FEDERAL WORKS AGENCY  
Public Works Administration

P. O. Box 3043  
Seattle, Washington

November 12, 1940

Docket 1870-F (Wash.)  
Tacoma Narrows Bridge  
Tacoma, Washington  
Eng. LRD:ej

AIR MAIL

TO: Director, Engineering Division

SUBJECT: Failure of Tacoma Narrows Bridge

Enclosures: Two Copies of statements by the following five men:  
Clark H. Eldridge, Bridge Engineer, Washington Toll  
Bridge Authority  
C. M. West, Bridge Designer, Washington Toll Bridge  
Authority  
R. E. Stewart, Instrumentman, W. T. B. A.  
Kenneth Arkin, Chairman, W. T. B. A.  
Professor F. B. Farquharson, University of Washington

The enclosures are copies of statements made by five different men who witnessed the disaster and were prepared and submitted by these men as soon afterward as we could get them. We thought it wise to secure as many different accounts as possible while the observed actions of the structure were still fresh in the minds of witnesses. The men submitting the statements all observed the failure; most of them from different points, and hence their reports may contain apparent discrepancies or contradictions. Professor Farquharson's statement should be particularly valuable as he was the last man off the structure and, in fact, was still on the east side span at the time a part of the center span fell into the Sound. He was also able to secure motion pictures of the bridge movement before failure. As you know, Professor Farquharson had been collaborating with the Owner's engineers in their studies of the bridge motion. He

had visited the project November 7th with his cameras with the intention of securing pictures of bridge movements during the day.

Our last report was dictated Saturday morning, November 9, and gave an account of the previous day's happenings. After dictating this memorandum, we visited the project, made several examinations of various members, and secured photographs taken at the time of failure which we forwarded you with our report.

We have made rather superficial inspections of both side spans and the two main towers, the latter from the roadway deck only. Owner's employees have examined the conditions at both the tops and bases of the towers. An important fact revealed by the tower top examinations is that there has been no slippage of the main cables at this point. Both towers are leaning approximately 12.8 feet toward the shores and many of the plates on the shore side have buckled, the worst condition of this appearing a few feet above the bases.

The worst damage to the main cables discovered to date appears at the west holddown point (Tower No.3) of the south cable. At this point, the cable is lying on the stub end of a broken lamp standard which has been forced some three inches into the cable, damaging several wires. A definite determination has not yet been made regarding the condition of the north cable near the center of the center span, where either the wrapping wire or the individual cable wires appear to be frayed. At our suggestion, a photograph was taken from the water which, upon development, may further reveal the amount of damage.

Crews are at work removing the concrete roadway deck of the two side spans and by the afternoon of November 12, approximately 75 linear feet of deck had been removed. It is, of course, important that the side spans be relieved of weight as soon as this can possibly be accomplished.

It is doubtful whether any definite information with respect to the salvage of the towers and main cable can be obtained for several days. In any event, we shall continue to keep you advised of developments.

L. R. Durkee  
Project Engineer

Authors' Note: In addition to the statements above mentioned, those of Walter F. Miles of the Pacific Bridge Co. and of Lt. W. C. Hogan, Commanding the Coast Guard Cutter "Atalanta" are attached.



November 7, 1940

Record of Observations of Narrows Bridge Failure  
Made by Clark H. Eldridge

Drove across the bridge at about 8:30 as usual to observe the behaviour of the bridge as the wind during the latter part of the night had been quite severe and was still blowing moderately. The bridge appeared to be behaving in the customary manner, the east side span being practically quiet, the main span oscillating in a four noded manner and the west span oscillating considerably from the temporary holddown to the tower. All of these motions, however, were considerably less than had occurred many times before so I came to the office at about nine o'clock. Yesterday in a conference with Mr. Andrews, Profeser Farquharson and Mr. Durkee, it had been determined that we would proceed immediately to streamline the southerly side of the main span. I was to prepare detailed sketches and secure prices on materials, etc. as quickly as possible and proceed with the work on the basis of owner's force account. Therefore, upon my return to the office at nine o'clock, I immediately undertook the preparation of these sketches and obtaining of information. At about ten o'clock Mr. Walter Miles called from his office to come and look at the bridge that it was about to go. This was the first indication I had that anything of an unusual or serious nature was occurring. I immediately drove with Mr. Miles to the dock from which we could see the bridge. The center span was swaying wildly, it being possible first to see the entire bottom side as it swung into a semi-vertical position and then the entire roadway. It was at once apparent that instead of the cables in the main span rising and falling together, they were moving in opposite directions thereby tilting the deck from side to side. I could observe one car, stationary, some distance east from the center of the span. It appeared that the center of the span was remaining about horizontal and the two halves were revolving about a longitudinal axis of the bridge. I then returned to the office, took my own car and went immediately to the bridge where I observed that all traffic had been stopped and that several people were coming off the bridge from the easterly side span. I walked to tower No. 5 and out onto the main span to about the quarter point observing conditions. The east side span was practically quiet, there being but a few inches of vertical motion. I looked over the side at the temporary tie down cables and observed they were perfectly quiet and functioning in the manner intended. Then I observed on the main span that the concrete sidewalk around the stiffeners of the girders was failing badly. The curbs at the construction joints were also failing. Adjacent to the girders, it appeared that the concrete was entirely free and the girders and the concrete were working back and forth continuously three or four inches. The concrete

roadway showed no signs of cracking. The main span was rolling wildly. It appeared to be in a true position horizontally. That is, the wind had not moved it a noticeable amount sidewise. The deck, however, was tipping from the horizontal to an angle approaching forty-five degrees. I could look beyond the center of the span at the lamp posts and see that the deck there was tilting in an opposite direction. The entire main span appeared to be twisting about a neutral point at the center of the span in somewhat the manner of a corkscrew. At tower No. 5, I met Professor Farquharson, who had his camera set up and was taking pictures. We remained there a few minutes and then decided to return to the east anchorage warning people who were approaching to get off of the span. From the east anchorage, I advised the officials of the Northern Pacific Railway that the span was in a dangerous condition and that train service should be discontinued. I also advised the Coast Guard and the U. S. Engineers. I then requested Mr. Stewart to go and examine the east holddown points and to go below into the cable tunnels to look at the anchorages. I then went from the administration building to the view plaza on the south side of the bridge to observe the behaviour. The main span was still rolling badly and the east side span was still quiet. The temporary tie downs were working as intended and appeared to be holding this span. At this moment, I felt very happy that these temporary guys were in. I requested Mr. Arkin, if possible, to determine whether the west tie downs were still in, he later reporting to me that they were. At that time, it appeared that should the wind die down, the span would perhaps come to rest and I resolved that we would immediately proceed to install a system of cables from the piers to the roadway level in the main span to prevent any recurrence. It appeared to me that this was extremely desirable so I returned to the administration building with the thought of making arrangements for crews to come in as quickly as the storm subsided for this purpose. I called Mr. Frincke of the Bethlehem Steel Company and requested him to send us a superintendent and a pusher immediately in order that we could undertake this work at once. I then called the Weather Bureau to determine whether or not the wind was dying down and was informed that the barometer was rising and in all probability the wind would quiet later in the day. During this interval, I talked with both Mr. Andrew in Seattle and Mr. Davis in Olympia who informed me they were proceeding to Tacoma at once. I was then informed that a panel of laterals in the center of the span had dropped out and a section of concrete slab had fallen. I immediately went to the south side of the plaza and from there could see that such was the case. The bridge was still rolling badly about the center as it had been doing previously. I returned to the toll plaza and from there observed the first section of steel fall out of the center. From then on successive sections towards each tower rapidly fell out. For fear that misfortune might come to the anchorage, I requested that all persons be cleared

from the administration buildings back to a safe distance. Shortly thereafter, during this time and coinciding with the dropping of the sections from the center span, I observed the side span settle rapidly and was momentarily expecting the towers to come down. I did not observe the exact time that the center section fell out although I was later informed that it was 11:10. Shortly thereafter, Mr. Andrew, Mr. Davis, Mr. Durkee and others arrived. We made what examination we could in the vicinity of the east anchorage determining that in all probability, there was no immediate danger of failure there. Instructions were issued to Mr. Arkin to determine the verticality of the towers and he reported they were leaning shoreward about twelve feet at the top. At about 1:30 P. M. Governor Martin arrived and Senator Bone and a good deal of time was consumed in talking with newspaper and radio men. After the Governor left, Mr. Davis and others left for a meeting with the County Commissioners in order to arrange for ferry service.

Mr. Andrew and I came to Sixth Avenue Dock and took the "Skylark" for an inspection tour of the piers and towers. It has been reported to us by Mr. Stewart that it appeared that the north cable was badly frayed at mid span. We proceeded around pier No. 5 examining the tower from the boat. The tower appeared to be buckled at a point ten or twelve feet above the base on the shore side. From there to the top, it appeared bent in a smooth curve. We then proceeded to the center of the channel to examine the cable with spy glasses. Because of the movements of the boat it was impossible for us to determine whether the mass of wires was wrapping wire which had been broken or whether portions of the main cable had failed. At this time two Foss Company tugs were sweeping the channel which had been arranged for some time before to determine whether or not it was safe for shipping to proceed. We boarded one of these tugs and talked with Mr. Orville Sund, who was in charge of operation. He advised us that he had swept the channel and was doing it for the third time and it seemed entirely clear. We then proceeded to intercept a Coast Guard boat and inquire whether or not they would patrol the channel the entire night. They informed us that they were waiting orders from a second boat which had gone to the Sixth Avenue Dock to receive instructions. We then proceeded around Pier No. 4 observing the condition of this tower. It was buckled in a similar point to No. 5 and also was bent to a smooth curve from this point upward. It appeared that both these towers were not damaged beyond repair and that it might be possible to salvage them. We then returned to the Sixth Avenue Dock passing on the way the second Coast Guard boat with whom we communicated and learned that they expected to patrol the vicinity all night. We returned to the office and Mr. Andrew returned to Seattle. It was apparent that nothing further could be done, arrangements having been made with the State Highway Department and Capt. Owens of the Toll Bridge

Authority for watchmen at both ends of the bridge to keep the public away.

November 7, 1940

## Record of Observations of Narrows Bridge Failure

Made by R. E. Stewart

The bridge was moving slightly when I arrived at the bridge at approximately 8 A.M. The motion kept building up and at about 9:30 I drove out and parked in center of main span and counted the number of cycles the bridge was making per minute and found it to be thirty-five. I then came in and reported it to Mr. Eldridge. I then started back to the bridge and at the top of the hill noticed the vibrations of cables were out of step and bridge was rocking from side to side badly, so I turned around and came back to report but in the meantime the news had reached the office, so I drove back to bridge arriving around 9:45. Mr. Eldridge, Carl West and I walked out to tower No. 5. The only noticeable damage was small chunks of concrete broken out of sidewalk at girder stiffeners. It was getting more violent so we decided we had better get off and we turned back several that were coming out. We got back to toll house about 10:00 and Mr. Eldridge asked me if I would go in anchorage and see what the effect was on anchor and Bent K tie down cable.

I first went in tunnel and noted anchor bars. I could feel a slight horizontal motion, the cables were moving as far horizontally as they could because they were hitting both sides of concrete where they went through the deck.

I then went up to look at the tie down cables at Bent K. The motion there of the shoes was about two inches. I then noticed a great jar and some concrete fell down and I got out. I didn't know that the bridge had failed until walking out and looking towards center span I saw a small portion gone near the center. I then went up on deck by toll house and watched until the final crash when most of center span fell into the bay.

Mr. Eldridge then wanted to know how much the towers were out of plumb. Mr. K. B. Arkin and I took transit and taking some readings figured tower No. 5 leaned approximately twelve feet east. We went out in "Skylark" and looked at tower No. 4 and it appeared to have the same lean as tower No. 5.

On the trip we noticed a place on the north cable that was frayed. We thought it was only wrapping. We then took a

level and set it up on shore and it appeared from there to be strands of main cable.

November 7, 1940

## Record of Observations of Narrows Bridge Failure

Made by C. M. West

The first warning I had of any unusual condition existing on the bridge was when I happened to glance up at the bridge while standing on the Sixth Avenue Ferry Dock which is situated about one and one-half miles from the bridge in a southwesterly direction. The deck at that time appeared to be tilting crazily about the longitudinal axis of the bridge and since no such motion had previously been observed I immediately called Mr. Eldridge to inform him of the situation so that he could observe it.

I immediately left the dock to go to the bridge site at the east anchorage. Upon reaching the bridge I observed a car that had been abandoned on the main span some distance from tower No. 5. At that time the deck in the main span did not appear to be deflected or swinging laterally but it was pitching about the longitudinal axis to about thirty degrees each way from the normal horizontal position. The east side span was normally quiet for such a windy day so I decided to go out to the main span to see if the concrete deck was showing any signs of breaking up. Upon reaching a point in the main span just west of tower No. 5 I observed that portions of the sidewalk slab were breaking badly next to the girder where vertical stiffeners occur. The sidewalk slab was also sliding back and forth along the shelf angle on the girder for a distance of approximately 2 1/2 inches. I did not at this point observe any disintegration of the deck slab nor curbs. Since the pitching of the deck did not appear to be abating, I did not consider it wise to go out further on the main span to see if any disintegration was occurring. I, therefore, turned and walked off the bridge to the east anchorage where I proceeded to the bluff just south of the bridge at approximately Station 72 and noticed that approximately at the mid point of the main span, the lateral bracing had come loose and was hanging suspended from the deck. Shortly thereafter portions of the concrete roadway dropped out and into the Narrows. I then noticed that the stiffening girders had been crimped in a horizontal direction indicating that with the lack of concrete deck at that point, the girders had failed in horizontal shear. I then turned my attention to the east side span particularly to the temporary guy cables and noticed that they were now becoming alternately slack and taut. However, the side span motion was still nothing to alarmed about

I then proceeded to the toll house and reported to Mr. Eldridge the failure of the girders at mid span. Coming back to the bluff to my previous vantage point, had not been there but a short time until approximately 700 feet of the deck at about the west quarter point of the main span fell into the Narrows. The violent motion of the deck seemed to abate somewhat after this portion of the deck had fallen. It seemed as though the motion might die out completely and the bridge remain in that condition. However, as I was standing and thinking of that possibility, the remainder of the deck, both that portion from the previous break toward Tower #4 and also toward Tower #5 started to fall into the Narrows progressively as the suspender ropes snapped. I remained on the bluff for a short time thereafter watching for the possible failure of the towers as they leaned shoreward due to the weight of the side spans which were remaining intact. As the last of the main deck had come off and there was no apparent tower failure, I went back up to the toll house to report to Mr. Eldridge.

November 7, 1940

#### Record of Observations of Narrows Bridge Failure

Made by Kenneth Arkin

The wind awakened me at 5:00 A.M., Thursday, November 7, 1940. Got up and drove to the Narrows Bridge to check the temporary guys. A fresh wind blowing directly from the south was having very little effect on the bridge. Went back home, had breakfast and came back to work at 7:30 A.M. Went to mid span and took velocity of wind - reading 38 miles an hour on anemometer. The bridge bounce was noticeable but of no great magnitude. Drove on to the west side span and noticed the temporary guy cables were whipping in a two foot arc. Drove underneath west side span and again looked at temporary guys noting same conditions as above. Drove back to toll plaza and asked Reynolds if any pile butts had come through. He informed me that three had gone across. I drove back across the bridge, parked my car on the hill and walked down to the bay and waited for Lewis to come ashore (approximately five minutes). He told me he had stayed aboard the scow all night and was going to breakfast and stated that at 3:30 A.M. the thrashing of the temporary guys had awakened him and upon investigating he had found that the bottom nuts had unscrewed approximately a half inch. We discussed the length of piling needed. He said he would wait in the restaurant until I had talked with Mr. Eldridge. Drove to the office and informed Mr. Eldridge of the condition of the temporary guys and was told by him to tighten the nuts,



also, was told that piling would be in at 11:00 A.M. Driving back across the bridges to the restaurant I noticed that the bounce had increased slightly in magnitude. Informed Lewis the piling would be in at 11:00 A.M., also, I might be held up at observation house and, if so, to go ahead and drive piling in position he had been shown. Drove back to mid span - anemometer reading showed wind velocity 42 miles per hour directly from the south. The magnitude of the bounce, in my opinion, did not appear to be the maximum recorded. Counted the r.p.m.'s and found a count of 38. Hurrying to the observation house, I encountered Mr. Farquharson. He asked me the velocity of the wind and the r.p.m.'s of the bounce and asked me to see him before he went back to Seattle. I climbed to the observation house and looking through the transit noticed about three feet of magnitude at mid span, the bridge having the same motion as hitherto recorded and the east side span having very little bounce. Then the mid span targets disappeared to the right of my vision. Looking over the transit, mid span seemed to have blown north approximately half the roadway width coming back into position in a spiral motion. I called quickly to Bill Ellis to stop traffic and call Mr. Eldridge. Turning back to transit I noticed the spiral motion had increased to great proportions. (Checking the call to the radio patrol office, the time was exactly 10:07 A.M. - this was approximately three minutes after the start of this peculiar motion.) Turning the transit on a tower leg, I noted that it was steady and turned back to the targets. I could not keep them in line of vision long enough to determine the exact amount of magnitude. The light poles crossing each other seemed to be almost horizontal at times. Observing the car parked beyond tower #5 on the right side of the roadway, noticed it suddenly careen to the left side and the driver fall out, grab the curb and hang on, for what appeared to be several minutes. The bridge calmed down sufficiently for him to struggle in. While this man was hanging onto the curb, Dolf came to me and asked if I thought it advisable for him to go out and help this man. I told him when the man had recovered from his fright, he would be able to make it. About this time Mr. Eldridge appeared and headed for Tower #5. Believing the bridge would right itself, I climbed down and walked after him. Stopping to look at the temporary guys, I noted they were intact. During this period the side span was moving very little. While walking out towards the pier, Hager called to me and said Eldridge was wanted on the phone. I ran on out with this information and we immediately came back together. I walked down on the parking area and noticed small chunks of concrete falling. I supposed they were the cracked sections around the light poles. I went to the observation house and noted the bridge had increased to as great a magnitude as previously observed. Hearing someone yell, I rushed down to the parking area and observed mid span was broken completely in two, the ends working against each other with a sidewise motinn. Shortly thereafter two or three sections of

roadway fell in. Then approximately 600 feet of girders and roadway fell in. I went back to toll house, saw Mr. Eldridge and as I came out the door, I heard a deafening crash and looking up saw clouds of dust and a snarled mass of suspenders and deck whipping. The east approach dropped below my vision and then everything became quiet. I went down into the tunnel. Not being able to see anything, I came back to the toll house and was told to determine the position of the top of the towers. Found them to be approximately twelve feet shoreward.

#### THE FAILURE OF THE TACOMA NARROWS BRIDGE

The following report covers a period starting just before 10:00 a.m. Thursday, November 7th, 1940, and lasting until the collapse of the bridge shortly after 11:00 a.m. of the same day.

A series of observations and photographs taken just before 10:00 a.m. showed the bridge to be moving with the usual vertical motion with a frequency of 36 cycles per minute. An examination of the films will be necessary before the mode of motion can be identified with certainty but it appeared to involve at least nine waves or eight nodes. Thru several observations this condition did not change. At the same time it was observed that the center of line of the bridge was deflected to the north. Observations of wind velocity at this time showed 42 mph. The lateral deflection of the center of the bridge was not measured but did not appear to be excessive, perhaps four times the width of the yellow center line (about 2 ft.)

While checking the frequency from a position on the roadway to the north of the toll plaza a few minutes later a violent change in motion was noted. This change appeared to take place without any intermediate stages and with such extreme violence that the span appeared to be about to roll completely over. The most startling condition arose out of the fact that from a line of sight very nearly parallel to the bridge the upper side of the roadway on the Gig Harbor side of the main span was visible while what appeared to be a nearly perpendicular view of the bottom of the roadway was offered on the Tacoma side. The motion, which a moment before had involved a number of waves (nine or ten) had shifted almost instantly to two. The frequency was not checked at this point, but a photographic record was taken from which this data may be taken after processing.

On reaching the axis of the bridge just in front of the toll house it was apparent that a motion of catastrophic proportions had developed the greater part of which was confined to the main span. As quickly as possible a station was reached just outside the towers from which a photographic record of the



motion of the main span was obtained. A considerable amount of shock motion of small amplitude was evident in the side span, enough to make operation of the motion picture camera difficult. It was quite impossible to operate a camera on the main span so violent was the motion.

At the moment of first observing the main span from the vicinity of the towers the motion had a frequency of 14 cycles per minute and consisted of a differential action of the cables with the two waves in each cable. The node was at the center of the main span and the structure was subjected to a violent torsional action about this point. The motion was not pure, however, and involved considerable motion at the center point. At times for a short period the motion changed over to a single wave on each cable but still with the cables out of step. This motion which never lasted long seemed to be of slightly greater amplitude than the single noded motion, but of the same frequency.

At this time, not more than 8 or 10 minutes after the start of this violent motion, a considerable amount of damage had been done to the concrete in the sidewalks and curbs, and several of the lamp posts in the vicinity of the Tacoma towers were already loose on their bases. At this time one lamp post was seen to fall on the Gig Harbor side of the span. At this same time it was noted that a considerable longitudinal motion had developed between the side walk and the girder. This motion was not measured at this time but seemed to be about two inches. No signs of fracture were observed in the concrete deck, but the curb had started to separate away from the deck at numerous places. Also rather large sections of concrete had broken out of the corners of the curb formed by the expansion joints. This loose concrete had, in some instances, been thrown to the center of the roadway by the violence of the motion.

It should be noted at this point that the amount of lateral deflection of the center of the bridge seemed to have increased to a marked degree but no measure of it could be obtained from a location near the towers.

A car was stalled some little distance beyond the quarter point towards the center of the bridge. An effort was made during a momentary decrease in the violence of the motion to drive this car to safety; but shortly after reaching the car the violence increased to a point where the acceleration must have reached that of gravity since the car began to shift about in a most alarming manner. While out on this portion of the span opportunity was taken to examine the state of the bridge. Close examination of the suspenders at the point where they entered the girder failed to disclose any evidence of excessive stress either in the form of heat or spalling of paint.

However, at two points one of the four suspenders in its group was permanently slack. As far as the eye and ear could detect the double amplitude of approximately 28 ft. was causing no distress in the girder. On the other hand the motion between the deck and the girder was noted all the way from the tower to well beyond the quarter point. This motion at once suggested that the lateral "K" bracing must at least have yielded. It appeared at this time that the velocity of the wind might have increased somewhat although no direct observations were available. While making observations in the vicinity of the stalled car several more lamp posts were seen to fall on the Gig Harbor side. Even at this time it seemed plain that the motion was more violent on the Gig Harbor side.

While returning to the Tacoma tower several observations were taken along the side of both girders looking towards the center of the span. It was plain that at this stage the lower limit of the undulation was almost exactly flattening the girder to a straight line from tower to tower while the upper limit must have been something over 25 ft. above this level.

It now became necessary to return to the toll house for a fresh supply of film and there ensued a gap of some ten or twelve inches in the observations. On returning to the Tacoma tower it was at once noted that the frequency had changed to 12 for single noded action and on one check the frequency of the single wave motion was also 12. A decided change had taken place by this time, however, in the character of the lateral deflection. It was no longer more or less steady but consisted of a very definite horizontal wave and the total amplitude of this lateral motion had very much increased. The complexity of these two superimposed motions was such that no adequate figure could be obtained regarding either the lateral frequency or amplitude. An immediate check was made on the motion between the concrete and the girder where this motion was found to have definitely increased to something over two and one half inches, suggesting that some further impairment in the lateral bracing system must have occurred. A further check on the lamp posts on the Gig Harbor side showed several more to have collapsed. Up until the final collapse of the bridge no lamp posts went down on the Tacoma side of the main span.

At about this time, while observing the lateral deflection by sighting along the outside edge of the girder a distinct lateral failure was observed at what appeared to be about the quarter point on the Gig Harbor side. This was observed first on the south side of the bridge but a quick check showed that it had also occurred on the north side at approximately the same point.

From this point on failure occurred so rapidly that

observation became difficult and impression and fact are somewhat mixed. While still observing from the tower quarter point suspenders on the Gig Harbor side on the north side of the span were seen to fly high in the air above the main cable. This failure progressed rapidly toward the center of the span until apparently a whole section fell out of the bridge near the quarter point.

After this first section fell the whole bridge almost at once ceased its violent torsional motion and fell into a much easier vertical motion which was photographed from the tower. Almost at once, however, the original motion resumed and this time also built up in the side span which had fallen from under the influence of the check reins when they sagged due to removal of weight from the main span. At once the failure became progressive along the main span the shock of each successive unloading of the main span producing a corresponding shock in the side span from which observations were being attempted. Two of these shocks were of sufficient force as to throw the observer violently to the deck. As the side spans sagged more and more they lost all of their torsional motion and finally came to comparative rest.

F. B. Farquharson

Nov. 7, 1940

#### NARROWS BRIDGE FAILURE

by

Walter F. Miles

The following is a report of my movements and observations prior to and after the failure of the Tacoma Narrows Bridge.

On the morning of November 7, 1940 I noted that the expected gale had arrived and was blowing rather hard so at breakfast asked my father Mr. F. M. Miles, who had just returned from Montana, if he wanted to see a bridge bounce. We arranged for him to meet me at the office after I arrived from picking up the mail.

I took my motion picture camera with me and after picking up the mail stopped at the Dept. of Labor and Industries requesting final audit, then stopped at the Eastman Kodak Company and picked up a 50 ft. roll of Kodachrome film, as I wanted to photograph the motion of the bridge.

Shortly after my arrival at the office my father arrived and at

about 8:45 A.M. we took my car and went up to inspect the bridge.

On driving across we went slowly past the east tie down and the east side span temporary guys and noted that the motion of the main span was being ironed out in the east side span indicating that the guys at this point were functioning satisfactorily. At mid-span we noticed that the mid-span guys were alternately tightening and loosening causing considerable snapping at this point. The west side span guys and holddowns were relatively quiet.

We went over to the west anchorage and down to Woodworth & Cornell operations where they were cleaning up on the removal of the west dock. There we talked with Earl Starbard and my father talked with Ralph Keys, the crane operator. We observed the vertical wave motion of the bridge and remarked about the satisfactory way the temporary guys on this side were dampening out the wave motion in the west side span.

We returned over the bridge at about 9:30 stopping at Tower #4 where I took pictures of the motion of the main span from this point. While taking pictures I noted that there was a slight horizontal motion of the tower at this point along the center line of the bridge and also that there was a slight horizontal motion of main span deck structure. A short count made estimate of wave frequency at about 35 per minute.

We stopped at mid-span to observe the action of the mid-span guys and I took pictures of the guys and deck motion from this point. From here we went directly to the office at Sixth Avenue.

At about 10:00 A.M. we drove to the Sixth Avenue Dock accompanied by Carl West. Upon arrival at the dock our attention was attracted to the bridge which had taken on a definite twisting motion, the deck apparently tilting from 30 to 40 degrees from horizontal, each half of the main span each side of mid-span being in opposite phase.

I rushed back to the office and picked up Mr. Clark Eldridge and returned to the Sixth Avenue Dock taking motion pictures from this point of the continued motion of the bridge.

From here all of the above party went directly to the east anchorage, Mr. Eldridge taking his car and arriving immediately behind us.

My father and I went to the edge of the bluff on the south side of the bridge and noted that the motion was still continuing the deflection being so great that we could almost completely see a car stalled a short distance west of Tower #5.

The motion continued for some time when the lateral bracing at mid-span failed, the beams dropping down so that they could be seen below the stiffening girders. After this some small pieces of concrete fell out at mid-span and a ray of light could be seen through the deck. Shortly after this I saw a lamp standard fall on the west side of main span. At this point I left leaving the camera with my father and went to the office at Sixth Avenue and called Mr. Keenan in San Francisco informing him that the bridge was failing and arranged to call him again at 1:00 P.M.

Upon arriving back at the east anchorage I saw that the stiffening girders had sheared completely at mid-span and the motion continuing, if anything, a little faster than previously. Carl West was here at the time and we discussed notification of N. P. Railroad and Coast guard. West informed me that Mr. Eldridge had previously notified them. My father, West and I observed the east side span temporary guys and saw that they were still effective. Noted however, that several lamp standards had fallen on the west side of the main span.

The motion continued and at about 11 o'clock a section west of mid-span fell out causing the remainder of the main span to raise and the side spans to slack. Noted that the temporary guys went completely slack and that considerable motion started up in the east side span.

State Patrolman and toll bridge men at this time and previously had been directing spectators away from the site of the bridge.

We remained here for a few minutes when the remainder of the main span suspended str. with the exception of about 250 feet adjacent to each main tower fell, causing the center cables to fly up and the side span to take on a terrific deflection. At this point I left for a safer position away from the anchorage.

The failure did not progress past this point so stopped and observed that the east side span was still in motion deflecting until it nearly touched the approach guard rail.

Shortly after this my father left the site and I went to the office to send a wire to the San Francisco Office informing them of the failure and confirming phone call to be made at 1:00 P.M.

I then returned to the east anchorage taking Mr. White, County Engineer, and another county man with me who were interested in resuming ferry service.

The above was rewritten from notes dictated at 5:10 P.M., November 7, 1940.

Signed: November 9, 1940  
Walter F. Miles

TREASURY DEPARTMENT  
United States Coast Guard

Coast Guard Cutter

ATALANTA

Seattle, Washington  
27 December, 1940

Mr. L. R. Durkee,  
Seattle, Washington.

Dear Sir:

Agreeable to your request, you are advised in writing that at the time the ATALANTA passed under the Tacoma Narrows Bridge, approximately one hour before it fell, I particularly noted that the north cable was loose in its mooring, at the center of the bridge, and was sliding back and forth for a distance of about 1 to 2 yards.

Signed: W. C. Hogan  
Commanding ATALANTA.

## APPENDIX VI

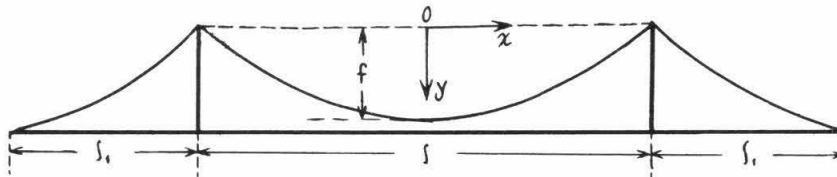
### THE VIBRATIONS OF SUSPENSION BRIDGES

By

W. D. Rannie, Graduate Assistant  
Guggenheim Aeronautical Laboratory  
California Institute of Technology

The exact theory of vibrations in suspension bridges involve difficulties similar to, but even more complex than those of the deflection theory for static loads. They arise from the fact that deflections of the cable are not exactly proportional to the loads, leading to non-linear equations for additional deflections from additional forces. In order to make the problem tractable, the equations may be linearized by assuming small deflections from the initial position and neglecting second and higher order terms. Certain additional simplifying assumptions are made to facilitate the solution and an estimate of the errors involved in these assumptions will be made as a later step.

Nomenclature. The case to be considered is a symmetrical suspension bridge with one center span and two side spans. The origin of the coordinate system is taken at the center of the line joining the tower tops, as shown.



$l$  = length of center span

$l_1$  = length of side spans



$f$  = sag of center span

$\alpha = \{, / \}$

$w$  = dead load per unit length (here assumed constant) on one cable.

$p$  = live or inertial load per unit length on one cable.

$T$  = tension in the cable.

$H_w$  = horizontal component of tension due to dead load.

$H$  = horizontal component of tension due to live load on inertia forces.

$h$  = maximum value of  $H$  in vibration.

$y'$  = vertical deflection of the cable at any point due to live load  
or inertial forces.

$\eta$  = amplitude of  $y'$  in vibration.

$\omega$  = circular frequency of vibration.

$N$  = frequency in cycles per minute.

$I$  = moment of inertia of one stiffening girder about its horizontal  
axis.

$W$  = total energy of oscillation in the bridge.

$W_g$  = energy of oscillation in the stiffening girders.

$$\mu = \sqrt{\frac{w}{H_w g}} \frac{l}{2} \omega$$

$$k^2 = \frac{H_w}{EI} \left(\frac{l}{2}\right)^2$$

Assumptions. The specific assumptions made in the preliminary analysis  
are as follows:

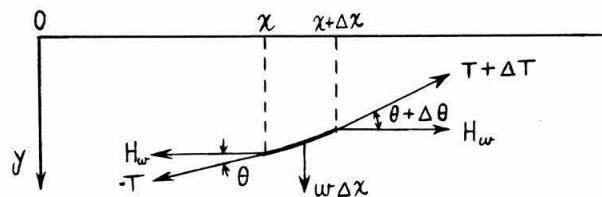
- a. The additional cable tension  $H$ , produced by the deflection,  
is small compared with the tension due to dead load  $H_w$ .
- b. The cable is inextensible.
- c. The suspenders are inextensible.

d. The towers are perfectly flexible to horizontal forces applied at the tops.

e. The bending stiffness of the suspended structure is neglected.

Assumption (a) implies that while the additional tension must be considered, it is small and results in small extensions of the cable. The effect of this extension is of the second order and therefore can be neglected without serious error, assumption (b). Assumption (c) is usual and implies that the cable and suspended structure always have the same vertical deflection. Assumption (d) is usual in suspension bridge design. The amount of error in assumption (e) will be discussed later and the necessary corrections applied.

Derivation of the Equations of Motion. Let the coordinates of the cable under dead load alone at any point be  $x$  and  $y$ . Considering the forces acting on an element of cable at  $x$  whose projection on the  $x$ -axis is  $\Delta x$  and which makes an angle  $\theta$  with the horizontal at  $x$ , and resolving the forces horizontally and vertically



$$H_w = T \cos \theta = \text{const.}$$

and

$$T \sin \theta = w$$

Since  $T = H_w / \cos \theta$  and  $\tan \theta = -dy/dx$ , eliminating  $T$ ,

$$H_w \frac{d^2 y}{dx^2} = -w \quad (1)$$

From this equation the well-known relation  $y_{\max.} = f = \frac{w \int^2}{8 H_w}$  is obtained.

If a live or inertial load  $p$  per unit length is added, causing a small vertical deflection  $y'$  and a small additional horizontal component of tension  $H$ , then  $w$  may be replaced by  $w + p$ ,  $y$  by  $y + y'$  and  $H_w$  by  $H_w + H$  in equation (1) giving

$$(H_w + H) \left( \frac{d^2 y}{d x^2} + \frac{d^2 y'}{d x^2} \right) = -w - p$$

Neglecting the term  $H \frac{d^2 y'}{d x^2}$  which is of the second order in small quantities, and using equation (1)

$$H_w \frac{d^2 y'}{d x^2} + p = \frac{H}{H_w} w \quad (2)$$

In the case of free steady vibrations, the effective live load results from the inertia of the vibrating mass. Assuming that the deflection at any point  $x$  at time  $t$  is given by  $y'(x, t) = \eta(x) \sin \omega t$  where  $\eta(x)$  is the amplitude at  $x$ , and  $\omega$  is the circular frequency,  $p = \frac{w}{g} \frac{\partial^2 y'}{\partial t^2} = \frac{w}{g} \omega^2 \eta(x) \sin \omega t$ . The additional horizontal tension  $H$  must also have a factor  $\sin \omega t$ , i.e.,  $H = h \sin \omega t$ , so canceling this factor from the three terms in equation (2) a differential equation is obtained for the maximum value of  $y'$  in each oscillation in terms of the corresponding maximum value of  $H$  and the frequency  $\omega$ .

$$\frac{d^2 \eta}{d x^2} + \frac{w}{H_w g} \omega^2 \eta = \frac{h}{H_w} w$$

It will be convenient to use, instead of the frequency  $\omega$  as an unknown, the dimensionless quantity  $\mu = \sqrt{\frac{w}{H_w g}} \frac{1}{2} \omega = \sqrt{\frac{2f}{g}} \omega$ . Then the differential equation above can be written in the form

$$\frac{d^2 \eta}{d x^2} + \left( \frac{2\mu}{l} \right)^2 \eta = 8 \frac{f}{l} \frac{h}{H_w} \quad (3)$$

Condition of Inextensibility of the Cable. The element of length of

the cable in the position of static equilibrium is  $ds = \sqrt{1 + y_x^2} dx$  where  $y_x = dy/dx$ . Hence, the length of the cable is  $L_0 = \int_{-l/2-l_1}^{l/2+l_1} \sqrt{1 + y_x^2} dx$

When deflected to maximum amplitude, the element of length becomes

$$ds = \sqrt{1 + (y_x + \eta_x)^2} dx \quad \text{and the total length of the cable becomes}$$

$$L = \int_{-l/2-l_1}^{l/2+l_1} \sqrt{1 + (y_x + \eta_x)^2} dx \quad . \quad \text{For } \eta \text{ much smaller than } y ,$$

$$\sqrt{1 + (y_x + \eta_x)^2} \cong \sqrt{1 + y_x^2 + 2 y_x \eta_x} \cong \sqrt{1 + y_x^2} \left( 1 + \frac{y_x \eta_x}{1 + y_x^2} \right)$$

which gives for the extension of the cable

$$L - L_0 = \int_{-l/2-l_1}^{l/2+l_1} \frac{y_x \eta_x}{\sqrt{1 + y_x^2}} dx$$

Integrating by parts,

$$L - L_0 = \left[ \eta \frac{y_x}{\sqrt{1 + y_x^2}} \right]_{-l/2-l_1}^{l/2+l_1} - \int_{-l/2-l_1}^{l/2+l_1} \eta \frac{d}{dx} \left( \frac{y_x}{\sqrt{1 + y_x^2}} \right) dx$$

Since the ends are assumed anchored,  $\eta = 0$  at  $x = \pm(l/2 + l_1)$ , and

$$L - L_0 = - \int_{-l/2-l_1}^{l/2+l_1} \eta \frac{y_{xx}}{(1 + y_x^2)^{3/2}} dx$$

If it is assumed that  $y_x^2$  is much smaller than unity, the denominator of the integrand can be replaced by unity. We also make use of the relation  $y_{xx} = -w/H_w = -8f/l^2$  to obtain a simple expression for the extension,

$$L - L_0 = \frac{8f}{l^2} \int_{-l/2-l_1}^{l/2+l_1} \eta dx \quad (4)$$

The condition for inextensibility of the cable is then simply

$$L = L_0 \quad \text{or} \quad \int_{-l/2-l_1}^{l/2+l_1} \eta dx = 0$$

Expressed otherwise, the condition requires that the algebraic sum of the areas between the deflection curve and the line of static equilibrium be zero. It may also be derived by the method of virtual displacements.

Symmetrical Modes. Since the ends of the cable are anchored at

$x = \pm (l/2 + l_1)$  and can have no vertical deflection at  $x = \pm l/2$ , the symmetrical modes will be solutions of equation (2),

$$\frac{d^2 \eta}{dx^2} + \left(\frac{2\mu}{l}\right)^2 \eta = 8 \frac{f}{l^2} \frac{h}{H_w}$$

each mode being given by an expression for the range  $0 < x < l/2$  which satisfies the boundary conditions  $\frac{d\eta}{dx} = 0$  at  $x = 0$ ,  $\eta = 0$  at  $x = l/2$ ; and an expression for the range  $l/2 < x < l/2 + l_1$ , which satisfies the boundary conditions  $\eta = 0$  at  $x = l/2$ ,  $\eta = 0$  at  $x = l/2 + l_1$ . The values of  $h$  and  $\mu$  are the same for both ranges because of the original assumptions.

The expressions for  $\eta$  which satisfy the above equation and conditions and are symmetrical about the centerline are,

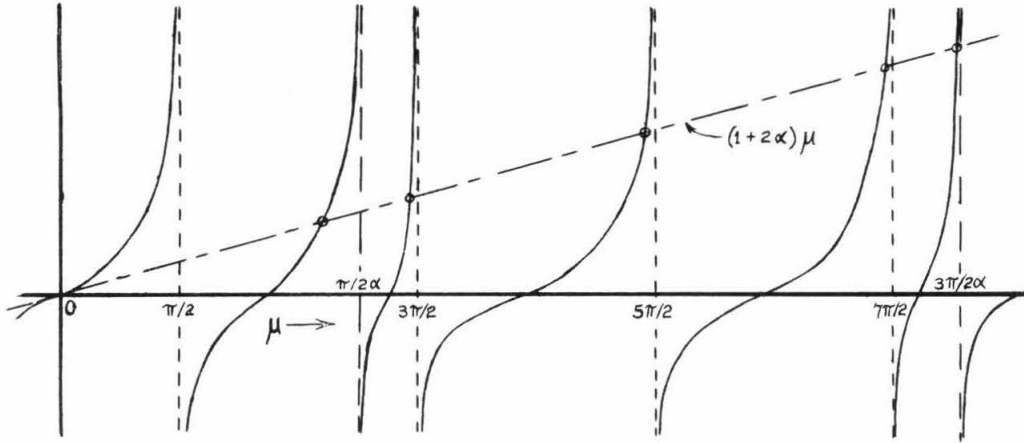
$$\left. \begin{aligned} \eta &= 2f \frac{h}{H_w} \frac{1}{\mu^2} \left[ 1 - \frac{\cos 2\mu x/l}{\cos \mu} \right], & 0 < x < l/2 \\ \eta &= 2f \frac{h}{H_w} \frac{1}{\mu^2} \left[ 1 - \frac{\cos \mu(1+\alpha - 2x/l)}{\cos \alpha \mu} \right], & l/2 < x < l/2 + l_1 \end{aligned} \right\} (5)$$

where  $\alpha = l_1/l$ .

The condition of inextensibility of the cable gives a relation determining  $\mu$ . Evaluating the integral  $\int_0^{l/2+l_1} \eta dx$  from the expressions for  $\eta$  and equating to zero, the following transcendental equation results,

$$\tan \mu + 2 \tan \alpha \mu = (1+2\alpha)\mu \quad (6)$$

Equation (6) has an infinite number of real roots  $\mu_1, \mu_2, \dots$  to each of which corresponds a frequency  $\omega$  and a mode (given by equation (5)). It is convenient to plot both sides of (6) as functions of  $\mu$ , then read off the intersections. Suppose that  $\alpha = l_1/l < \frac{1}{2}$ . Then the left side of (6) becomes infinite for  $\tan \mu = \infty$ , i.e.,  $\mu = \frac{2n+1}{2} \pi$ , and for  $\tan \alpha \mu = \infty$ , i.e.,  $\mu = \frac{2n+1}{2} \frac{\pi}{\alpha}$ . A plot of  $\tan \mu + \tan \alpha \mu$  appears as shown by the solid lines.



The right side of equation (6) is a straight line of slope  $1+2\alpha$ .

If  $S_1 = 0$ , that is, if the bridge consists of center span only, the

roots are given approximately by  $\mu = \frac{2n+1}{2} \pi$ ,  $n = 1, 2, \dots$  and if  $S_1/S = \infty$

i.e., the bridge consists of side spans only, the roots are approximately

$\mu = \frac{2n+1}{2} \frac{\pi}{\alpha}$ ,  $n = 0, 1, 2, \dots$ . The combination of center and side spans

gives roots only slightly different from those due to the sections

separately, and save possibly for the first one or two, the roots close

to  $\mu = \frac{2n+1}{2} \pi$  correspond to modes where the major portion of the energy

is in the center span, the other roots to the modes where the major

portion of the energy is in the side span. This tendency becomes more

pronounced in the higher modes.

Asymmetric Modes. In the asymmetric modes the center point of the cable must move back and forth longitudinally in each oscillation. Two cases will be considered:

(1) The longitudinal components of the inertial forces of the masses are neglected. This assumes that the cable is free to move relative to the suspended structure and that the horizontal effects from the cable inertia are negligible.

(2) The cable is tied to the mid-point of the suspended structure

so that the entire mass of the center span oscillates longitudinally in phase with the vertical oscillations. Therefore, in this second case the additional horizontal cable tension  $h$  is also asymmetric, which means that the difference of the cable tensions on the two sides of the center line (i.e.,  $2h$  in magnitude) must equal the inertial force due to the mass moving longitudinally.

#### Case (1)

Since the inertial forces in the longitudinal direction are neglected, the additional tension  $h$  is zero. The differential equation for the modes becomes, on putting  $h=0$  in equation (3)

$$\frac{d^2 \eta}{dx^2} + \left(\frac{2\mu}{l}\right)^2 \eta = 0$$

with the boundary conditions that  $\eta$  at  $x=0$ ,  $x=\pm l/2$  and  $x=\pm(l/2+l)$

Since the additional cable tension is zero, there can be no interaction between center and side spans and the vertical oscillations of the side spans must be independent of those of the center span. Hence, in the center span the modes and frequencies are given by

$$\left. \begin{aligned} \eta &= \eta_0 \sin n\pi \frac{2x}{l} \\ \mu &= \pi n \end{aligned} \right\} n = 1, 2, 3, \dots$$

and

and in the side spans by

$$\left. \begin{aligned} \eta &= \eta_0 \sin n\pi \frac{1}{l_1} (x - l/2) \\ \mu &= \pi n / 2\alpha \end{aligned} \right\} n = 1, 2, 3, \dots$$

and

where  $\eta_0$  is the amplitude of the mode in the span under consideration.

Since  $\eta(-x) = -\eta(x)$ , the condition for inextensibility of the cable is automatically satisfied.

#### Case (2)

The mass fixed to the center point of each cable is  $wl$  and if the

amplitude of the displacement of the center point longitudinally is  $\xi$  (in the x-direction) then the force which must act on the mass to make it oscillate with a frequency  $\omega$  is  $-\frac{\omega^2}{g} \xi$ . This force is given by the difference in cable tensions in the two halves, that is, if  $h$  is the maximum additional horizontal component of tension in the part of the cable for  $x > 0$ ,

$$2h = -\frac{\omega^2}{g} \xi = -4H_w \mu^2 \xi / l \quad (7)$$

The differential equation (3) must be solved with boundary conditions  $\eta = 0$  at  $x=0$ ,  $x=l/2$  and  $x=l/2+l_1$ . The solution satisfying the conditions is given by

$$\left. \begin{aligned} \eta &= 2f \frac{h}{H_w} \frac{1}{\mu^2} \left[ 1 - \frac{\cos \mu \left( \frac{1}{2} - \frac{2x}{l} \right)}{\cos \frac{1}{2} \mu} \right] & 0 < x < l/2 \\ \eta &= 2f \frac{h}{H_w} \frac{1}{\mu^2} \left[ 1 - \frac{\cos \mu (1 + \alpha - 2x/l)}{\cos \alpha \mu} \right] & l/2 < x < l/2 + l_1 \end{aligned} \right\} \quad (8)$$

The condition of inextensibility is satisfied since  $\eta(-x) = -\eta(x)$ , but there is the additional relation (7) to determine the frequencies. From equation (4), considering the part of the bridge for  $x > 0$  only,

$$\xi = L - L_0 \approx \frac{\omega^2}{H_w} \int_0^{l/2+l_1} \eta dx$$

Carrying out the integration and using equation (7) gives a transcendental equation for  $\mu$ ,

$$\tan \frac{1}{2} \mu + \tan \alpha \mu = \left( \frac{1}{2} + \alpha + \frac{l^2}{32f^2} \right) \mu \quad (9)$$

This equation can be solved for  $\mu$  as in the symmetrical case. The roots are slightly less than those values of  $\mu$  which make the left hand side infinite, i.e.,  $\mu = (2n+1)\pi$  and  $\mu = (2n+1)\pi/2\alpha$  where  $n = 0, 1, 2, \dots$

In addition to these frequencies and modes there are other possible asymmetric modes where the center point does not move and hence where



the additional cable tension is zero. Thus they are identical with those of case (1) which satisfy these conditions and are given by

$$\left. \begin{aligned} \mu = 2\pi n \quad & \left\{ \begin{array}{ll} \eta = \eta_0 \sin 4n\pi x/l & 0 < x < l/2 \\ \eta = 0 & l/2 < x < l/2 + l_1 \end{array} \right. \\ \mu = \pi n/\alpha \quad & \left\{ \begin{array}{ll} \eta = \eta_0 \sin 2n\pi (x/l_1 - 1/2\alpha), & l/2 < x < l/2 + l_1 \\ \eta = 0 & 0 < x < l/2 \end{array} \right. \end{aligned} \right\} n = 0, 1, 2, \dots$$

i.e., every second mode of case (1) occurs also in case (2).

#### Application to the Mode of Oscillation of the Tacoma Narrows Bridge.

In order to determine the numerical values of the frequencies and modes for a particular bridge of the type under consideration, it is to be noted that the only data required are the length  $l$ , the sag ratio  $f/l$  and the span ratio  $\alpha = l_1/l$ . In the case of the Tacoma Bridge,  $l = 2800$  ft.,  $f/l = 1/12$  and  $\alpha = 11/28$ . Hence, the frequency  $\omega = 0.2625 \mu$  radians/sec. or  $N = 2.506 \mu$  cycles/min.

The approximate values for the roots of the transcendental equations mentioned in the previous sections give the frequencies as accurately as is required in most cases but since the modes are very sensitive to slight changes in  $\mu$ , the values of  $\mu$  must be determined very carefully.

##### (A) Symmetrical modes.

The values of  $\mu$  which make the left hand side of equation (6) infinite, are in increasing order of magnitude

$$\text{or } \mu = \frac{14}{11}\pi, \frac{3}{2}\pi, \frac{5}{2}\pi, \frac{7}{2}\pi, \frac{42}{11}\pi, \frac{9}{2}\pi, \frac{11}{2}\pi$$

$$\mu = 4.00, 4.71, 7.86, 11.00, 12.00, 14.15, 17.30$$

A more accurate determination of the roots gives

$$\mu = 3.118, 4.645, 7.784, 10.928, 11.763, 14.102, 17.227$$

or  $N = 7.82, 11.63, 19.50, 27.4, 29.5, 35.4, 43.3$

The modes are sketched in Fig. VI-1, taking the amplitude of the center as one unit except in the case corresponding to  $N = 29.5$ .

(B) Asymmetric modes.

Case (1). The appropriate values of  $\mu$  in increasing order of magnitude are

$$\mu = \pi, \frac{14}{11}\pi, 2\pi, \frac{28}{11}\pi, 3\pi, \frac{42}{11}\pi, 4\pi$$

or  $N = 7.88, \underline{10.03}, 15.73, \underline{20.05}, 23.6, \underline{30.1}, 31.5$

In this approximation the deflection curves of all the modes are simple sine curves, those corresponding to the underlined frequencies involving motion of the side spans only, the others, center span only. (See Fig. VI-2)

Case (2). The appropriate values of  $\mu$  are given by the solutions of equation (9) and in addition the values from Case (1) which correspond to modes where the center point of the cable does not move. The values of  $\mu$  which make the left hand side of equation (9) infinite, along with those from Case (1) are, in increasing order of magnitude, exactly the same as the tabulated values for Case (1). A more accurate determination of the roots gives

$$\mu = 2.999, 3.892, 2\pi, \frac{28}{11}\pi, 9.385, 11.956, 4\pi, 15.685$$

$$N = 7.52, 9.76, 15.73, 20.05, 23.5, 30.0, 31.5, 39.2$$

These modes are sketches in Fig. VI-3.

The Effect of Flexural Rigidity of the Stiffening Girders. In order to consider the flexural rigidity of the girder, a term  $-\frac{EI}{H_w} \frac{d^4\eta}{dx^4}$  must be added to equation (3) where  $I$  is the moment of inertia of one

FIG. VI-1  
CALCULATED SYMMETRIC MODES

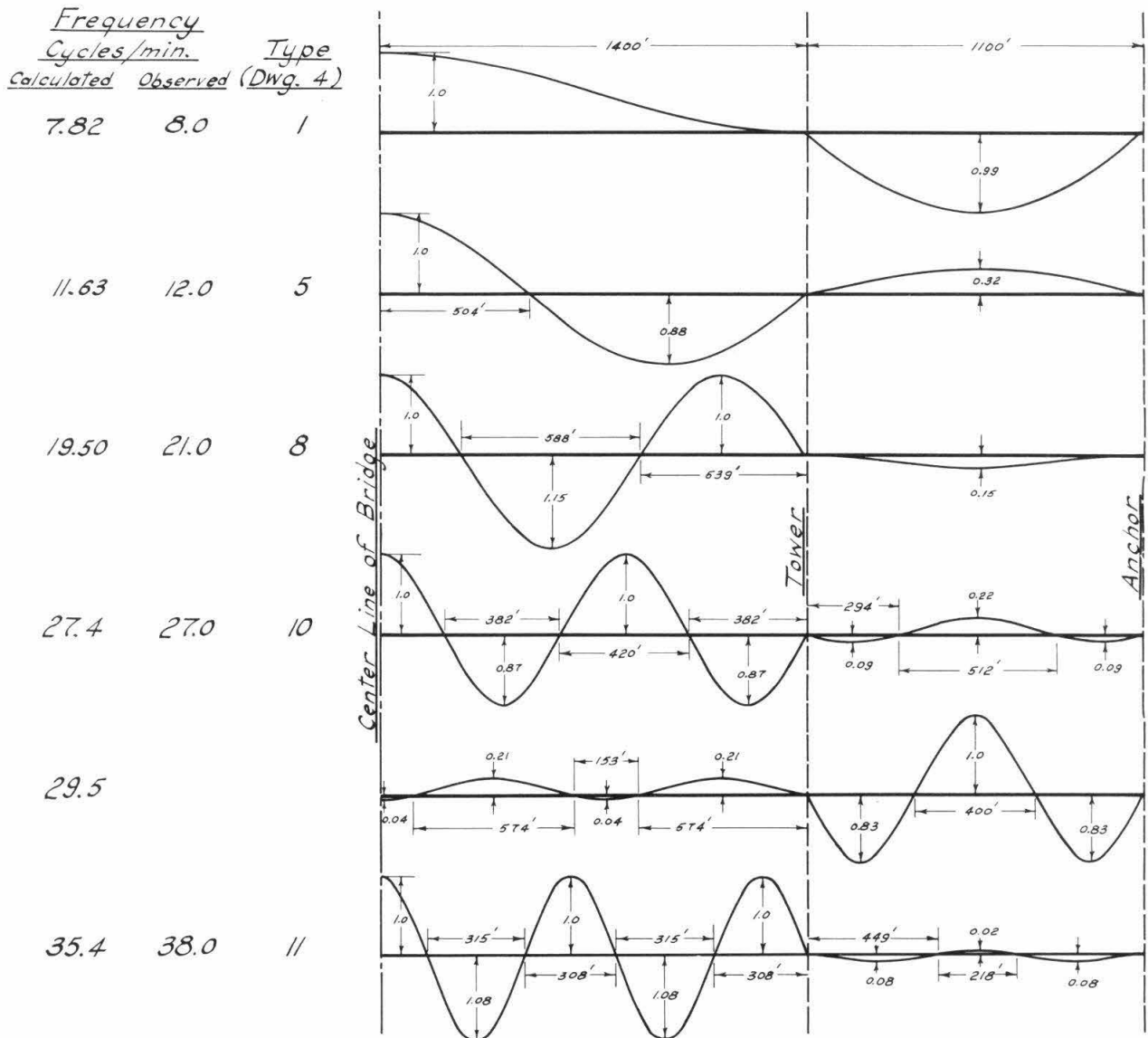


FIG. VI-2  
CALCULATED ASYMMETRIC MODES 1  
(MIDPOINT OF CABLE FREE)

| <u>Frequency</u>  |                                       | <u>Type</u><br><u>(Dwg. 4)</u> |
|-------------------|---------------------------------------|--------------------------------|
| <u>calculated</u> | <u>Cycles/min.</u><br><u>observed</u> |                                |

|      |     |   |
|------|-----|---|
| 7.88 | 8.7 | 2 |
|------|-----|---|

|       |  |   |
|-------|--|---|
| 10.03 |  | 4 |
|-------|--|---|

|      |      |   |
|------|------|---|
| 15.7 | 16.5 | 7 |
|------|------|---|

|      |  |  |
|------|--|--|
| 20.0 |  |  |
|------|--|--|

|      |      |   |
|------|------|---|
| 23.6 | 24.0 | 9 |
|------|------|---|

|      |  |  |
|------|--|--|
| 30.1 |  |  |
|------|--|--|

|      |      |    |
|------|------|----|
| 31.5 | 34.0 | 11 |
|------|------|----|

|      |  |  |
|------|--|--|
| 39.4 |  |  |
|------|--|--|

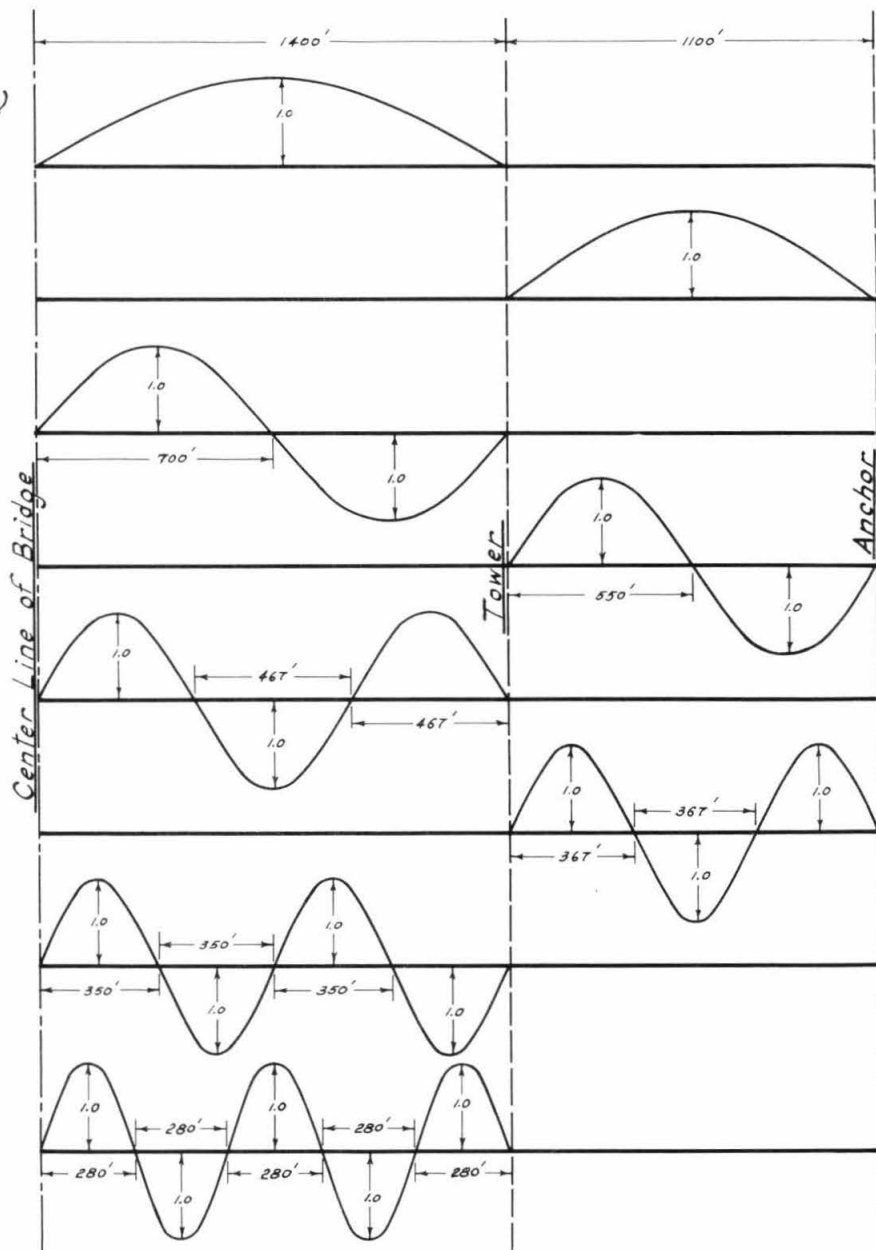
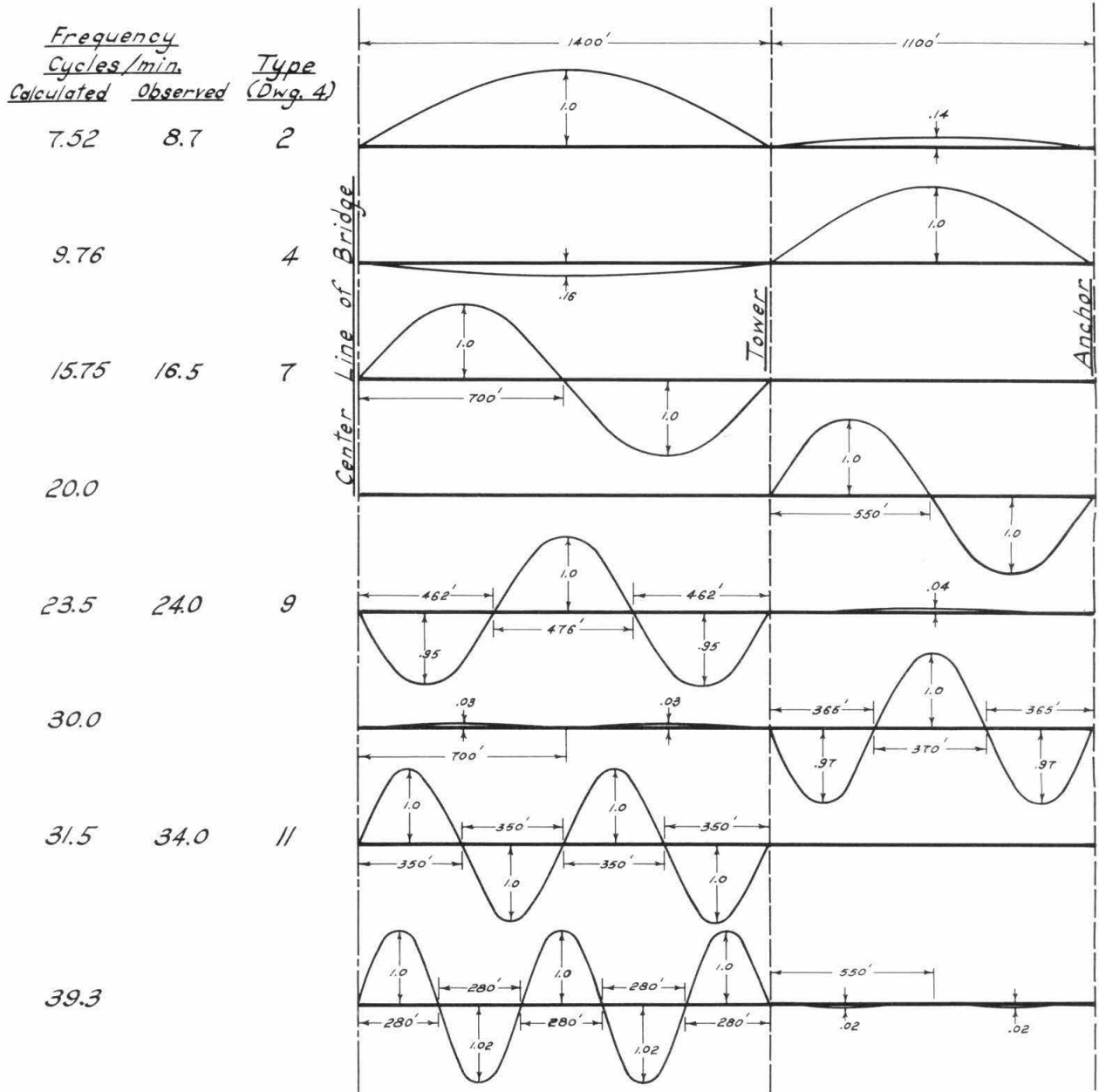


FIG. VI-3

CALCULATED ASYMMETRIC MODES 2  
(MIDPOINT OF CABLE TIED)



girder about its horizontal axis. The differential equation to be solved is then

$$-\frac{EI}{H_w} \frac{d^4 \eta}{dx^4} + \frac{d^2 \eta}{dx^2} + \left(\frac{2\mu}{l}\right)^2 \eta = 8 \frac{f}{l^2} \frac{h}{H_w} \quad (10)$$

with appropriate boundary conditions.

The method of solution will be sketched very briefly for the cases considered above.

(A) Symmetric modes.

Equation (10) is to be solved with the following conditions on  $\eta$ ,

$$\eta(-x) = \eta(x), \quad \eta = \frac{d^2 \eta}{dx^2} = 0 \quad \text{at} \quad x = l/2, \quad x = l/2 + l,$$

For convenience the following dimensionless quantities are introduced.

$$\begin{aligned} k^2 &= \frac{EI}{H_w} \left(\frac{l}{2}\right)^2 \\ k_i^2 &= k^2 [1 + (\mu/k)^2] \\ \mu_i^2 &= \mu^2 [1 - (\mu/k)^2] \end{aligned}$$

Consider only the cases where  $\mu/k$  is small compared with unity. The solutions satisfying the differential equation and the boundary conditions are then

$$\eta = 2f \frac{h}{H_w} \frac{1}{\mu^2} \left[ 1 - \frac{1}{1 + (\mu_i/k_i)^2} \frac{\cos 2\mu_i x/l}{\cos \mu_i} - \frac{(\mu_i/k_i)^2}{1 + (\mu_i/k_i)^2} \frac{\cosh 2k_i x/l}{\cosh k_i} \right]$$

for  $0 < x < l/2$ , and

$$\eta = 2f \frac{h}{H_w} \frac{1}{\mu^2} \left[ 1 - \frac{1}{1 + (\mu_i/k_i)^2} \frac{\cos \mu_i(1 + \alpha - 2x/l)}{\cos \alpha \mu_i} - \frac{(\mu_i/k_i)^2}{1 + (\mu_i/k_i)^2} \frac{\cosh k_i(1 + \alpha - 2x/l)}{\cosh \alpha k_i} \right]$$

for  $l/2 < x < l/2 + l$ .

By making  $k$  approach infinity, corresponding to zero bending stiffness of the roadway, the above expressions reduce to the former solution (5).

Applying the condition of inextensibility of the cable, the following transcendental equation for the frequencies results.

$$\tan \mu_1 + 2 \tan \alpha \mu_1 = (1 + 2\alpha) \mu_1 - \left(\frac{\mu_1}{k_1}\right)^3 (\tanh k_1 + 2 \tanh \alpha k_1) + (1 + 2\alpha) \frac{\mu_1^3}{k_1^2} \quad (11)$$

This equation differs from equation (6) in that  $\mu$  is replaced by  $\mu_1$  and small correction terms are added. The roots of (11) differ from those of (6) by terms of order equal to or less than  $(\mu/k)^4$ , so that for  $(\mu/k)^2$  small the result is

$$\mu = \mu_1 \left[1 - (\mu/k)^2\right]^{-\frac{1}{2}} \cong \mu_1 \left[1 + \frac{1}{2} (\mu_1/k_1)^2\right]$$

where  $\mu$  is now a root of equation (6). Hence, to take account of the bending stiffness of the suspended structure, the frequency for zero stiffness must be multiplied by a factor  $1 + \frac{1}{2} (\mu/k)^2$ , or in terms of parameters of the bridge, by a factor  $\left(1 + 4 \frac{fEI}{l^2 g} \omega^2\right)$  where  $\omega$  is the circular frequency calculated for zero bending stiffness.

#### (B) Asymmetric modes.

##### B.1. Center free.

In this case we have as before  $h=0$  and the expressions for  $\eta$  are

$$\left. \begin{aligned} \eta &= \eta_0 \sin 2n\pi x/l & 0 < x < l/2 \\ \eta &= \eta_0 \sin n\pi (x/l - 1/2\alpha), & l/2 < x < l/2 + l_1 \end{aligned} \right\} n = 1, 2, \dots$$

The values of  $\mu$  are given by

$$\mu = n\pi \left[1 + \frac{1}{2} (\mu/k)^2\right] \quad \text{for motion of center span only,}$$

and  $\mu = \frac{n\pi}{2\alpha} \left[1 + \frac{1}{2} (\mu/k)^2\right]$  for motion of side spans only.  
as long as  $(\mu/k)^2$  is small.

##### B.2. Center tied.

The correction term to be applied to the frequencies to take

account of bending stiffness of the roadway will be the same as in the above cases.

For the Tacoma Narrows Bridge the value of  $I$  for one girder was about 1000 ft.<sup>2</sup> in.<sup>2</sup> Hence, the correction factor to be applied to the frequencies already calculated is  $(1 + .00925 \omega^2)$  or  $(1 + .000101 N^2)$  where  $N$  is the frequency in cycles per minute.

Approximate Formulae for the Frequencies. It is possible to deduce from the transcendental equations (6) and (9), approximate formulae for the roots by considering the terms due to side spans separately from those due to the center span. For instance, in place of equation (6) consider the equations  $\tan \mu = \mu$  for the center span and  $\tan \alpha \mu = \alpha \mu$  for the side spans. The roots of  $\tan \chi = \chi$  are given to a close degree of approximation by the formula  $\chi = \frac{2n+1}{2} \pi \left[ 1 - \frac{4}{(2n+1)^2 \pi^2} - \frac{32}{3(2n+1)^4 \pi^4} \dots \right]$  where  $n = 1, 2, 3, \dots$ . Hence, the results can be written for the various cases considered as follows, where the correction factor for flexural rigidity of the roadway is added. Denoting by (a) the modes involving center span only and by (b) the modes involving side spans only.

(A) Symmetric modes.

$$(a) \quad N = 15\sqrt{g/2f} \, n \left( 1 - \frac{4}{\pi^2 n^2} - \frac{32}{3\pi^4 n^4} \dots \right) \left( 1 + \frac{4\pi^2 E I f}{w \int^4} n^2 \right)$$

$$(b) \quad N = \frac{15}{\alpha} \sqrt{g/2f} \, n \left( 1 - \frac{4\alpha^2}{\pi^2 n^2} - \frac{32\alpha^4}{3\pi^4 n^4} \dots \right) \left( \quad \quad \quad \right)$$

where  $n = 3, 5, 7, \dots$

(B) Asymmetric modes.

(1) Center Free.

$$(a) \quad N = 30\sqrt{g/2f} \, n \left( 1 + \frac{16\pi^2 E I f}{w \int^4} n^2 \right)$$

$$(b) \quad N = \frac{15}{\alpha} \sqrt{g/2f} \, n \left( 1 + \frac{4\pi^2 E I f}{w \int^4 \alpha^2} n^2 \right)$$

where  $n = 1, 2, 3, \dots$



(2) Center fixed to the roadway.

$$(a) \quad N = 30\sqrt{g/2f} \, n \left( 1 + \frac{16\pi^2 E I f}{w l^4} n^2 \right)$$

$$(b) \quad N = \frac{15}{\alpha} \sqrt{g/2f} \, n \left( 1 + \frac{4\pi^2 E I f}{w l^4 \alpha^2} n^2 \right)$$

where  $n = 1, 3, 5 - - -$

Note that the frequency of fundamental symmetric mode is not given by these formulae. The reason is that, in deriving the formulae it was assumed that there was no interaction between center spans and side spans. This is approximately true for all but the fundamental mode.

These results are very similar to expressions given recently by Westergaard (21). He considers a single span with rigid towers, and uses an energy method to determine the natural periods. His results (Equations (126) and (127)) written in the notation used above and expanded in powers of small quantities to be more directly comparable are, for the symmetric modes,

$$N = 15\sqrt{g/2f} \, n \left( 1 - \frac{1}{2n^2} + \frac{11}{8n^2} \right) \left[ 1 + \frac{4\pi^2 E I f}{w l^4} n^2 \left( 1 + \frac{1}{n^2} \right) \right]$$

$n = 3, 5, 7 - - -$

and for the asymmetric modes,

$$N = 30\sqrt{g/2f} \, n \left( 1 + \frac{16\pi^2 E I f}{w l^4} n^2 \right)$$

$n = 1, 2, 3 - - -$

Comparison of Theoretical Results with Observations on the Tacoma Narrows Bridge. The results are tabulated below, column I giving the calculated frequencies corrected for rigidity of the stiffening girders, column II, the frequencies from the approximate formulae, and column III the frequencies from Westergaard's formulae (all frequencies in cycles per minute). (a) refers to modes where the largest amplitude is in the center span, (b) to modes where the largest amplitude is in the side spans. The extreme observed values of the frequencies of the modes

which have been identified are given in Column IV, and column V lists the type numbers as designated in Drawing 4 of the modes as observed in the model.

| (A) Symmetrical modes | I     | II    | III   | IV        | V  |
|-----------------------|-------|-------|-------|-----------|----|
| Fundamental           | 7.82  | --    | --    | 7.5/8.6   | 1  |
| (a)                   | 11.82 | 11.42 | 11.50 | 11.6/12.0 | 5  |
|                       | 20.24 | 20.14 | 20.10 | 19.7/21.7 | 8  |
|                       | 29.5  | 29.5  | 29.4  | 27.0      | 10 |
|                       | 39.9  | 39.7  | 39.7  | 38.0      | 12 |
|                       | 51.5  | 51.5  | 51.5  | --        | -- |
| (b)                   | 32.1  | 32.8  | --    | --        | -- |
| (B) Asymmetric modes  |       |       |       |           |    |
| (1) Center Free       |       |       |       |           |    |
| (a)                   | 7.92  | 7.92  | 7.92  | 8.7       | 2  |
|                       | 16.15 | 16.15 | 16.15 | 16.2/17.7 | 7  |
|                       | 24.92 | 24.92 | 24.92 | 24.6      | 9  |
|                       | 34.6  | 34.6  | 34.6  | 34.0      | 11 |
|                       | 45.6  | 45.6  | 45.6  | --        | -- |
| (b)                   | 10.12 | 10.12 | --    | --        | 4  |
|                       | 20.86 | 20.86 | --    | --        | -  |
|                       | 32.9  | 32.9  | --    | --        | -  |
|                       | 46.8  | 46.8  | --    | --        | -  |
| (2) Center Tied       |       |       |       |           |    |
| (a)                   | 7.56  | 7.92  | --    | 8.7       | 2  |
|                       | 16.15 | 16.15 | --    | 16.2/17.7 | 7  |
|                       | 24.8  | 24.9  | --    | 24.6      | 9  |
|                       | 34.6  | 34.6  | --    | 34.0      | 11 |
|                       | 45.6  | 45.6  | --    | --        | -- |
| (b)                   | 9.86  | 10.12 | --    | --        | 4  |
|                       | 20.86 | 20.86 | --    | --        | -  |
|                       | 32.8  | 32.9  | --    | --        | -  |
|                       | 46.8  | 46.8  | --    | --        | -  |

The observed asymmetric modes have been listed in both B,1 and B,2 as the assumptions for these cases lead to results so similar that the observations alone cannot determine which assumption more nearly fits the facts.

The agreement of the theoretical results with the observations of frequencies and modes on both prototype and model is remarkably good.

Since the observations of deflection curves for the modes on the prototype were not obtained very accurately, especially in the higher modes, which were seldom maintained in pure form for any length of time, this is rather surprising.

The only observed types not identified with the calculated modes are Types 3 and 6. Although no frequency is given for the latter, it is probably the same as Type 5, where the side span amplitude was too small to be noticed. Type 3 as shown by Drawing 4 violates the condition of inextensibility of the cable since the deflection has the same sign over the entire bridge length. It is unlikely that it could be explained by cable elongation as the frequency is too low. Because the frequency is the same as that of Type 7, and because in the three observations in the prototype identified as Type 3 the wind speed was about the same as that for Type 7, it is more likely an error in observation, or perhaps a combination of modes. It is not understood how Type 3 could be observed on the model unless it was not a pure mode.

As can be seen from Figs. VI-1, and VI-3, the oscillations of the center span and the side spans can be considered almost independent, the exception being the fundamental symmetric mode. If the main motion is in the center span, the side spans oscillate with a very small amplitude as if a small oscillating force acted on them. If the main motion is in the side spans, the center span is forced into a vibration of relatively small amplitude. Hence, in the case where the main motion is in the center span, the tower tops move through only a very small amplitude for all but the fundamental symmetric mode. The higher modes involving side span motion mainly have the same characteristic.

Thus in the calculation of frequencies of the higher modes, it is

sufficient to assume that the towers are rigid and that the spans vibrate independently. This assumption is the basis of Westergaard's formulae and of the equivalent approximate formulae developed above, both of which give very accurate results.

Effect of Tower Rigidity. When there is interaction of the three spans, the vibration of the system involves that of the towers, and for extreme precision the effects of the rigidity of the towers must be considered. Formulae for the correction to account for this effect have been developed. Substituting numerical values, it was found that for the Tacoma Narrows Bridge, the correction for the fundamental mode would be smaller than that for the stiffening girders and is therefore of a negligible order. With the higher modes, it has been stated in the previous section that the amplitude of motion of the tower tops is small. Therefore, except for towers of unusual rigidity, the effect of their rigidity on the frequency is negligible.

Calculation of Energies of the Modes. Since it is assumed that the vibrating bridge is a conservative system, the total energy in a vibration is equal to the maximum value of the kinetic energy, or to the maximum value of the potential energy. In our case it is easier to determine the maximum kinetic energy.

#### A. Symmetric modes.

The maximum kinetic energy per unit length of the bridge is

$$2 \cdot \frac{1}{2} \cdot \frac{w}{g} \cdot \left( \frac{\partial y}{\partial t} \right)^2 = \frac{w}{g} \omega^2 \eta$$

Hence, for the whole bridge, if the total energy is  $W$ ,

$$W = 2 \frac{w}{g} \omega^2 \int_0^{1/2 + l_1} \eta^2 dx = 4 f w \frac{h^2}{H_w^2} \frac{1}{\mu^2} \left[ \int_0^{1/2} \left( 1 - \frac{\cos 2\mu x/l_1}{\cos \mu} \right)^2 dx + \int_{1/2}^{1/2 + l_1} \left\{ 1 - \frac{\cos \mu(1 + \alpha - 2x/l_1)}{\cos \alpha \mu} \right\}^2 dx \right]$$

$$W = 4fw \frac{h^2}{H_w^2} \left[ \frac{1}{4} (3 + \tan^2 \mu - \frac{3}{\mu} \tan \mu) + \frac{1}{2} (3\alpha + \alpha \tan^2 \alpha \mu - \frac{3}{\mu} \tan \alpha \mu) \right]$$

and making use of the transcendental equation (6) this simplifies to

$$W = flw \frac{h^2}{H_w^2} \frac{1}{\mu^2} (\tan^2 \mu + 2\alpha \tan^2 \alpha \mu)$$

It is convenient to determine the energies in terms of the amplitude

$$\text{at } \alpha = 0, \text{ i.e., } \eta_0 = 2f \frac{h}{H_w} \frac{1}{\mu^2} \left(1 - \frac{1}{\cos \mu}\right)$$

Then  $\frac{4fW}{lw\eta_0^2} = \frac{\mu^2}{(1 - 1/\cos \mu)^2} (\tan^2 \mu + 2\alpha \tan^2 \alpha \mu)$ , so the energies of the modes can be determined in terms of the deflection at center  $\eta_0$ .

on substituting for  $\mu$  the roots of equation (6). For the higher modes

$\tan^2 \mu \cong (1/\cos \mu)^2 \cong (1 - 1/\cos \mu)^2$  and the approximate formula  $W \cong \frac{wl}{f} \eta_0^2 \left(\frac{\mu}{2}\right)^2$  is satisfactory for the modes which involve the center span mainly. The corresponding approximate formula for the higher modes involving mainly side span motion is  $W \cong 2\alpha \frac{wl}{f} \eta_0^2 (\mu/2)^2$ , where  $\eta_0$  is now the maximum deflection in the side spans.

#### B. Asymmetric modes.

Case (1), (Center free). The corresponding expressions for the energies are  $W = \frac{wl}{f} \eta_0^2 \left(\frac{\mu}{2}\right)^2$  for the center span and  $W = 2\alpha \frac{wl}{f} \eta_0^2 \left(\frac{\mu}{2}\right)^2$  for the side spans, where the formulae are now exact.

Case (2), (Cables tied to center of bridge). The expression for the energy is

$$W = 2 \frac{wl}{g} \omega^2 \int_0^{1/2+l_1} \eta^2 dx + \frac{wl}{g} \omega^2 \xi^2$$

Substituting the appropriate expressions for  $\eta$  and  $\xi$  and integrating, the expression for the energy becomes

$$W = flw \frac{h^2}{H_w^2} \frac{1}{\mu^2} \left( \tan^2 \frac{1}{2} \mu + 2\alpha \tan^2 \alpha \mu - \frac{1}{16} \frac{\mu^2}{f^2} \right)$$

This expression can be written in terms of the maximum deflection in

either center span or side spans. Again for the higher modes the approximate formulae for the total energy are  $W = \frac{w f}{f} \eta_0^2 (\mu/2)^2$  for the maximum amplitude in the center span and  $W = 2\alpha \frac{w f}{f} \eta_0^2 (\mu/2)^2$  for the maximum amplitude in the side spans. The approximations are quite close for all but the first mode, which involves a considerable amplitude of the center point and hence considerable energy in the longitudinal motion of the roadway.

Energy Stored in Stiffening Girders. The elastic energy of bending in the girders during oscillations is of interest in a treatment of the effect of structural damping. It will be assumed that the deflection of the girders is given with sufficient accuracy by the modes already determined for zero bending stiffness. The potential energy of a unit length of one girder is  $1/2 E I \left( \frac{d^2 \eta}{dx^2} \right)^2$ , so that for the entire lengths of both girders the energy is given by  $W_g = 2 E I \int_0^{1/2+l} \left( \frac{d^2 \eta}{dx^2} \right)^2 dx$

A. Symmetric modes.

Substituting the appropriate values of  $\eta$  into the integral and carrying out the calculation, the result is

$$W_g = 32 \frac{E I f^2}{l^3} \frac{h^2}{H_w^2} \left[ \tan^2 \mu + 2\alpha \tan^2 \alpha \mu + 2(1+2\alpha) \right]$$

Hence, the ratio of the energy in the girders to the total energy of the structure calculated in the previous section is

$$W_g/W = \frac{32 E I f}{w l^4} \mu^2 \left[ 1 + \frac{2(1+2\alpha)}{\tan^2 \mu + 2\alpha \tan^2 \alpha \mu} \right]$$

For all modes except possibly the fundamental, the second term in the square bracket is small compared with unity and the approximate expression  $W_g/W = \frac{32 E I f}{w l^4} \mu^2$  is quite sufficient. The ratio for the various modes increases proportionally to the square of the frequency as might be

expected since the energies of the modes of a vibrating string or cable for given amplitude are proportional to the square of the frequency and the energies of the modes of a vibrating beam are proportional to the fourth powers of the frequencies. The formulae above, of course, do not hold for frequencies so high that the beam alone determines the deflection curve, so they cannot be considered accurate for values of  $W_g/W$  above about 1/4.

#### B. Asymmetric modes.

Case (1), Center Free. The result in this case is  $W_g/W = \frac{32fEI}{w l^4} \mu^2$  for modes involving either center span alone or side spans alone. In this case the expression is accurate for all frequencies and modes, since the modes and frequencies of the girders alone are exactly the same as the modes and frequencies of the cables alone.

Case (2), Cable tied to center of roadway. The expression for the potential energy of the girder in this case is

$$W_g = 32EI \frac{f^2}{l^3} \frac{h^2}{H_w^2} \left[ \tan^2 \frac{1}{2} \mu + \frac{\alpha}{2} \tan^2 \alpha \mu + \frac{l^2}{16f^2} + 2(1+2\alpha) \right]$$

The ratio of energy in the girders to total energy is

$$W_g/W = \frac{32fEI}{w l^4} \mu^2 \left[ 1 + \frac{2(1+2\alpha) + \frac{l^2}{8f^2}}{\tan^2 \frac{1}{2} \mu + 2\alpha \tan^2 \alpha \mu - \frac{l^2}{16f^2}} \right]$$

and from this the approximate expression  $W_g/W = \frac{32fEI}{w l^4} \mu^2$  may be derived as in the previous cases.

It is interesting to note the connection between the energy expression and the correction factor applied to the frequencies to allow for flexural rigidity of the girders. This factor was  $\left[ 1 + \frac{1}{2} \left( \frac{\mu}{k} \right)^2 \right]$  and on substituting the value of  $k$ , it is equal to  $1 + \frac{16EI}{w l^4} \approx 1 + \frac{1}{2} W_g/W$

Tabulated below are the values of  $\frac{W f}{w l \eta_o^2}$  and of  $W_g/W$  for the

modes already considered for the Tacoma Narrows Bridge, as calculated from the exact and approximate expressions.

A. Symmetric modes, ( $\eta_0$  = maximum deflection at the center)

|                                |  |       |       |       |      |       |      |
|--------------------------------|--|-------|-------|-------|------|-------|------|
| N , cycles/min.                |  | 7.82  | 11.63 | 19.50 | 27.4 | 29.5* | 35.4 |
| $\frac{Wf}{w\eta_0^2}$ (exact) |  | 3.21  | 5.00  | 16.35 | 26.5 | 23.05 | 55.0 |
| " (app.)                       |  | 2.43  | 5.39  | 14.34 | 29.8 | 27.2  | 49.7 |
| $W_g/W$ (exact)                |  | .0124 | .0275 | .077  | .153 | .176  | .254 |
| " (app.)                       |  | .0124 | .0275 | .077  | .153 | .176  | .254 |

\*For this mode  $\eta_0$  is the maximum deflection in the side span.

B. Asymmetric modes, ( $\eta_0$  = maximum deflection)

1. Center free.

|                                |  |       |       |      |       |      |      |      |      |
|--------------------------------|--|-------|-------|------|-------|------|------|------|------|
| N , cycles/min.                |  | 7.88  | 10.0  | 15.7 | 20.0  | 23.6 | 30.1 | 31.5 | 39.4 |
| $\frac{Wf}{w\eta_0^2}$ (exact) |  | 2.465 | 3.14  | 9.87 | 12.53 | 22.2 | 28.2 | 39.5 | 61.6 |
| $W_g/W$ (exact)                |  | .0125 | .0200 | .050 | .081  | .113 | .183 | .201 | .314 |

2. Center tied.

|                                |  |       |      |       |       |      |      |      |      |
|--------------------------------|--|-------|------|-------|-------|------|------|------|------|
| N , cycles/min.                |  | 7.52  | 9.76 | 15.75 | 20.0  | 23.5 | 30.0 | 31.5 | 39.3 |
| $\frac{Wf}{w\eta_0^2}$ (exact) |  | 2.536 | 3.23 | 9.87  | 12.53 | 21.0 | 27.1 | 39.5 | 62.5 |
| " (app.)                       |  | 2.25  | 2.98 | "     | "     | 22.0 | 28.1 | "    | 61.4 |
| $W_g/W$ (exact)                |  | .013  | .020 | .050  | .081  | .112 | .184 | .201 | .304 |
| " (app.)                       |  | .012  | .020 | "     | "     | "    | "    | "    | "    |

#### TORSIONAL OSCILLATIONS OF THE STRUCTURE

In the foregoing, only vertical oscillations of the roadway have been discussed. However, the torsional modes may be obtained immediately by considering them as vertical oscillations with the cables  $180^\circ$  out of phase. If the entire mass of the roadway were concentrated at the stiffening girders, the frequencies of the torsional modes would be exactly



those of the vertical modes from which they are derived. If the weight of the roadway is  $2w_r$  per unit length, the weight of the cables and suspenders  $w-w_r$  per unit length on each side, the radius of gyration of the roadway cross section about its center is  $r_r$  and the distance between the cables  $b$ , the inertial load  $\frac{w}{g} \omega^2 \eta$  in the vertical oscillation must be replaced by  $\frac{1}{g} (w_r \frac{4r_r^2}{b^2} + w - w_r) \omega^2 \eta = \frac{w}{g} (\frac{2r}{b})^2 \omega^2 \eta$  where  $r$  is the effective radius of gyration of the cables and roadway together and

$$(\frac{2r}{b})^2 = 1 - \frac{w_r}{w} (1 - \frac{4r_r^2}{b^2})$$

The resulting equation for the deflection is the same as (3) if the factor  $\frac{2\mu}{1}$  is replaced by  $\frac{4\mu r}{b^2}$ . The solutions will be of the same form, and the frequencies may be obtained from those above calculated for vertical oscillations by multiplying them by the factor  $\frac{b}{2r}$

For the Tacoma Bridge,  $w = 2860$  lb./ft.,  $w_r = 747$  lb./ft.,

$r_r = 13.0$  ft.,  $b = 39$  ft. Hence,  $2r/b = 0.770$  and the frequencies obtained previously must be multiplied by the factor 1.300 to obtain the corresponding torsional frequencies. Of particular interest is the torsional mode which led to failure of the structure, i.e., with one node at the center. For the case where the cable is free to move relative to the roadway the frequency is from the previous section  $7.92 \times 1.300 = 10.30$  cycles/min. This mode involves no motion of the side spans and no additional cable tension.

The case where the cables are tied to the center of the stiffening girders involves lateral motion of the roadway since the forces due to additional cable tensions acting on the two sides of the roadway at the center are equal and opposite, giving an oscillating moment about a vertical axis. Calculations which will not be reproduced here show that the effect is very nearly as if the lowest points of the cable in the

center span were anchored in space. The lateral motion of the roadway is negligible. The frequency in such a case is, for the torsional oscillation  $9.24 \times 1.300 = 12.00$  cycles/min. This motion involves a maximum deflection in the side span about 25% greater than that in the main span. The additional cable tension is approximately 69,000 lb. per ft. amplitude of the center span deflection.

In these calculations the effect of torsional rigidity of the roadway has been neglected, as well as the torsional rigidity of the towers in the second case considered. The torsional mode observed prior to failure of the bridge had a large amplitude in the center span and no comparable amplitude in the side spans. The frequency was, according to reports, 14 cycles per minute at first and changed to 12 cycles per minute after the structure began to fail. It is rather difficult to compare the calculated frequencies with the observations since the towers probably have much more influence in torsional modes than in the vertical modes and further the amplitudes go beyond the range of small deflection theory. Nevertheless, it is thought that certain conclusions can be drawn from the results of the calculations above.

Originally the cables were tied by guys to the stiffening girders so relative motion longitudinally was prevented. It seems certain that these guys, and probably the short suspenders at the center as well, failed or came loose on one or both sides before the torsional oscillation reached the observed amplitude of 13.5 ft. Otherwise, as the calculations show, the side spans must have torsional oscillations of about 17 ft. amplitude, the towers must twist and the additional cable tension would amount to about 1,000,000 lb. This latter requires that

the guys and short suspenders on each side must exert a maximum longitudinal pull of 2,000,000 lb. since the additional cable tensions are of opposite sign in the two halves of the bridge. Although the extension of the cable has been neglected, it could not be sufficient to reduce the loads to a figure which the guys and suspenders could support.

The assumption that the cables are free at the center leads to results which fit the observations much better. In this case there is no additional cable tension, no motion of the towers and no interaction between side spans and center span. The mid-point of the cable oscillates longitudinally relatively to the suspended structure through an amplitude given approximately by  $\frac{2}{3\pi}$  times the maximum vertical amplitude. This gives a longitudinal amplitude of about 3 ft. for the vertical amplitude of 13.5 ft. It was found on examining the mid-point of one of the cables after the failure that it was worn at the mid-point over a length of 68 in. where the band clamping the guys to the cable had slipped. On making allowance for the width of the cable band a double amplitude of 41 in. is obtained for the relative motion.

The question as to when failure of the guys and suspenders took place probably cannot be answered. It is interesting to notice that the ratio of the frequencies based on the two assumptions of cables tied and cables free is exactly the same as the ratio of the two observations of the frequencies after torsional oscillations began.

## APPENDIX VII

### EXPERIMENTAL INVESTIGATIONS IN CONNECTION WITH THE TACOMA NARROWS BRIDGE AT THE UNIVERSITY OF WASHINGTON.

The considerations leading to the various experimental tests made in connection with the Tacoma Narrows Bridge by Professor F. B. Farquharson at the University of Washington have been mentioned in Chapter I. The experiments covered two rather distinct fields, the first concerned itself with the dynamic characteristics of the bridge, and the second with the aerodynamics of its suspended structure considered as an airfoil. The tests were made possible by cooperation of the University of Washington, the Washington State Highway Department and the United States Public Roads Administration. No attempt is made in this Appendix to completely describe the experiments nor to reproduce the large amount of data secured. Instead, the results of the experiments which contribute directly to our investigation will be summarized.

#### DYNAMIC MODEL TESTS

The dynamic model tests may be considered under the following headings.

- a. Periods and Modes of Oscillations.
- b. Energy Dissipation as affected by weight, rigidity of the stiffening girders and by various damping appliances.

The dynamic model has been described elsewhere (7). In the series of tests this model was partially rebuilt to introduce the variables considered.

Periods and Modes of Oscillations. One of the first steps in the in-

vestigation was to correlate the periods and modes of oscillations with those actually observed on the prototype. The various types of motions and their frequencies as determined on the model are reproduced on Drawing 4.

The extent to which the various elements of a suspension bridge contributes to its periods has been shown mathematically in Appendix VI. Data from the experiments are as follows:

Frequency of Tacoma Narrows Bridge as Determined by Model Tests

| Type.<br>Dwg.<br>4 | No. of<br>Nodes in<br>Main Span | Frequency -- Cycles per Minute (x)<br>for Condition |      |      |      |           |       |
|--------------------|---------------------------------|---|------|------|------|-----------|-------|
|                    |                                 | a   | b    | c    | d    | e         | f     |
| 1                  | 0                               | 8.0   | 7.8  | 7.2  | 8.0  | 7.5/8.6   | 7.81  |
| 2                  | 1*                              | 8.0   | 8.0  | 8.0  | 8.7  | g         | 7.94  |
| 5                  | 2                               | 12.0  | 12.0 | 12.0 | 12.0 | 11.6/12.0 | 11.62 |
| 8                  | 4                               |   | 20.4 |      | 20.6 | 20.1/21.7 | 20.1  |

Condition a. - Cables, Suspenders and Towers only.

b. - Full Dead Load on Suspenders.

c. - Full Dead Load on Suspenders - Towers Restrained at Roadway Level.

d. - Bridge As Built with Full Dead Load and Stiffening Girders.

e. - Observed on Bridge.

f. - As calculated, Appendix VI.

g. - Not Given in Tables 1 and 2.

(x) - For Prototype, Frequency on Model was 10 Times as great.

\* - With Towers in Phase, i.e., Tower Tops moving in same direction.

While the data does not cover all the modes observed on the model and on the prototype, it permits certain conclusions and confirms that the errors arising from the assumptions made in Appendix VI are of a minor nature.

The principal conclusion is, within the modes considered, that the frequencies are controlled primarily by the cables. A comparison of columns b and d shows that the influence of the stiffening girders is comparatively small.

The results of fixing the towers at the roadway level, thereby increasing the stiffness of the cable, is also small. It would be expected that the effect, if any, would be to slightly increase the frequency. The only difference occurs in the fundamental mode and reduces the frequency. Stiffening the towers and thereby increasing their frequency evidently has little, if any, effect in the third mode, Type 5, in which mode the natural frequency of the unstiffened towers corresponds with the natural frequency of the combined system.

The calculated frequencies are slightly below those observed either on the model or on the prototype. The difference is not greater than may be explained by errors of observation, but it is consistently in the same direction. This indicates that the factors adding to the rigidity of the system which have been neglected in the calculation have noticeable influence.

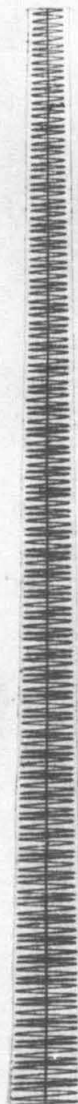
Effect of the Stay Cables. After it had been found that the center ties and the hydraulic buffers at the towers were insufficient to keep the amplitudes of the oscillations within satisfactory limits, the next logical step was to investigate various systems of diagonal ties from

the tops of the towers to the roadway. In making these tests, it was assumed that the effects of the various devices on the structure would be reflected, at least qualitatively, in the rate of decrease of the amplitudes. In the series with single stays each stay was given an initial tension which would produce a vertical deflection of 20 ins., on the prototype, at the point of attachment. For comparative purposes an initial amplitude of 60 ins. (0.6 ins. on the model) was assumed.

The results shown by Figs. VII-2 and VII-2 copied from Prof. Farquharson's records, are typical of those secured for several different arrangements of stays. Others included stays from the roadway level at the towers to the cables and from the roadway level to the base of the piers. The variables in this series include also stays fixed to the top of the towers, or to a fixed rocker, stays passing over a movable rocker and different degrees of friction applied to the rocker.

As a result of these tests Prof. Farquharson concludes in respect to the Tacoma Narrows Bridge, "One can conclude from these curves that at least for the arrangements tested, diagonal stays fixed at either top or bottom of the towers are only moderately effective (at least with respect to the two-noded motion which was applied to each of these conditions). In addition, one can conclude that the attachments of these stays to a rocker at either the top of the tower or at roadway level, depending on the type of stay, very greatly improves the damping rate. It is further apparent that varying degrees of friction applied on these rockers once again effect a drastic change in the damping characteristics of the main span."

The tests are not sufficiently comprehensive to permit conclusions



BRIDGE AS BUILT  
406 SECS. TO DAMP FROM 0.6" AMP.

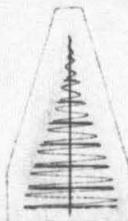


FIXED ROCKER AT TOWER TOP  
152 SECS.



FREE ROCKER AT TOWER TOP  
32 SECS.

0 100 200  
VERTICAL DEFLECTION SCALE  
INCHES ON PROTOTYPE



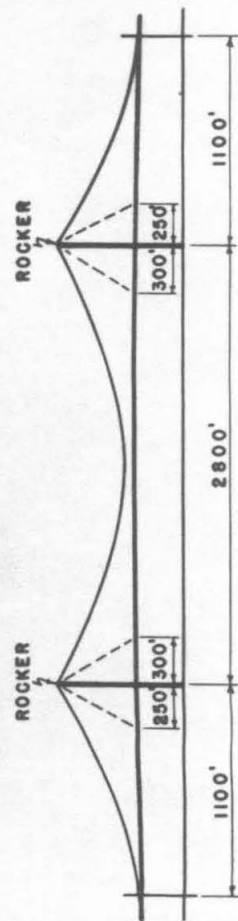
HEAVY FRICTION ON ROCKER  
8 SECS.



TIME IN SECONDS

FIG. VII-1  
TACOMA NARROWS BRIDGE

TIME - AMPLITUDE RECORDS WITH VARIOUS  
CONDITIONS OF DIAGONAL STAYS  
TESTS RUN ON MODEL





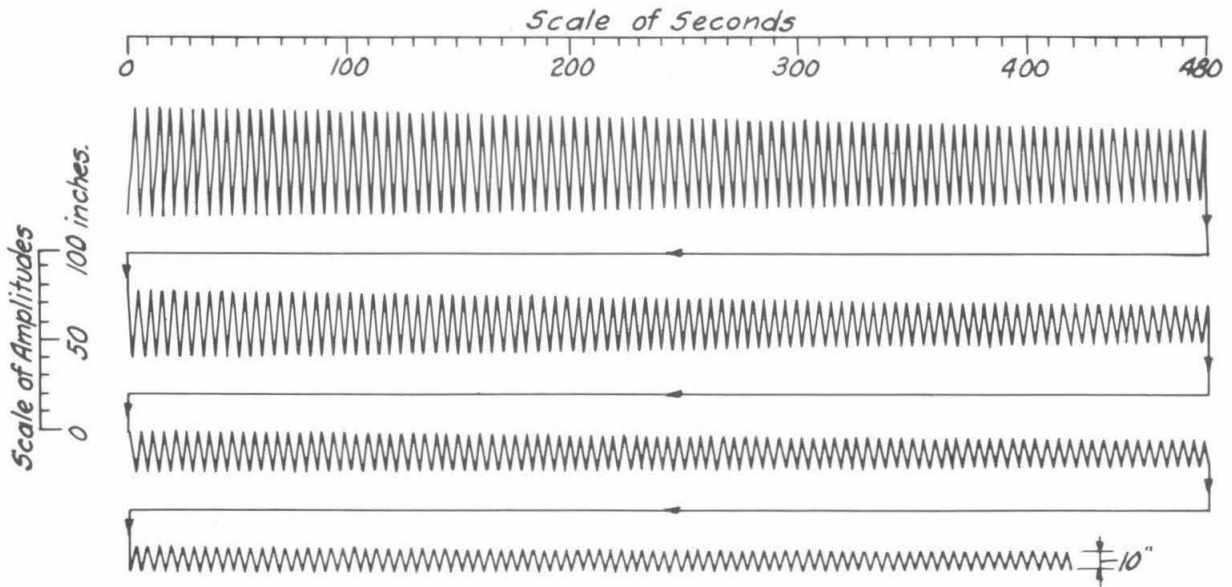
on the quantitative effect of different systems of stays on dynamic oscillations. Further experiments are required to clarify this question.

Energy Dissipation. Another series of tests on the dynamic model of the Tacoma Narrows Bridge had the purpose of determining dynamic characteristics of different weights of the structure and of different moments of inertia of the stiffening girders. The amplitudes were varied for the different conditions of weight and girder stiffness so that at the time of release the model would have a potential energy of 0.218 ft. lbs. This corresponds to the energy in the prototype when vibrating in the two-noded condition with a double amplitude of 60 ins.

Typical oscillation dissipation curves, reproduced from Prof. Farquharson's records, are shown in Fig. VII-3 for three girder stiffnesses, the bridge as built ( $I = 2566 \text{ in.}^2 \text{ ft.}^2$ ), 10.4 times stiffer and 22.8 times stiffer, respectively, and for approximately equal weights.

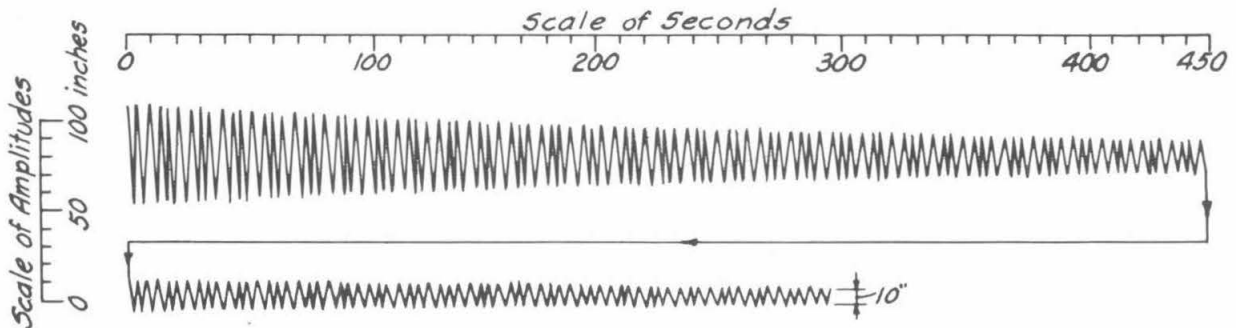
Energy dissipation curves calculated from these tests are shown graphically on Fig. VII-4. From these data the effects of the variables on the damping may be estimated. The total damping may be assumed to be the sum of the air and the structural damping. The structural damping is the sum of that in the suspended floor system, the cables and suspenders, the towers and any friction in the system. It may be assumed that the changes in the model structure from changes of weight or of girder dimensions do not affect the air damping.

As a convenience in the calculations the data from Fig. VII-4 was plotted in Fig. VII-5 on semi-logarithmic graph so as to reduce the curves to approximately straight lines, the variations therefrom being of a minor nature. The straight lines, the variations therefrom being of a minor nature. The straight lines indicate that the logarithmic



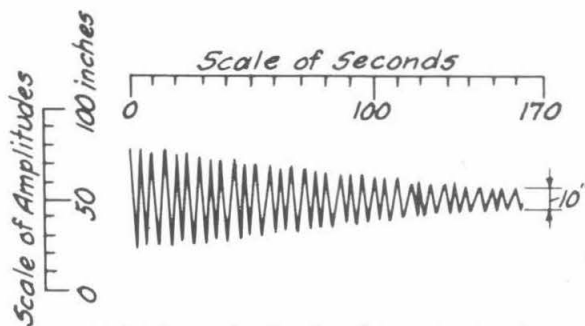
Weight per lineal foot of bridge = 5700 lbs.  
Amplitudes diminish to 10" in 1860 seconds.

BRIDGE AS BUILT. MOM. OF INERTIA OF GIRDERS =  $2,566 \text{ IN}^2 \text{ FT}^2$ .



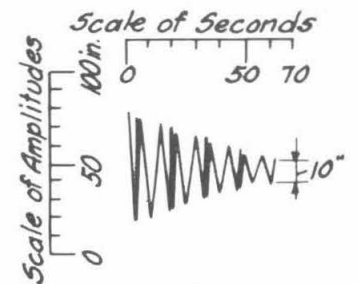
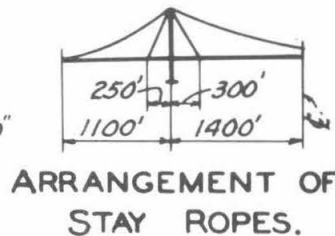
Weight per lineal foot of bridge = 5700 lbs.  
Amplitudes diminish to 10" in 730 seconds.

STAY ROPES WITH FIXED CONNECTIONS TO TOWER.



Weight per lin. ft. of bridge = 5700 lbs.  
Amplitudes diminish to 10" in 160 seconds.

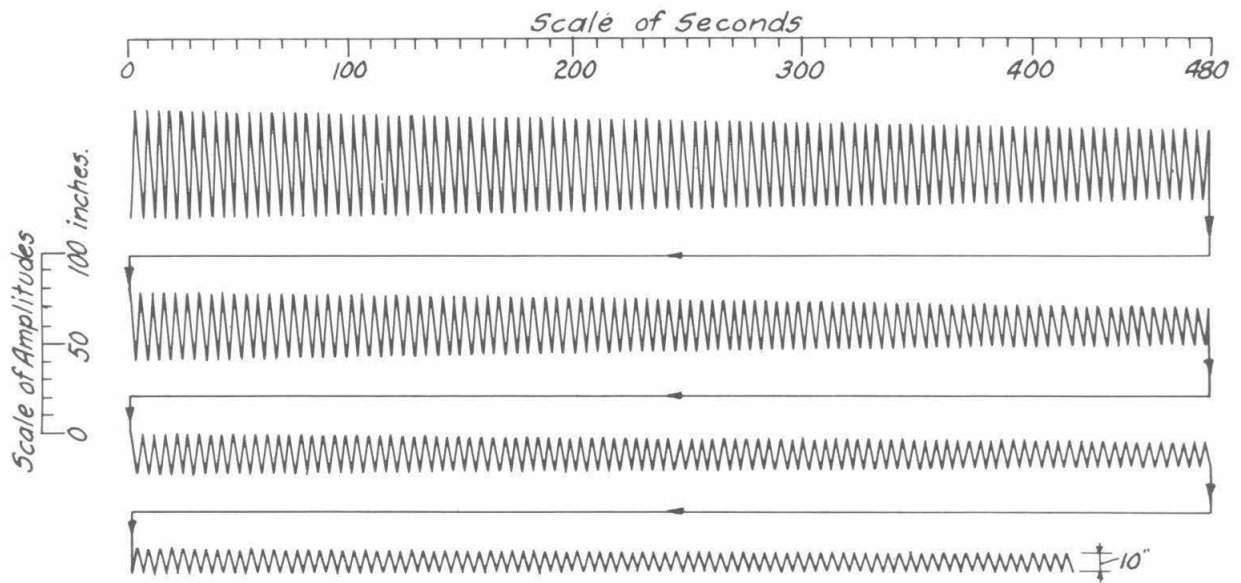
STAY ROPES OVER FRICTIONLESS  
ROCKER AT TOP OF TOWER.



Wt. per lin. ft. bridge = 5700 lbs.  
Amp. diminish to 10" in 60 seconds.

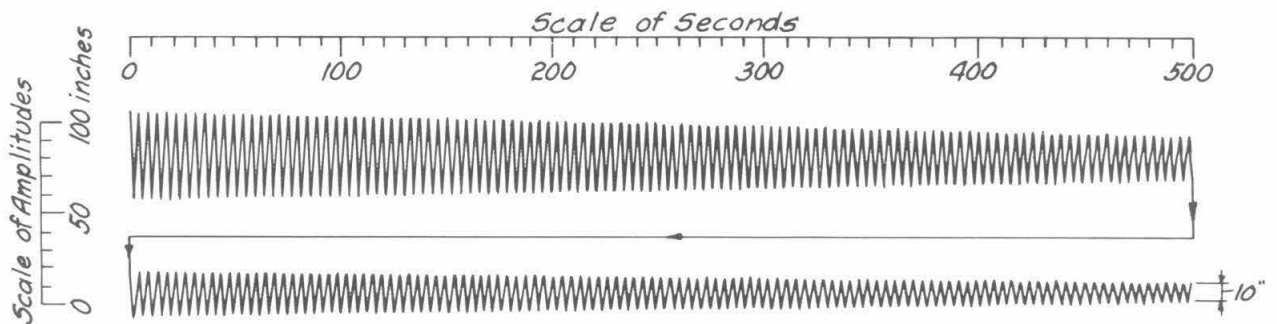
STAY ROPES OVER ROCKER  
WITH HEAVY FRICTION AT  
TOP OF TOWER.

FIG. VII-2 - OSCILLATION DISSIPATION CURVES FOR DIFFERENT STAY ROPE ARRANGEMENTS OF THE TACOMA NARROWS BRIDGE.



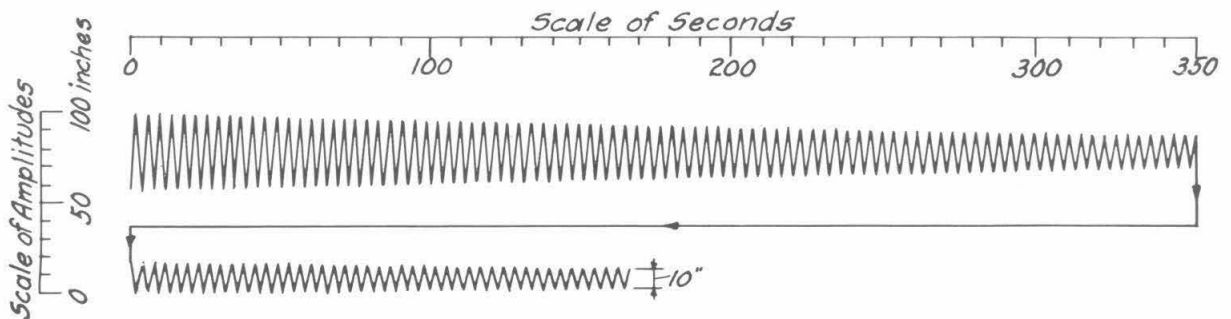
Weight per lineal foot of bridge = 5700 lbs.  
Amplitudes diminish to 10" in 1860 seconds.

BRIDGE AS BUILT. MOM. OF INERTIA OF GIRDERS =  $2,566 \text{ IN}^2 \text{ FT}^2$



Weight per lineal foot of bridge = 6180 lbs.  
Amplitudes diminish to 10" in 1230 seconds.

MOM. OF INERTIA OF GIRDERS =  $25,500 \text{ IN}^2 \text{ FT}^2$



Weight per lineal foot of bridge = 6600 lbs.  
Amplitudes diminish to 10" in 520 seconds.

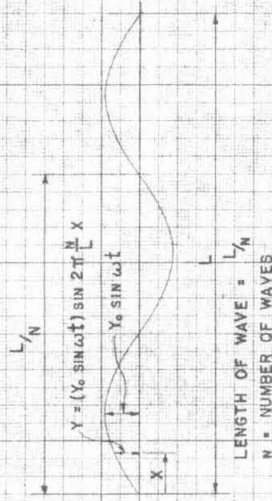
MOM. OF INERTIA OF GIRDERS =  $56,200 \text{ IN}^2 \text{ FT}^2$

FIG. VII-3 — OSCILLATION DISSIPATION CURVES  
FOR THREE GIRDER STIFFNESSES  
OF THE TACOMA NARROWS BRIDGE.

FIG. VII-4

# TIME - ENERGY DISSIPATION CURVES - FROM MODEL

TWO NODED MOTION ON MAIN SPAN



WHEN  $X = \frac{L}{2N}$ ;  $Y = 0$   
 "  $X = \frac{L}{N}$ ;  $Y = 0$   
 "  $X = \frac{L}{4N}$ ;  $Y = Y_0$

VELOCITY =  $\frac{dY}{dt} = Y_0 \omega \cos \omega t (\sin 2\pi \frac{N}{L} X)$

LET WEIGHT PER FOOT =  $C$

THEN: ENERGY =  $\int \frac{C dx}{2g} V^2$

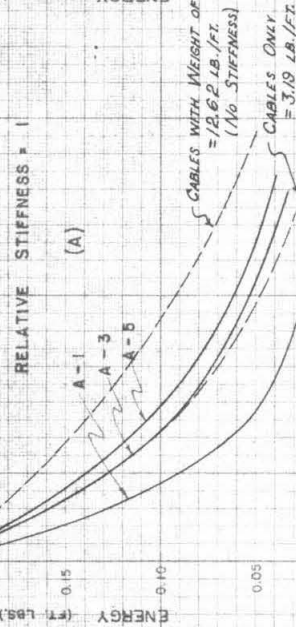
$$E = \frac{C}{2g} Y_0^2 \omega^2 \int_{X=0}^{X=L} \cos^2 \omega t (\sin^2 2\pi \frac{N}{L} X) dx$$

$$E_{MAX} = \frac{C}{2g} Y_0^2 \omega^2 \int_{X=0}^{X=L} (\sin^2 2\pi \frac{N}{L} X) dx$$

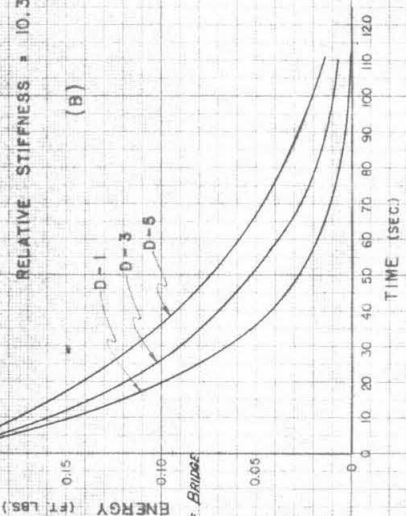
$$= \frac{C}{2g} Y_0^2 \omega^2 \left[ \frac{X}{2} - \frac{L}{8\pi N} \sin 4\pi \frac{N}{L} X \right]_{X=0}^{X=L}$$

$$E_{MAX} = \frac{C}{2g} \pi^2 f^2 L Y_0^2$$

TRUSS "A" - STIFFNESS CONSTANT  
 WEIGHT VARIED  
 A-1 = 9.66 LB./FT.  
 A-3 = 12.62 "  
 A-5 = 14.84 "



TRUSS "D" - STIFFNESS CONSTANT  
 WEIGHT VARIED  
 D-1 = 10.72 LB./FT.  
 D-3 = 13.68 "  
 D-5 = 15.90 "



TRUSS "E" - STIFFNESS CONSTANT  
 WEIGHT VARIED  
 E-1 = 11.78 LB./FT.  
 E-3 = 14.74 "  
 E-5 = 16.96 "

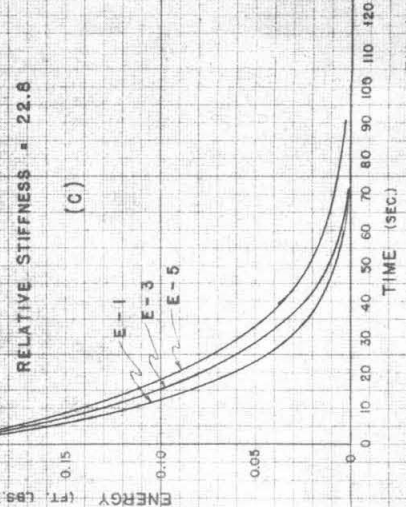


FIG. 1

decrement of amplitude is almost constant during the decay of oscillation for all 11 cases. The logarithmic decrement is defined as the natural logarithm of the ratio of two consecutive amplitudes of a free damped oscillation. If the amplitude at any time  $t$  is expressed by the formula  $a = a_0 e^{-\lambda t}$  where  $a_0$  is the initial amplitude at  $t = 0$ , the logarithmic decrement  $\delta$  is equal to  $\frac{60\lambda}{N}$ . The energy of the oscillating system is proportional to the square of the amplitude.

The equation for the decay of the energy  $W$  is therefore  $W = W_0 e^{-2\lambda t}$  and the rate of dissipation per cycle is given by  $\frac{\Delta W}{W} = 1 - e^{-\frac{120\lambda}{N}} \approx \frac{120\lambda}{N} = 2\delta$ . If the logarithmic decrement is constant, amplitude and energy give straight line in a semi-logarithmic diagram; the amount of energy dissipated per cycle is proportional to the energy of the system at any time.

In any vibrating system the energy continually passes back and forth between its two forms, kinetic and potential. The potential energy may consist of elastic strain energy or of energy stored by raising masses in the field of gravity. Dissipation of energy due to hysteresis occurs whenever the potential energy is in the form of elastic strain, and it has been determined experimentally that the logarithmic decrement as defined above is a property of the materials which depends mainly on the stress limits involved in the vibrations. For instance the values  $\delta = 0.005 - 0.012$  are given in the literature for various steels within a stress limit  $< 7500$  lb. per sq. in. In most engineering structures, however, the larger part of the energy dissipation is probably due to solid friction at joints or interfaces where relative motion is possible. Bernhard (23) gives  $\delta = 0.3$  for beam type bridges

as a result of dynamic tests.

Suspension bridges differ from ordinary bridges and similar structures in that the potential energy in vibration is largely stored in the form of work done against gravity, that is, the bridge behaves somewhat as a pendulum. A certain amount of energy goes into strain energy in the cables and suspended structure, but in suspension bridges this amount is not large. Hence, the dissipation of energy is small since most of it is not available for dissipative forces in the form of internal or sliding friction. It has been shown in Appendix VI that for slender girders the potential energy in the girder is proportional to its rigidity ( $EI$ ). It may be assumed that the logarithmic decrement is proportional to the rigidity of the girder.

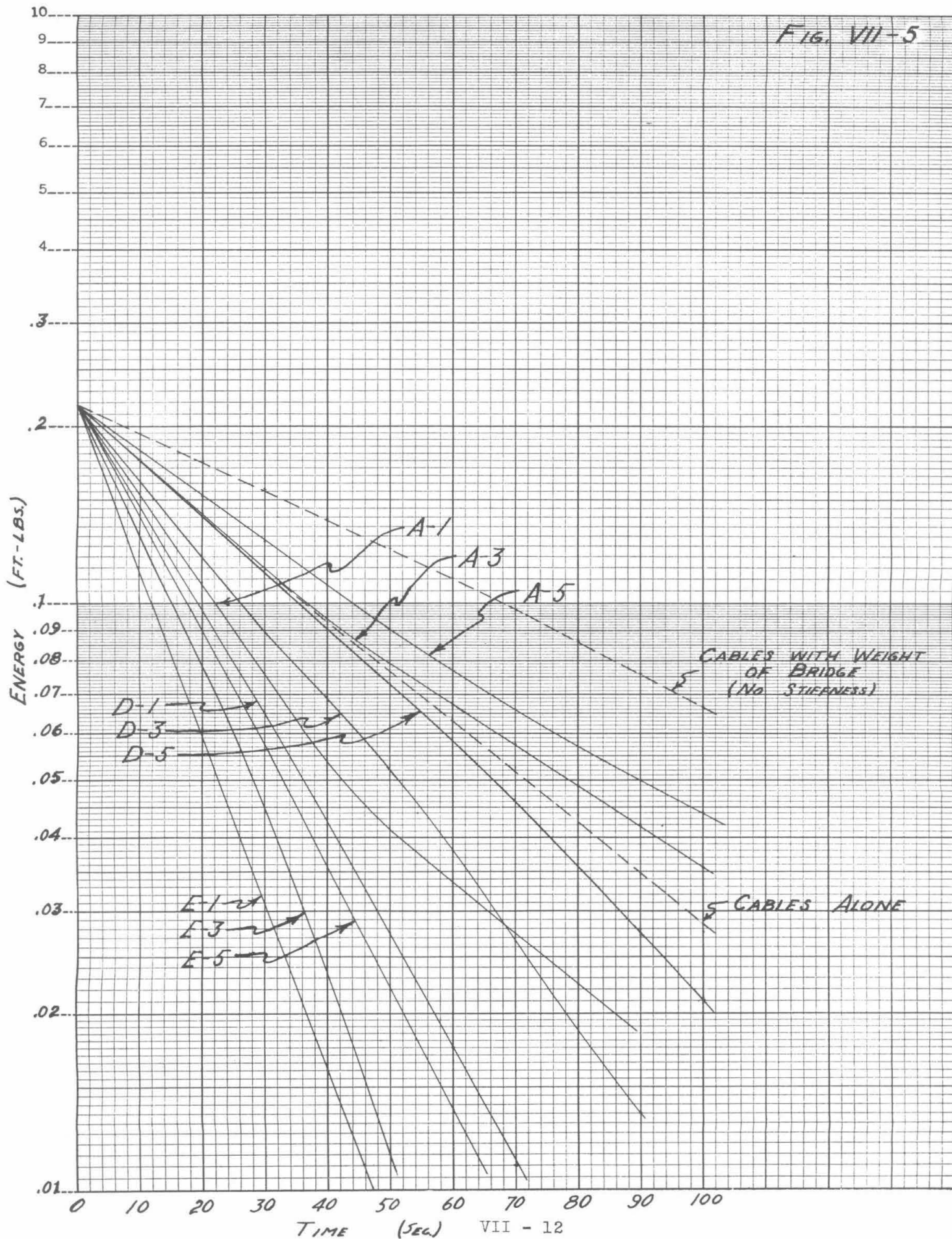
If the weight is increased without increase in rigidity, the dissipative forces are less effective and since the amount of potential energy stored in the girder in each oscillation is inversely proportional to the weight, the logarithmic decrement should also be inversely proportional to the weight.

The values of  $\delta$  derived from the eleven curves are shown plotted against  $w$ , the weight per foot, in Fig. VII-6. The frequencies were determined from Fig. VII-3. The dotted curves indicate approximately the effect of changes in weight for constant stiffness of the girder. As can be seen from the table below, where the product  $w\delta$  is taken from three points on each of the curves, they indicate that the decrement is approximately inversely proportional to the weight.

|           |       |   |      |      |      |
|-----------|-------|---|------|------|------|
| Stiffness | 1     | : | .062 | .073 | .086 |
| Stiffness | 10.35 | : | .077 | .086 | .094 |
| Stiffness | 22.8  | : | .140 | .143 | .143 |



FIG. VII-5



The effect of changing stiffness and keeping weight constant does not appear as would be expected, but here the girder of stiffness 22.8 cannot be considered as "slender", and the influence of other factors as well is not certain.

In the model, the roadway of stiffness 1 was made up of a relatively thick wooden strip stiffened by brass strips on the sides. The stiffer roadways were made by adding steel strips on the sides. Since the internal damping of wood is much higher than that of metal, it is probable that the difference in decrement between the roadway of stiffness 1 and the weighted cable of zero stiffness is largely due to the wooden strip.

To estimate the relative contributions of the various components of the suspension system the decrement for cable alone is reduced in the weight ratio  $3.19/12.62$  to determine the decrement for a cable of weight 12.62 lb/ft. This point is shown in the figure. The difference between zero stiffness and cable alone is probably due to the rope joints which support the weight.

The values of the logarithmic decrement are seen to be quite small for a suspension bridge, even in the case of stiffness 22.8, much smaller than the values obtained by Bernhard in dynamic tests of other types of bridges. Although the girder of stiffness 22.8 has an important effect on the energy distribution, the stiffness was increased by adding solid strips of steel, so that the logarithmic decrement probably tends more toward the value for hysteresis effect alone rather than to the value obtained for other bridges where other forms of frictional forces are so important.



A rough verification of this statement can be made by using the results of the energy calculations from Appendix VI. It was shown there that for girder stiffness 1 the ratio of the energy in the girders to the total energy of the vibrating structure is 0.0275 for the second symmetrical mode. Then for stiffness 22.8 the ratio becomes approximately 0.6. Hence, the addition of the steel girders would be expected to raise the value of  $\delta$  for the whole structure an amount given approximately by 60 per cent of the value of  $\delta$  for steel alone. Using the values quoted above this means an increase in  $\delta$  for the entire bridge of .003 to .007 depending on the type of steel. From Fig. VII-6 the experiments show that the addition of the girders increases the value of  $\delta$  by about .006, i.e., comparable with the calculated increase.

The various tests on energy dissipation are only a preliminary approach to the problem. The results should be considered only as indicative of general trends rather than as furnishing definite quantitative values. This is especially true insofar as the effect of rigidity of the stiffening girders are concerned and they do not touch on the relative merits of stiffening girders vs. stiffening trusses. They indicate possible methods of producing greater damping in suspension systems,

#### WIND TUNNEL TESTS

The essential results of the University of Washington wind tunnel tests have been described elsewhere (9). Further discussion of these tests and a comparison with similar tests made at the California Institute of Technology is given in Appendix VIII.

# LOGARITHMIC DECREMENT OF ENERGY DISSIPATION OF MODEL

W = DEAD LOAD / FT.

| STIFFNESS | 1     | FREQUENCY | 120 CYCLES / MIN |
|-----------|-------|-----------|------------------|
| A         | 1035, | "         | 140              |
| a         | 228,  | "         | 157              |

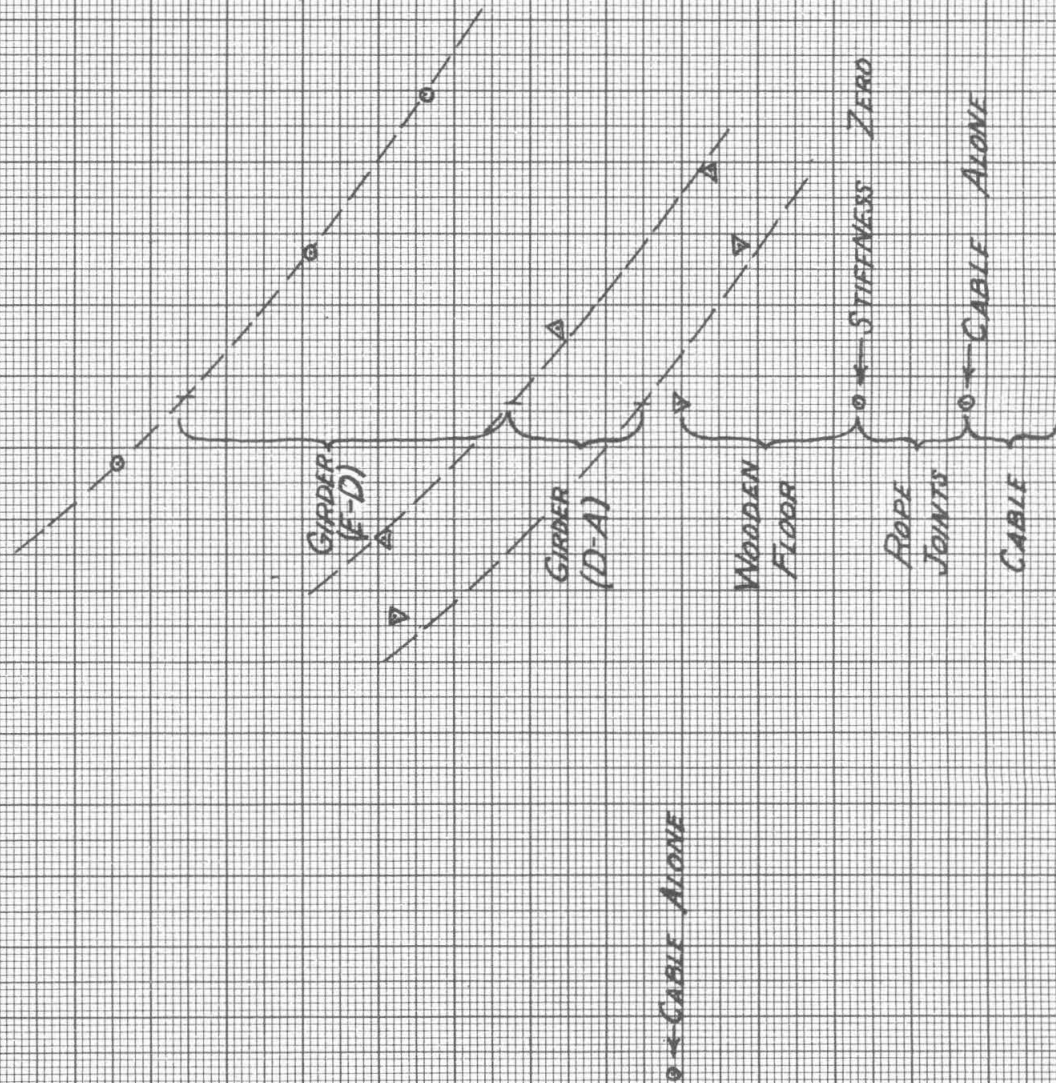


FIG VII-6

APPENDIX VIII

EXPERIMENTAL INVESTIGATIONS ON THE AERODYNAMIC CHARACTERISTICS  
OF THE SUSPENDED STRUCTURE OF THE TACOMA NARROWS BRIDGE

By  
Dr. Louis G. Dunn  
Instructor in Aeronautics  
Guggenheim Aeronautical Laboratory  
California Institute of Technology

Before the appointment of this Board, wind tunnel tests were made at the University of Washington to determine the static wind forces acting on the suspended structure of the bridge. In these tests the angle of inclination between the direction of the wind-stream and the bridge floor was varied, the angle being held constant during each measurement. From the results of such tests certain conclusions can be drawn concerning the damping of the vertical oscillations of the bridge (see Chapter IV). It has been suggested that aerodynamic instability (negative damping) may result when the lift curve is "inverted", i. e., a positive angle of attack produces negative lift. However, since the failure of the bridge was undoubtedly caused by torsional oscillations whose damping characteristics cannot be determined by any kind of static wind tunnel tests, it was deemed necessary to conduct oscillation tests on models simulating sections of the floor structure suspended properly in a wind stream. Part A of this Appendix contains the description and results of these experiments. In these tests the section of the floor structure was suspended as a rigid body on springs and cables. In addition a model of the entire structure with elastic floor was built and placed in a 20 ft.

section of the wind tunnel. The purpose of these tests was a further verification of the results obtained by the tests with the individual sections. These tests are described in part B. Since some of the results of the earlier static tests appeared to be contradictory, it was deemed desirable to repeat the static wind tunnel tests on a model with a larger length-width ratio. The comparison between the two series of tests is given in part C of this Appendix.

A. Experiments with Oscillating Models Simulating Sections of the Bridge.

The primary purpose of these experiments was to determine the damping characteristics of the suspended structure in a windstream as a function of the wind velocity. A particular object of the experiments was also to determine whether or not the large amplitudes of the torsional oscillations of the bridge were caused by negative aerodynamic damping (aerodynamic instability). For the proper interpretation of these tests with respect to the prototype, a dimensional analysis of the damped vibration of a body in a uniform wind stream was necessary.

Dimensional Analysis. Assume that a cylindrical body with constant cross section is suspended elastically in a parallel wind stream directed normal to the axis of the body; and that it oscillates about its axis so that the center of gravity is at rest. The moments acting on a unit length of the body consists of the following:

a) the moment of the inertia forces, expressed by the product of the moment of inertia  $I$  and the angular acceleration  $\frac{d^2\theta}{dt^2}$ .

b) the moment of the damping forces, depending on the angular velocity  $\frac{d\theta}{dt}$

c) the moment of the restraining forces depending on the angular displacement  $\theta$ .

In the model experiments the restraining forces were furnished by springs. The weight was transferred to the springs by a cable suspension system (Fig. VIII-23). The contribution of the aerodynamic forces to the restraining moment is determined by the static moment of the lift and drag forces corresponding to the geometrical angle of attack ( $\theta$ ) at any instant. This moment is small in comparison to the spring forces and was considered negligible for the purpose of the present investigation.

The damping consists of structural and aerodynamic damping. The main purpose of the tests was the determination of the damping moment; i.e., the moment proportional to the angular velocity  $\frac{d\theta}{dt}$  (1/sec.). The quantities affecting damping in addition to  $\frac{d\theta}{dt}$  are the air density  $\rho$  (lb. sec.<sup>2</sup>/ft.<sup>4</sup>), the linear dimension of the section, for example, the width  $b$  (ft.) and the wind velocity  $V$  (ft./sec.). The dimensions of the damping moment  $M$  per unit length are obviously that of a force. Hence, the only dimensionally correct combination for which  $M$  is proportional to  $\frac{d\theta}{dt}$  is given by

$$M = K_m \rho b^3 V \frac{d\theta}{dt} \quad (1)$$

where  $K_m$  is a dimensionless quantity.

The physical meaning of equation (1) can be illustrated by the simple example of a flat plate oscillating in a windstream. If the plate, figure a, is in a torsional oscillating motion of small amplitude, then at any instant of time,  $t$ , an element at a distance  $x$  from the axis of rotation has a component of velocity in a direction perpendicular to AB, which is equal to,  $w_x = x \frac{d\theta}{dt}$

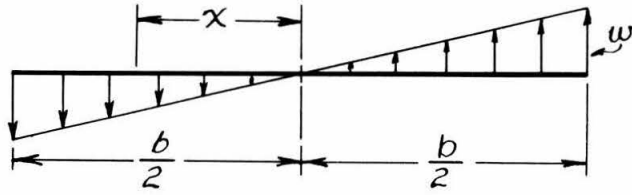


Figure a

Inasmuch as we are concerned with small oscillations, it is sufficiently accurate to consider  $w_x$  to be equal to the vertical component of velocity. If the structure is now considered to be oscillating in a horizontal wind velocity of magnitude  $V$ , the relative wind velocity at a distance  $x$  from the center of rotation will be at an angle  $\alpha_{\epsilon x}$  to the line AB and is given by

$$\alpha_{\epsilon x} = \frac{w_x}{V} \quad (2)$$

where  $\alpha_{\epsilon x}$  is generally referred to as the effective angle of attack.

The damping moment is the moment resulting from the lift forces acting on the plate; the lift forces are proportional to the dynamic pressure  $\frac{\rho V^2}{2}$ , hence the resultant lift force is proportional to  $b \frac{\rho V^2}{2}$  and the resultant moment to  $b^2 \frac{\rho V^2}{2}$ . Hence, the moment per unit length can be expressed in the form

$$M = C_m b^2 \frac{\rho V^2}{2} \quad (3)$$

The proportionality factor  $C_m$  depends on the magnitude and distribution of the local angles of attack. Now at any point along the width of the plate,  $w_x$  varies proportionally with the velocity at the tip  $\frac{b}{2} \frac{d\theta}{dt}$  and therefore the angle of attack and the lift force at any element varies proportionally with  $\frac{b}{2} \frac{d\theta}{dt}$ . In other words  $C_m$  has the form

$$C_m = \text{const.} \frac{b \frac{d\theta}{dt}}{V} \quad (4)$$

Substituting (4) in equation (3) the form (1) results.

Equation (1) applies to bodies having a geometrically similar cross section. For slow oscillations the factor  $K_m$  is constant. However, it has been shown in the theory of unsteady fluid motions that in general it is a function of the dimensionless ratio  $\frac{V}{bn}$ , where  $n$  is the frequency of oscillation. This ratio is called the reduced velocity  $V_r$ . It has the following physical meaning: If the plate is oscillating with a frequency  $n$ , then any disturbance at, say, the leading edge, will in a time  $\frac{b}{V}$  be carried passed the plate, and the period of the disturbance will be proportional to  $\frac{1}{n}$ . If the plate is oscillating with a low frequency, the period of disturbance will be large in comparison with the time  $\frac{b}{V}$ , and the disturbance will be carried downstream passed the plate before the influence of the next disturbance is felt. In this case, the assumption of quasi-steady forces is justified, and the quantity  $K_m$  will be a constant. However, if the frequency of oscillation is relatively high, the period of disturbance will be small in comparison with  $\frac{b}{V}$  and  $K_m$  will be a function of  $\frac{V}{bn}$ .

The expression  $K_m \rho V b^3 \frac{d\theta}{dt}$  is the aerodynamic damping per unit length. It should be noted that the aerodynamic damping is a maximum when the angular velocity is a maximum, i.e., when the amplitude is zero, and is zero when the amplitude is a maximum.

The general differential equation of motion for free torsional oscillations with damping is:

$$I \frac{d^2\theta}{dt^2} + K \frac{d\theta}{dt} + k\theta = 0 \quad (5)$$

Substituting,  $K \frac{d\theta}{dt} = K_m \rho V b^3 \frac{d\theta}{dt} + K_s \frac{d\theta}{dt}$   
 $I = \frac{9}{g} i^2$



we have,

$$\frac{d^2\theta}{dt^2} + \left( \frac{K_m P V b^3 g}{q i^2} + \frac{K_s g}{q i^2} \right) \frac{d\theta}{dt} + \frac{k g}{q i^2} \theta = 0 \quad (6)$$

where  $q$  = weight per unit length.

$i$  = radius of gyration.

$K_s \frac{d\theta}{dt}$  = structural damping per unit length.

$k$  = spring constant of the structure.

Using the notation,

$$\begin{aligned} p^2 &= \frac{K_s g}{q i^2} \\ 2\lambda &= \frac{K_m P V b^3 g}{q i^2} + \frac{K_s g}{q i^2} \end{aligned} \quad (7)$$

the general solution of equation (6) is

$$\theta = e^{-\lambda t} \left( \theta_0 \cos \omega t + \frac{\left( \frac{d\theta}{dt} \right)_0 + \lambda \theta_0}{\omega} \sin \omega t \right) \quad (8)$$

where  $\omega = \sqrt{p^2 + \lambda^2}$

$\theta_0$  = displacement at  $t=0$ .

$\left( \frac{d\theta}{dt} \right)_0$  = angular velocity at  $t=0$ .

The terms in the bracket of equation (8) depend only on the initial displacement and the initial velocity. The quantity  $\lambda$  is generally referred to as the damping coefficient. It is of interest to note that as long as  $\lambda$  is positive the structure will, after a certain time, come to rest at its initial position of equilibrium if disturbed from this position. However, if  $\lambda$  is negative and the structure is given a finite displacement, the angular rotation increases with time. It does not mean, however, that the amplitude builds up indefinitely, for, the structural damping coefficient will increase with increasing amplitude. In other words, the structural damping coefficient is a function of the amplitude, whereas the aerodynamic damping coefficient is



a function of the wind velocity. Hence, with every wind velocity there is associated an amplitude such that the structural and negative aerodynamic damping coefficients are numerically equal. For example, the structural damping coefficient will in general vary with the amplitude as indicated by curve "a" of figure b, whereas the aerodynamic damping coefficients are independent of amplitude and vary only with wind velocity, as shown by curve b, c and d. The vertical lines, figure b, indicate equal and opposite values of structural and aerodynamic damping coefficients. Hence,  $a_1$ ,  $a_2$  and  $a_3$  indicate the respective amplitudes of the steady state conditions corresponding to the wind velocities  $V_1$ ,  $V_2$  and  $V_3$ .

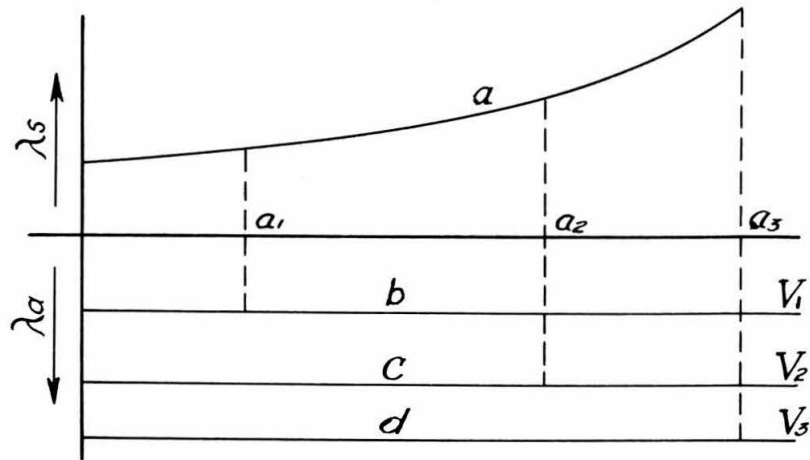


Figure b

Dividing each side of equation (7) by the frequency  $n$ , and noting that  $K_m = f\left(\frac{V}{bn}\right)$  we have

$$\frac{2\lambda}{n} = \left(\frac{\rho b^2}{q/g}\right) \left(\frac{b}{t}\right)^2 f\left(\frac{V}{bn}\right) + \frac{K_s g}{q t^2 n} \quad (9)$$

where  $\frac{\rho b^2}{q/g}$  is the density-mass ratio  $\mu$ , and  $\left(\frac{b}{t}\right)^2$  is a quantity which depends only upon the geometry of the cross-section. The quantity

of its equivalent  $\lambda\tau$ , is called the logarithmic decrement,  $\tau$  being the period of vibration. (In other sections of this report, the ratio  $\frac{\lambda}{n}$  has been designated by  $\delta$  or  $\log \Delta$ , where  $\Delta$  is the ratio between two consecutive amplitudes.)

Now let

$$\begin{aligned}\frac{\lambda_s}{n} &= \text{the decrement due to the structural damping.} \\ \frac{\lambda_a}{n} &= \text{the decrement due to the aerodynamic damping alone.} \\ \frac{\lambda}{n} &= \text{decrement due to both structural and aerodynamic damping.}\end{aligned}$$

Then

$$\frac{2\lambda_a}{n} = \frac{2\lambda}{n} - \frac{2\lambda_s}{n} \quad (10)$$

Substituting for  $\frac{2\lambda}{n}$  from equation (9):

$$\frac{2\lambda_a}{n} = \left[ \mu \left( \frac{b}{l} \right)^2 f \left( \frac{V}{bn} \right) + \frac{K_s g}{q l^2 n} \right] - \frac{2\lambda_s}{n}$$

but by definition  $\frac{2\lambda_s}{n} = \frac{K_s g}{q l^2 n}$ , hence

$$\frac{\lambda_a}{n} = \mu \left( \frac{b}{l} \right)^2 f \left( \frac{V}{bn} \right) \quad (11)$$

We see that the logarithmic decrement of the aerodynamic damping is a function of three quantities, namely:

- a) The density-mass ratio,  $\mu$ , of the structure.
- b) A quantity  $\left( \frac{b}{l} \right)^2$  which depends only upon the geometrical cross-section of the structure.
- c) The reduced velocity ratio  $V_r = \frac{V}{bn}$ .

The logarithmic decrement  $\frac{\lambda_a}{n}$  can be determined experimentally for the structure at any given wind velocity,  $V$ , and will be further discussed in the section on "Evaluation of Data".

Model Data. Table I contains the data of the models tested. All models were constructed from brass. The cross-sections of all models were the

same and represent to scale the cross-section of the Tacoma Narrows Bridge. No attempt was made to reproduce all details on these models, only such details as were considered important in their aerodynamic effects were incorporated. Photographs of a typical model are shown in Figure VIII-23.

The models were suspended in the working section of the wind tunnel in such a manner that the suspension cables and suspenders were in this airstream. Since these models represented only a small section of the center span, the ends of the suspension cables were attached to springs in order to simulate the elastic continuity of the suspension system. The springs were attached to rigid supports outside the tunnel walls. The spring constants of the models did not represent to scale the spring constant of the prototype. However, the spring constants were varied through a wide range in order to determine the influence of spring constants upon the dynamic behaviour of the structure when subjected to aerodynamic forces resulting from a steady wind. The spring constants of the various springs used in the tests are listed in Table I.

From equation (11) it is apparent that the mass-density ratio of the structure has some influence on the wind velocity at which the structure becomes aerodynamically unstable. For this reason two series of tests were made in which the weight per unit length of the model was increased by the addition of weights.

In general, in the construction of dynamically similar models, both the force reduction factor  $m$ , and the linear scale reduction factor  $l$  can be chosen arbitrarily. However, in order to maintain aerodynamic similarity it is necessary that  $m$  be equal to  $l$ . This can be shown

from the following considerations.

The aerodynamic force  $F$  in lbs., produced on an area  $A$  of the model is given by the equation

$$F = C_L \rho v^2 A$$

Now to get the corresponding quantities on the prototype  $F$  is multiplied by  $m l^2$ ,  $v$  by  $\sqrt{l}$  and  $A$  by  $l^2$ ,  $C_L$  is a non-dimensional constant, and the air density remains the same, hence, for aerodynamic similarity it is necessary that,

$$m l^2 = l^3$$

or  $m = l$

It can also be shown that to maintain similarity in the density-mass ratio it is necessary that  $l = m$ , for,

$$\mu = \frac{\rho b^2}{q/g} = \frac{l^2}{m l} = \frac{l}{m}$$

Hence, in order that  $\mu$  be the same for model and prototype,  $m$  must equal  $l$ . In models 4a and 4b an attempt was made to obtain an equality between these two reduction factors. Due to difficulties of construction an exact equality was not possible, however, it was felt that the two values were close enough to approach aerodynamic similarity.

Test Procedure. The model was suspended in the wind tunnel working section with a pointer indicator attached to one end. Hence, when the model oscillated in torsion, the pointer indicated the amplitude of the motion on a graduated scale placed on the glass wall of the wind tunnel. The model was then given a forced oscillation in still air, the applied force removed, and the resulting motion recorded by photographing the pointer indicator and scale with a high speed motion picture camera. Inasmuch as the film speed was known, this provided an accurate

determination of the decrement in amplitude due to aerodynamic and structural damping. The same procedure was then followed for various wind speeds. At the higher wind speeds it became unnecessary to apply an external force to the system, and the oscillations built up of their own accord due to the aerodynamic forces. In this case, of course, the camera recorded a constantly increasing amplitude instead of a decreasing amplitude. (See Figs. VIII-5, VIII-6 and VIII-7)

It was desired to separate as nearly as possible from the aerodynamic damping, the structural damping due to the suspenders, cables and springs. This was accomplished in the following manner. First, a small streamlined weight was suspended from one of the springs and started in vibration. The resulting amplitude decrement was then measured photographically. This determined the damping due to the springs alone, the aerodynamic forces acting on the weight being of a negligible order. Next, the bridge was suspended from the spring at four points only without benefit of cables or suspenders. The bridge was then set into torsional vibrations, the applied force removed, and the resulting motion photographed. This, then permitted the determination of damping due to the springs and the aerodynamic forces acting on the bridge. Hence, the aerodynamic damping alone was calculable for still air. Inasmuch as the damping obtained in the regular test procedure at zero wind velocity was made up of both structural and aerodynamic damping, it was then possible to determine the structural damping alone.

Evaluation of Data. From the films, curves of maximum amplitude were plotted vs. time. (See Figs. VIII-5 to VIII-7) It can be seen from the general solution, equation (8), that the amplitude of vibration diminishes

after every cycle in the ratio  $e^{-\lambda\tau} : 1$  where  $\tau$  is the period of vibration. The quantity  $\lambda\tau$  is called the logarithmic decrement and is merely equal to the difference between the logarithms of two consecutive amplitudes measured at the instants  $t$  and  $t + \tau$ . Hence, it was only necessary to plot the logarithm of the amplitude against time and calculate the ratio of the ordinates between a time interval equal to the period. (See Figs. VIII-3 to VIII-16) This ratio determined  $\lambda\tau$  or its equivalent,  $\frac{\lambda}{n}$ . This, of course, was the decrement due to the total damping in the system, both structural and aerodynamic. To determine the aerodynamic damping alone, logarithmic curves were drawn up for the springs alone, (Fig. VIII-15) the bridge and springs without the suspension system (Fig. VIII-16), and the bridge with the springs and suspension system (Figs. VIII-13 and VIII-14), all at zero wind velocity. The decrements for these separate conditions were calculated, and from these, the decrement for the structural damping alone (i.e., suspension system of springs) was determined. This decrement ( $\frac{\lambda_s}{n}$ ) turned out to be nearly a constant for the models in which the suspension system was the same. This structural damping increment was then subtracted from the total giving the increment due to aerodynamic damping,  $\frac{\lambda_a}{n}$ . This value was corrected for density-mass ratio and geometrical properties, see equation (11), and the final corrected value plotted against  $\frac{V}{bn}$  (Fig. VIII-3).

Test Results. The results of the tests on the oscillating models are shown in Figs. VIII-1, VIII-2 and VIII-3. The test results for the models having a density-mass ratio of 0.00514 and 0.01825 are shown in Figs. VIII-1 and VIII-2, respectively. The experimental values shown in Fig. VIII-2 lie very close to a common curve, while those of Fig. VIII-1

show some scatter, especially for small values of  $\frac{\lambda a}{n}$ . The main reason for this scatter is, that the value of  $\frac{\lambda a}{n}$  is obtained by subtracting  $\frac{\lambda_s}{n}$  from  $\frac{\lambda}{n}$  and at small values of  $\frac{\lambda a}{n}$  these two quantities are of the same order of magnitude. Hence, any small experimental inaccuracies are considerably magnified in the value obtained for  $\frac{\lambda a}{n}$ . In obtaining the test results shown in Fig. VIII-2 great care was exercised to avoid any test conditions which might lead to inaccuracies.

A final plot of  $\frac{\lambda a}{n} \frac{1}{\mu} \left(\frac{l}{b}\right)^2$  as a function of  $\frac{V}{bn}$  is shown in Fig. VIII-3, where  $V$  is in ft. per sec.,  $b$  is the width in ft. and  $n$  is the frequency in cycles per sec. The test results for all the models are included in this figure. The fact that all the experimental results scatter rather closely about a common curve proves without doubt the validity of the parameters which were obtained from the dimensional analysis. The resulting curve indicates that for values of  $\frac{V}{bn}$  less than 4.1 the aerodynamic damping is positive. For values of  $\frac{V}{bn}$  greater than 4.1 the aerodynamic damping becomes negative and increases rapidly (negatively) with an increasing velocity ratio  $\frac{V}{bn}$ . Physically this means that for values of  $\frac{V}{bn}$  less than 4.1 the structure imparts energy to the air, that is, if the structure is set in a torsional motion by some external force the air passing over it takes energy from the structure and consequently the air acts as a damping medium. However, when  $\frac{V}{bn}$  becomes greater than 4.1 the air passing over the structure imparts energy to it, or, in other words, the structure absorbs energy from the air. If there were no structural damping present, it is obvious that this would then immediately constitute a condition of aerodynamic instability. However, as previously pointed out, the condition of aerodynamic instability does not occur until the negative aerodynamic damp-

ing coefficient becomes larger than the structural damping coefficient. This is equivalent to saying that it is necessary that the logarithmic decrement due to the air damping must be greater than the logarithmic decrement due to the structural damping. From the above discussion it follows that the wind velocity causing a condition of aerodynamic instability is not necessarily that at which the aerodynamic damping becomes negative, but depends on the amount of structural damping present in the structure.

From the experimental data contained in Fig. VIII-3, a damping curve, Fig. VIII-4, was constructed for the Tacoma Narrows Bridge. The values of  $\mu$  and  $\left(\frac{1}{b}\right)^2$  for the prototype was used in these calculations. It is seen that the aerodynamic damping becomes negative wind velocity of approximately 25 miles per hour. In order for the structure to remain aerodynamically stable up to a wind velocity of 40 miles per hour, it requires that the logarithmic decrement due to structural damping be equal to 0.16. These facts bring out several important points in regard to structural damping, namely:

- a) The marked dependence of the critical wind velocity, i.e., the wind velocity at which the structure becomes aerodynamically unstable, on the amount of structural damping present in the structure.
- b) The desirability of incorporating high structural damping in structures such as the Tacoma Narrows Bridge.
- c) The possibility of designing structures which will be aerodynamically stable, throughout the anticipated range of wind velocities, by incorporating in the structure the necessary amount of structural damping.



Very little is known about structural damping in the present types of suspension bridges. An extensive investigation of this problem may be well worth while as it may become one of the important phases in the design of relatively light suspension bridges.

Models with Deep Girders. Several models were constructed with deep truss-type girders in order to investigate the influence of aerodynamic forces on this type of construction. Two of the models are shown in the photograph, Fig. VIII-24. The first model corresponded to an actual floor width of 53 ft., and a girder depth of 20 ft. The second corresponded to a floor width of 39 ft. and a girder depth of 24 ft. In both cases, the linear scale reduction factor was 80.

The primary object of these tests was to determine (a) whether negative aerodynamic damping would also be induced in this type of structure, and (b) the magnitudes of the critical wind velocities if negative damping was present.

The models were suspended in the wind tunnel in the same manner as has been described under "testing procedure". Both models exhibited the same negative aerodynamic damping characteristics which were present in the previously tested models. The critical wind velocities at which aerodynamic instability occurred were found to be 16 miles per hour for the first model and 14 miles per hour for the second. When the critical wind velocities were computed from the data given in Fig. VIII-3 using the structural damping coefficient previously obtained, the values were found to be approximately 25% less than the observed speeds. However, there is no reason to expect this type of structure to give the same aerodynamic damping curve as was obtained for the simple plate girder

type. It is reasonable to assume that the moment due to the high drag forces on the truss girder would result in increased positive aerodynamic damping, thus delaying the point at which the net aerodynamic damping became negative. Also, there are higher inertia forces to be overcome due to the particular mass distribution in the structure. These considerations would account for the higher observed critical velocities.

From these tests it appears that this type of construction offers the following advantages in the design of suspension bridges:

- a) Higher structural damping due to increased torsional rigidity.
- b) More favorable mass distribution.
- c) Higher critical wind velocity.

It is felt that it would be of great value to conduct a systematic investigation of the aerodynamic damping characteristics of various types of construction. It is possible that such an investigation would yield sufficient data to greatly aid engineers in the design of efficient and aerodynamically stable structures.

## B. Tests on Scale Model

To further substantiate the results obtained and the conclusions drawn from the dynamic tests on the section models, a 1/234 scale model of the Tacoma Narrows Bridge was constructed and tested. The roadbed of this model consisted of a hard rubber extrusion. This material not only gives the desired flexural and torsional flexibility, but can be readily extruded in any desired shape. The cross-section of the bridge could therefore be accurately reproduced with the exception of the wind bracing which was omitted. The total length of the model was 19 ft., and the roadbed was 2 ins. wide. The center span was 11 ft. 9.5 ins. long and was exactly to scale. Space limitations in the wind tunnel required the side spans to be 14 ins. shorter than was required for exact correspondence to the prototype. The sag ratio corresponded to that of the prototype thus insuring the proper tension in the suspension cables. The towers were freely pivoted at the "water-level" and were balanced by weights below this level as shown in Fig. VIII-25. Additional weights were added to the side spans to obtain a proper balance in the cable tension at the towers. The complete assembly, mounted in the 20 ft. section of the Guggenheim laboratory wind tunnel is shown in the photographs, Fig. VII-25. This is not the working section of the tunnel, but is the section immediately aft of the propeller.

The first test on this model indicated that, although the vertical motions were evident at low speeds, it was not possible to induce torsional oscillations at moderate speeds due to the extremely high damping in this model. This damping came from the following two sources, (a) the rubber roadbed itself has a very high damping coefficient and, (b) the inertia

forces in the towers were very large compared to the inertia forces in the roadbed, the towers being approximately 10 times as heavy as the suspended structure. Therefore, it was decided to fix the towers at their pivot points and to simulate the towers and side spans by inserting a coil-spring between the towers and the suspension cables of the center span. Two sets of springs were used in the remaining tests. The first set had a rather low spring constant which corresponded to a much more elastic structure than that in the actual bridge structure. The second set of springs had a much higher spring constant and corresponded quite closely to the elastic constant of the bridge structure. The elastic correspondence can be readily obtained by measuring the static deflection due to a distributed load on the prototype. For example, a distributed load of 200 lbs. per ft. on the center span of the bridge caused a deflection of 1.58 ft. at the center. A corresponding load on the model caused a deflection which would correspond to 1.86 ft. on the prototype. Comparing these deflections show that the prototype was 17.5% stiffer than the model. In the case of the softer springs the prototype was 50% stiffer than the model. A comparison between the torsional stiffness of the model with the stiff springs and prototype was also obtained by loading the model as shown in Figure 23 of Chapter III. In this case the ratio of the deflections at the quarter points was 1.20 which indicates about the same relative torsional stiffness between the model and prototype as in the bending stiffness.

A test of the model with the softer springs exhibited, at low speeds, the same type of galloping motions as were observed in the actual bridge. At a wind velocity of 5.5 miles per hour the fundamental torsional mode

of vibration was induced. By merely increasing the wind velocity it was not possible to excite any of the higher modes of vibration. However, by holding the floor structure at either the center or quarter points the higher modes of torsional vibration were easily excited. The difficulty in obtaining the higher modes of vibration at higher wind velocities is probably due to the fact that as the wind velocity is increased the torsional amplitudes become very large, with a vertical motion superimposed on the torsional motion. Since the rubber has a high damping coefficient a considerable amount of energy is involved in changing from the violent motion in the fundamental mode to one of the higher modes. It was observed that the torsional oscillation in conjunction with the vertical motion gave the effect of a traveling torsional wave.

In the tests on the model with the stiffer springs the same types of motion were observed as in the model with the softer springs. The only essential difference was that the wind velocity increased to 5.7 miles per hour before the torsional oscillations were observed. This slight increase in wind velocity required to excite the torsional oscillation would indicate that the difference between the model rigidity and that of the actual bridge would not have caused an appreciable difference in the critical wind velocities. By critical wind velocity is meant the wind velocity at which torsional oscillations are induced. As the available time was limited it was not possible to conduct a test in which the elastic constants of both model and prototype were the same.

The fundamental vertical frequencies were observed and found to be 116 and 126 cycles per minute for the soft and stiff springs, re-

spectively. This would correspond to 7.6 and 8.3 cycles per minute for the prototype. The fundamental vertical frequency observed on the prototype was about 8 cycles per minute which indicates a very close agreement between the frequency of the model with the stiff springs and the actual bridge.

To convert the model wind velocities to prototype wind velocities, the model wind velocity is multiplied by  $\sqrt{\ell}$ , where  $\ell$  is the linear scale reduction factor. Hence, the critical wind velocities for the prototype would be 83 and 86 miles per hour for the soft and stiff spring, respectively. These velocities are about twice as high as that of the actual bridge. Since the structural damping of the model is not known, the results of the full-scale model tests can only be considered as qualitative rather than quantitative. The influence of the negative aerodynamic damping could be readily observed. If a torsional oscillation was induced at zero wind velocity, the time required to come to rest was 5 seconds, whereas at about 3 miles per hour the time required was 12 seconds. This definitely shows the presence of negative aerodynamic damping.

Motion pictures were taken of the various types of motion, which allows a later study of the various types of motion and also a comparison with the types of motions observed on the prototype. A number of photographs, reproduced from the motion picture, are shown in Fig. VIII-26.

### C. Aerodynamic Characteristics of Various Sections.

When a body such as the suspended structure of the Tacoma Narrows Bridge is subjected to a transverse wind, the air passing over the structure will exert a force on it. The magnitude, direction and line of action of this resultant force can be readily determined by wind tunnel tests.

The components of the force perpendicular and parallel to the relative wind direction are called the lift and drag forces, respectively. The line of action of the resultant force is determined by measuring the moment exerted around a suitably chosen trunnion axis. The moments given in this report are referred to the longitudinal axis passing through the mid-point of the roadbed.

In a structure such as the Tacoma Narrows Bridge wind tunnel tests are performed on a scale model representing a certain length of the span. From the model tests the section characteristics are determined, that is, the lift, drag and moment coefficients as a function of the angle of inclination between the direction of the wind stream and the bridge floor. This angle of inclination is generally referred to as the angle of attack  $\alpha$ . The coefficients are defined by the following three equations:

$$\begin{aligned}C_L &= \frac{L}{\rho/2 V^2 A} \\C_D &= \frac{D}{\rho/2 V^2 A} \\C_M &= \frac{M}{\rho/2 V^2 b A}\end{aligned}$$

where:

$$\begin{aligned}C_L &= \text{lift coefficient (non-dimensional)} \\C_D &= \text{drag coefficient ( " " )} \\C_M &= \text{moment coefficient ( " " )}\end{aligned}$$

$L$  = lift in lbs.

$D$  = drag in lbs.

$M$  = moment in ft. lbs. (generally referred to some given axis)

$\rho$  = air density - slugs/cu. ft.

$V$  = wind velocity in ft./sec.

$b$  = width of the roadbed in ft.

$A$  = surface area in square ft.

From the wind tunnel tests these coefficients can be calculated and plotted as functions of the angle of attack. If it is assumed that the aerodynamic characteristics of the model and prototype are the same, that is, the influence of Reynolds Number is neglected, then by means of the above three equations, the aerodynamic forces acting on the prototype can be calculated for any angle of attack  $\alpha$  and wind velocity  $V$ .

The aerodynamic characteristics for a number of models of various cross-sections were determined and will be discussed in the following paragraphs.

In Figs. VIII-17, VIII-18 and VIII-19, the drag, lift and moment coefficients are plotted as functions of the angle of attack  $\alpha$  for a number of variations in cross section. First, a model of the suspension structure as built was tested. Then the influence of streamlining the girders and of increasing the girder depth was determined. As would be expected, streamlining decreases the drag coefficient, whereas increasing the girder depth increases the drag coefficient. The influence of the shape on the lift coefficient is more important, for here the effect is not merely a quantitative difference in the numerical values. The so-called aerodynamic stability of the structure depends on the slope of the



lift coefficient curve.

It is seen that although the lift curve for the section representing the bridge as built is by no means a smooth curve, especially in the range of  $+7^\circ$  and  $-7^\circ$ ; nevertheless, the slope is positive for all angles of attack. This means stability or positive aerodynamic damping for vertical oscillations. However, when the girder depth is increased, the slope changes from positive to negative and then back to positive again in the range of  $+10^\circ$  and  $-10^\circ$ , the maximum variation in the lift coefficient being  $+0.22$  and  $-0.22$ .

This type of curve indicates the possibility of instability, i.e., negative damping. In the case in which the girders are streamlined the lift curve has a nearly constant positive slope in the range between  $+7^\circ$  and  $-7^\circ$ .

The moment coefficients shown in Fig. VIII-19 indicate that in all cases except the streamlined section, the resulting moments in the range of  $+10^\circ$  and  $-10^\circ$  are stable, whereas streamlining the girder results in unstable moment characteristics. An unstable moment means that the position of the resultant force is such as to tend to rotate the section in the direction of the increasing angle of attack. (Increase here refers only to magnitude and may be either positive or negative.) The unstable moment does not produce in this case increasing oscillation amplitudes; it merely reduces somewhat the restoring moment of the cables.

The tests represented in Fig. VIII-17 to VIII-19 were conducted at the GALCIT\*. As previously stated, wind tunnel tests were conducted at the University of Washington before the appointment of this Board. A number of curves are included showing a comparison between the data ob-

\*Guggenheim Aeronautical Laboratory, California Institute of Technology.

tained at the University of Washington and that obtained at the GALCIT.

In Fig. VIII-20 the lift curves, representing the bridge structure as built, are plotted. Curve No. 1 is the lift curve obtained from the tests at the GALCIT, whereas curve No. 2 gives the lift curve obtained from the tests at the University of Washington. This curve exhibits unstable lift characteristics such as were obtained at the GALCIT for the models with increased depth of girder. The model used in the GALCIT tests differs from that used in the Washington tests in the so-called aspect ratio, i.e., the ratio of length to width. The model used in the GALCIT test had an aspect ratio of 9.5 and a very small gap between the ends of the model and the wind tunnel walls, whereas the model used in the Washington tunnel had an aspect ratio of 4 with a considerable gap between the ends and the wind tunnel walls. In the case of the actual bridge the aspect ratio is 72, i.e., the flow over most of the span is unaffected by end effects. It is believed that the conditions in the GALCIT tests approximate closer the actual conditions. This is confirmed by further Washington tests in which thin plates were attached to the ends of the bridge section in order to approximate the conditions prevailing in the case of an infinite aspect ratio.

Curve No. 3 gives the results for the model with 27 in. diameter end-plates. (The model width was 23.6 ins.) It is seen that the addition of end plates causes a marked change in the character of the lift curve. In the range of  $+4^\circ$  and  $-4^\circ$  this curve is quite similar to that obtained at the GALCIT for a model with an aspect ratio of 9.5 (Curve No. 1). However, an abrupt change in  $C_L$  occurs at  $+8^\circ$  and  $-4^\circ$  (Curve No. 3); this permits at present of no logical explanation.

All of the lift curves obtained at the GALCIT, together with two curves obtained at the University of Washington (No. 5, bridge as built, and No. 2, streamlined model), are shown in Fig. VIII-21. It can be seen that there is a good correlation between the curves for the streamlined models obtained at the University of Washington, and at the GALCIT.

Lift curves for models with truss type girders are shown in Fig. VIII-22, curves No. 2 and No. 3. Curve No. 3 corresponds to a floor width of 53 ft. and a girder depth of 20 ft. (see Fig. VIII-24 for general construction); curve No. 2 is the lift curve for the Golden Gate Bridge, a model of which was tested at Stanford University. Curve No. 4 is again the lift curve for the Tacoma Narrows Bridge as built, obtained at the GALCIT. It is obvious that the lift curves for the truss type girders are quite smooth, whereas the curve for the Tacoma Bridge has several inflection points.

Summarizing the results it is felt that Curve No. 1 gives the correct lift characteristics for the model tested, however, it cannot be positively stated that the prototype would have exactly the same lift characteristics, as not enough is known about the Reynolds Number effect on sections of this type. It is evident that in general plate girders have the tendency to produce unstable lift characteristics. The exact influence of the ratio between the height of the girders and the width of the roadbed has to be determined by further investigations.

The significance of the aerodynamic characteristics relative to the behavior of the bridge has been discussed in detail in Chapter IV.

TABLE VIII - 1

| <u>Model Data</u> |    |                       |        |               |                                       |                              |
|-------------------|----|-----------------------|--------|---------------|---------------------------------------|------------------------------|
| Bridge            | n  | Weight<br>per<br>foot | m<br>* | Spring<br>No. | Torsional<br>Frequency<br>cycles/sec. | Density-<br>mass ratio<br>** |
| 1                 | 80 | 3.55                  | 20     | 4             | 4.3                                   | 0.00514                      |
| 1a                | 80 | 3.55                  | 20     | 3             | 2.73                                  | 0.00514                      |
| 1b                | 80 | 3.55                  | 20     | 4             | 4.3                                   | 0.00514                      |
| 1c                | 80 | 3.55                  | 20     | 5             | 6.5                                   | 0.00514                      |
| 2                 | 80 | 5.53                  |        | 4             | 3.53                                  | 0.00330                      |
| 3                 | 80 | 6.48                  |        | 4             | 3.32                                  | 0.00282                      |
| 4a                | 80 | 1.00                  | 71.25  | 1             | 2.78                                  | 0.01825                      |
| 4b                | 80 | 1.00                  | 71.25  | 2             | 3.27                                  | 0.01825                      |

| Spring No.                       | 1   | 2   | 3    | 4    | 5   |
|----------------------------------|-----|-----|------|------|-----|
| Spring constant<br>in lbs./inch. | 1.0 | 1.6 | 1.86 | 5.26 | 9.3 |

= linear scale reduction factor.

m = force reduction factor.

\*m is based on prototype weight of 5700 lbs./ft.

\*\* density-mass ratio =

Fig. VIII-1 Results of Experiments on  
Damping of Angular Oscillations for  
Models Having the Same Value  
of  $\frac{1}{\mu}(\frac{l}{b})^2$

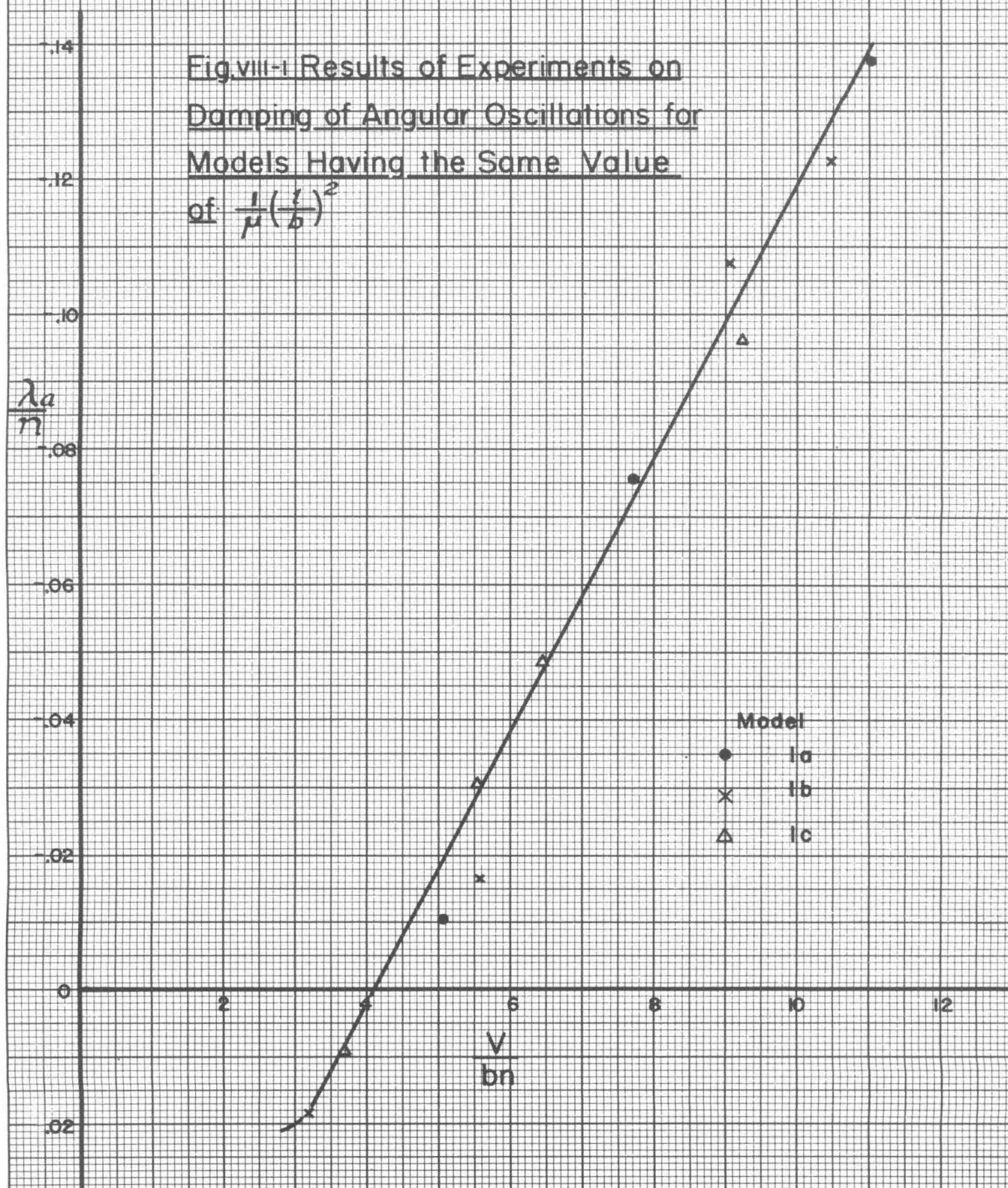
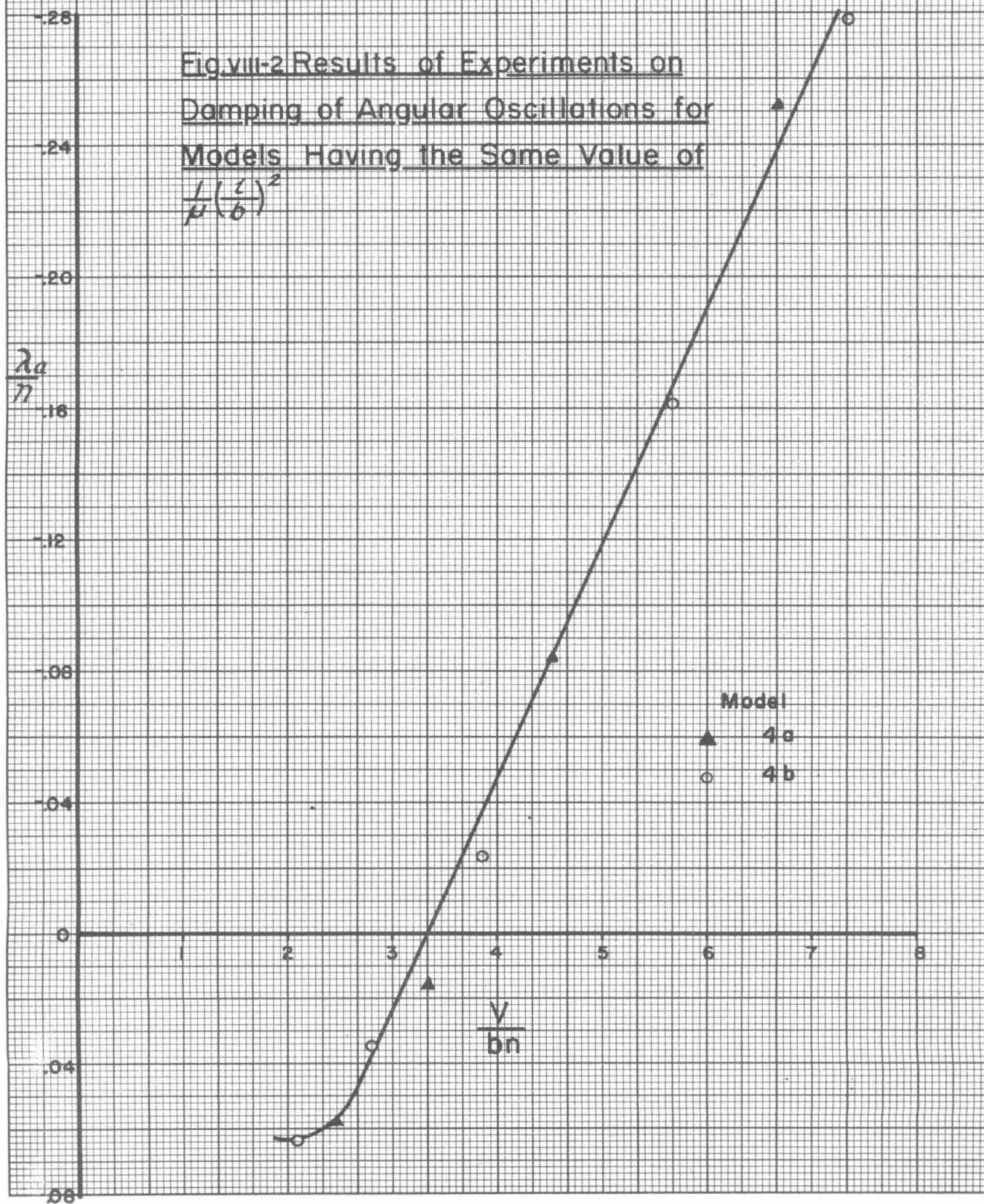
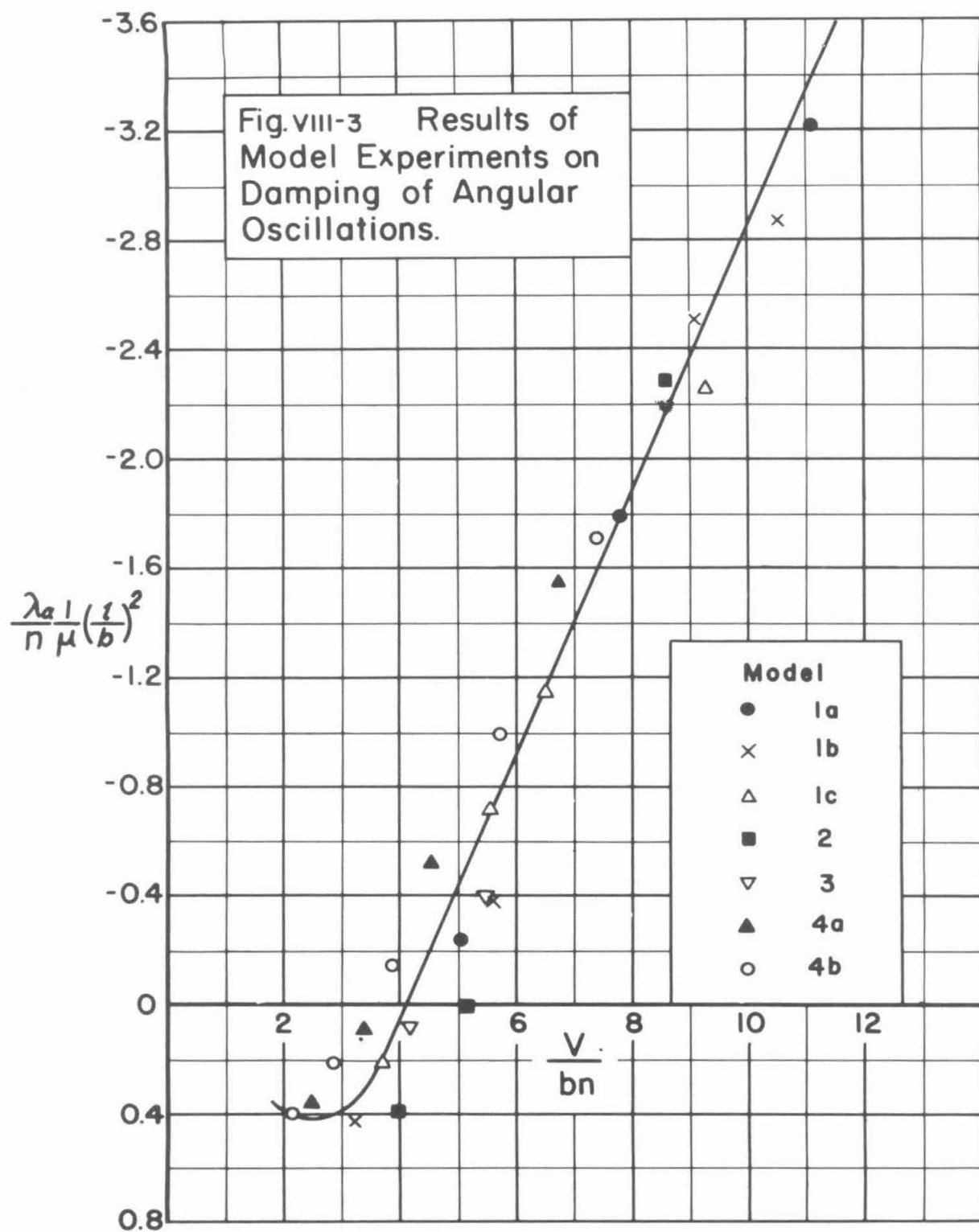




Fig. VIII-2 Results of Experiments on  
Damping of Angular Oscillations for  
Models Having the Same Value of  
 $\frac{1}{\mu} \left( \frac{l}{b} \right)^2$





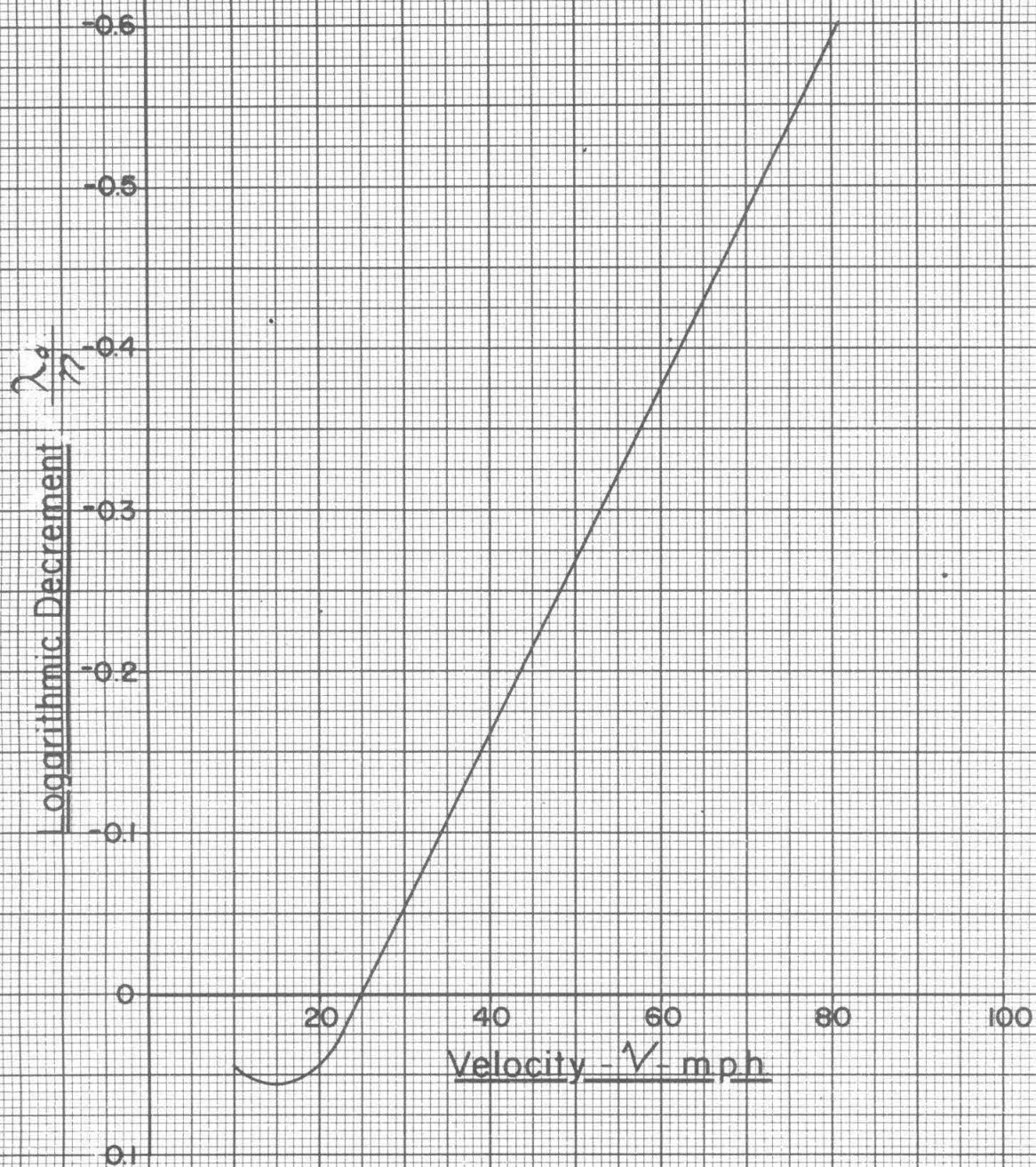


Fig.viii-4Application of Results of Model Experiments to the Prototype - Tacoma Narrows Bridge



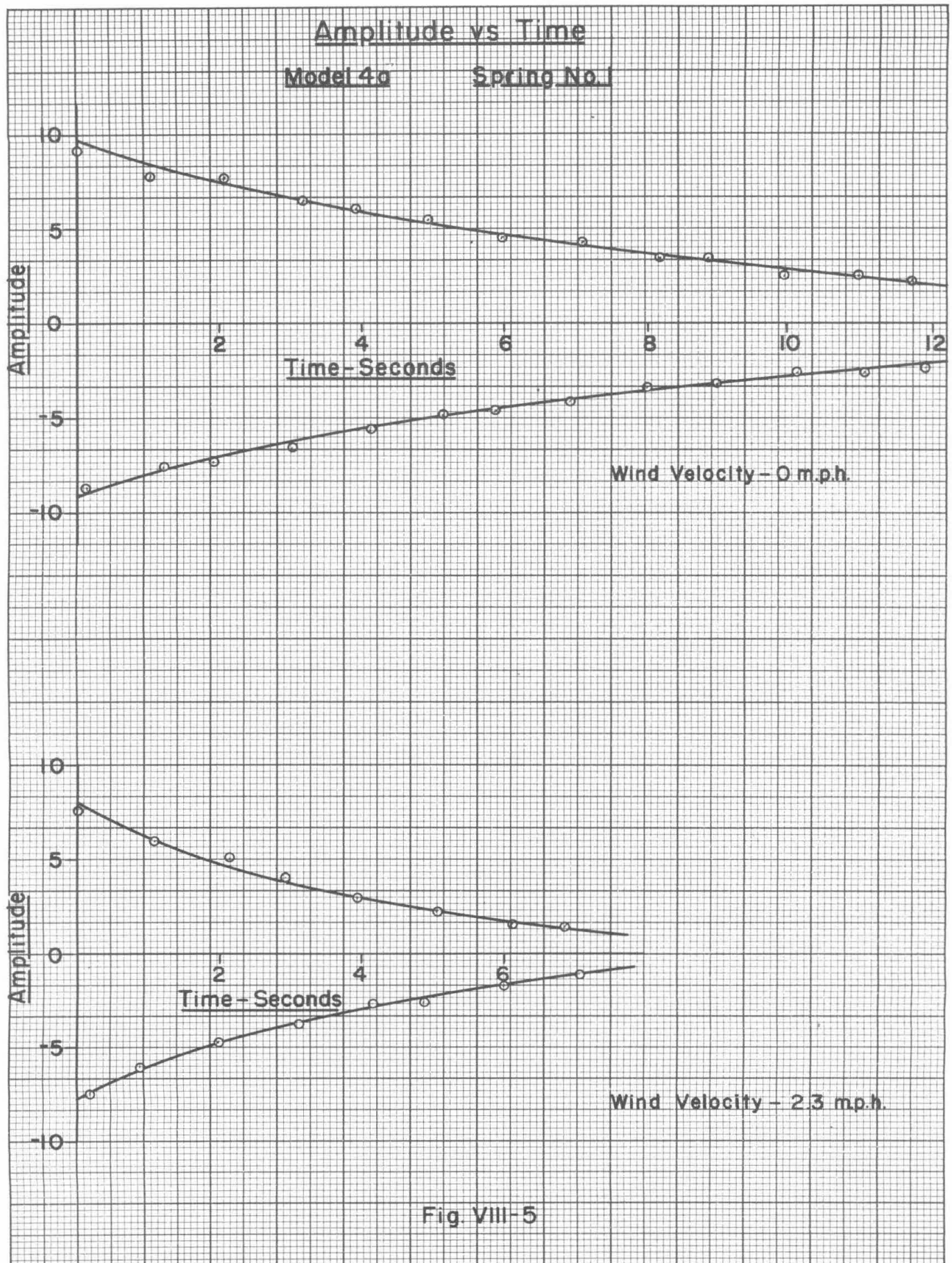
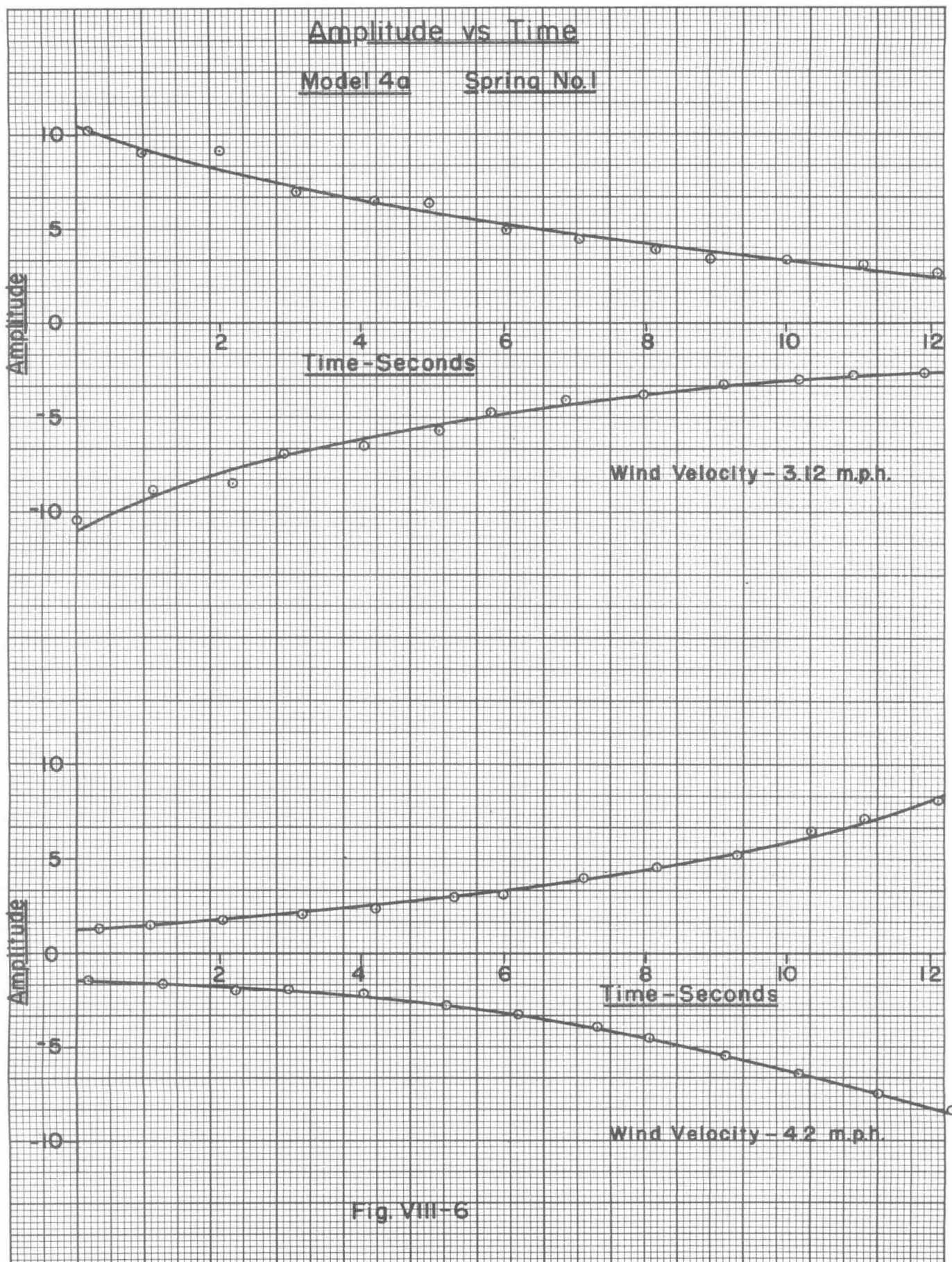


Fig. VIII-5





Amplitude vs Time  
Model 4a Spring No.1

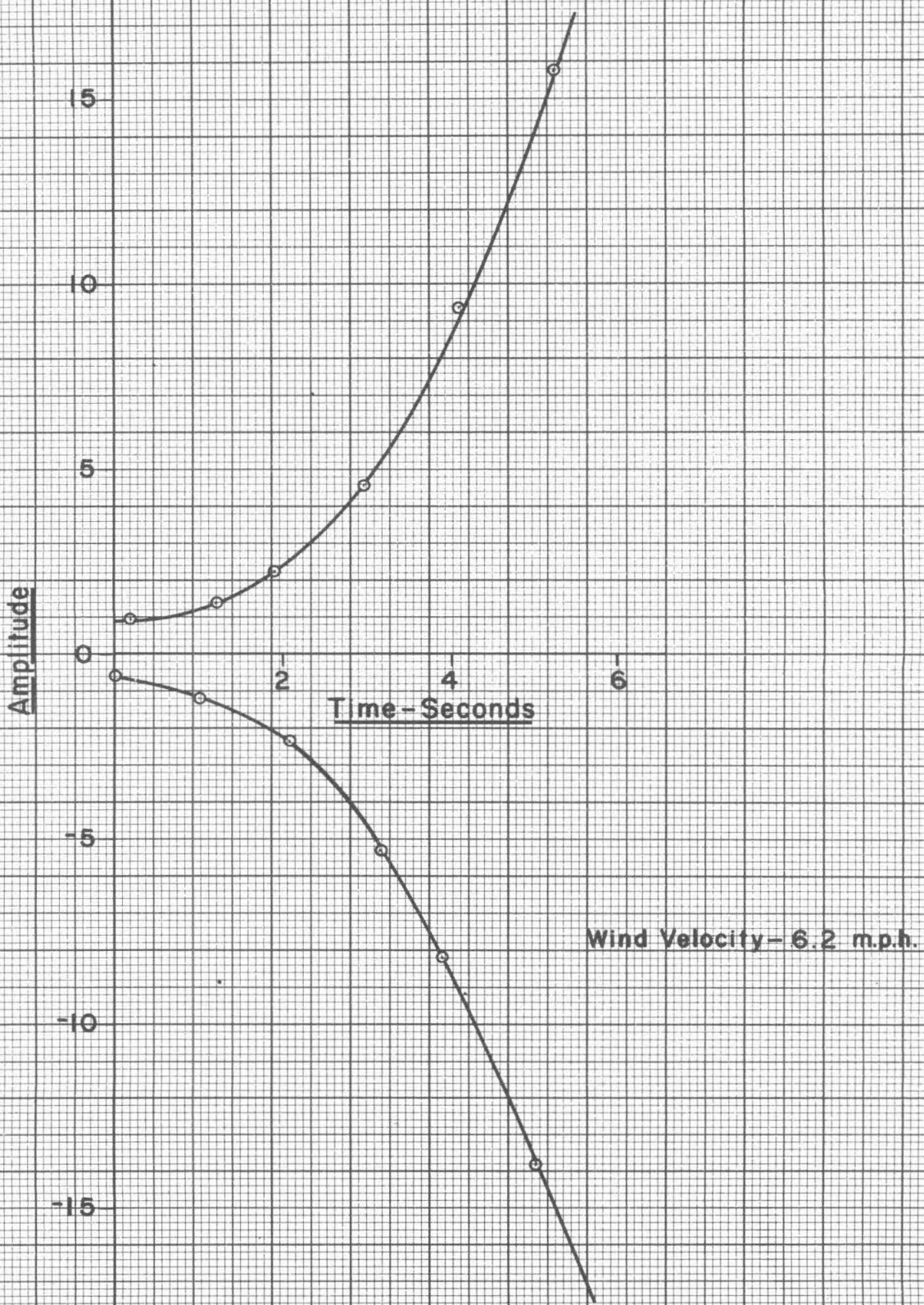


Fig. VIII - 7

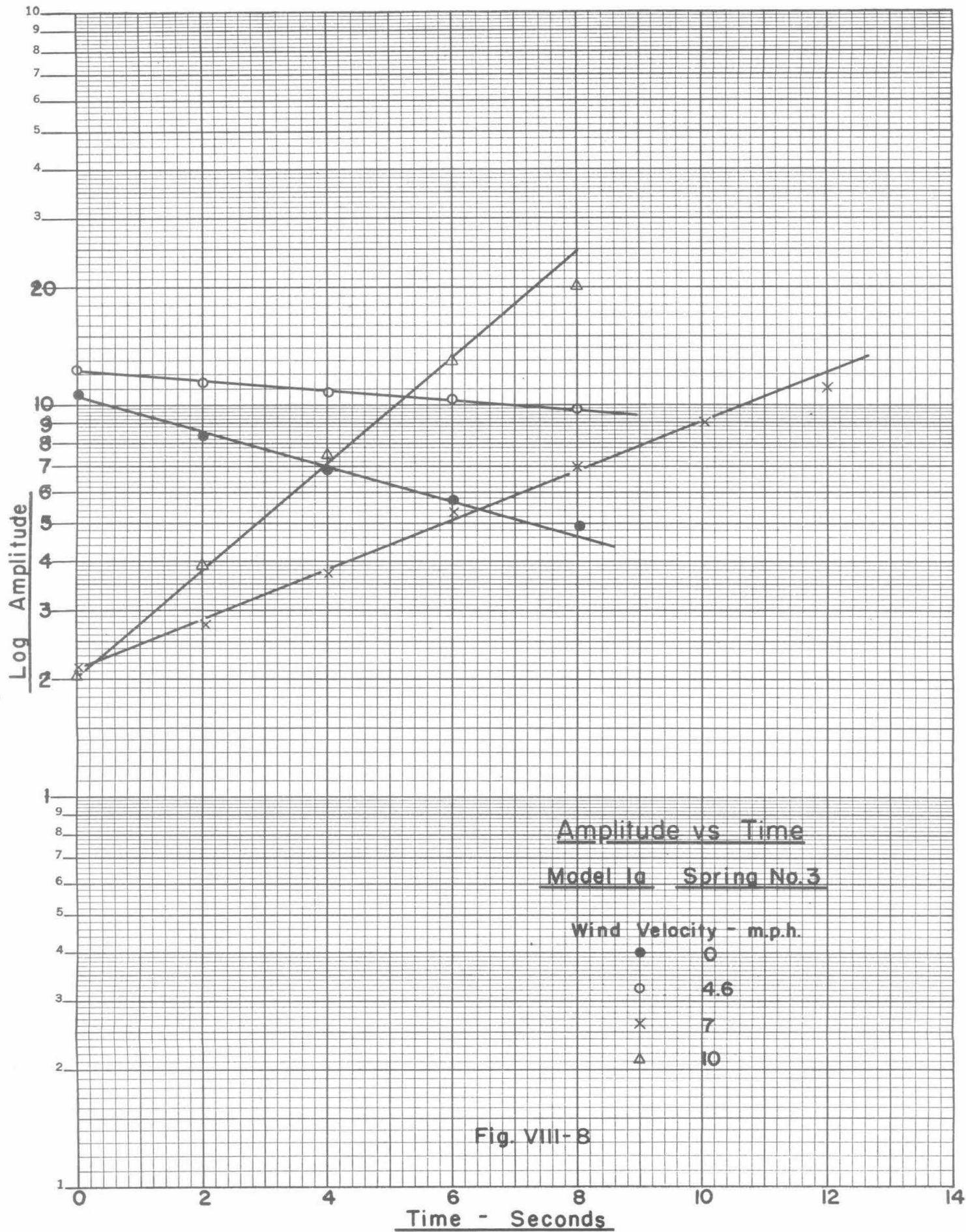
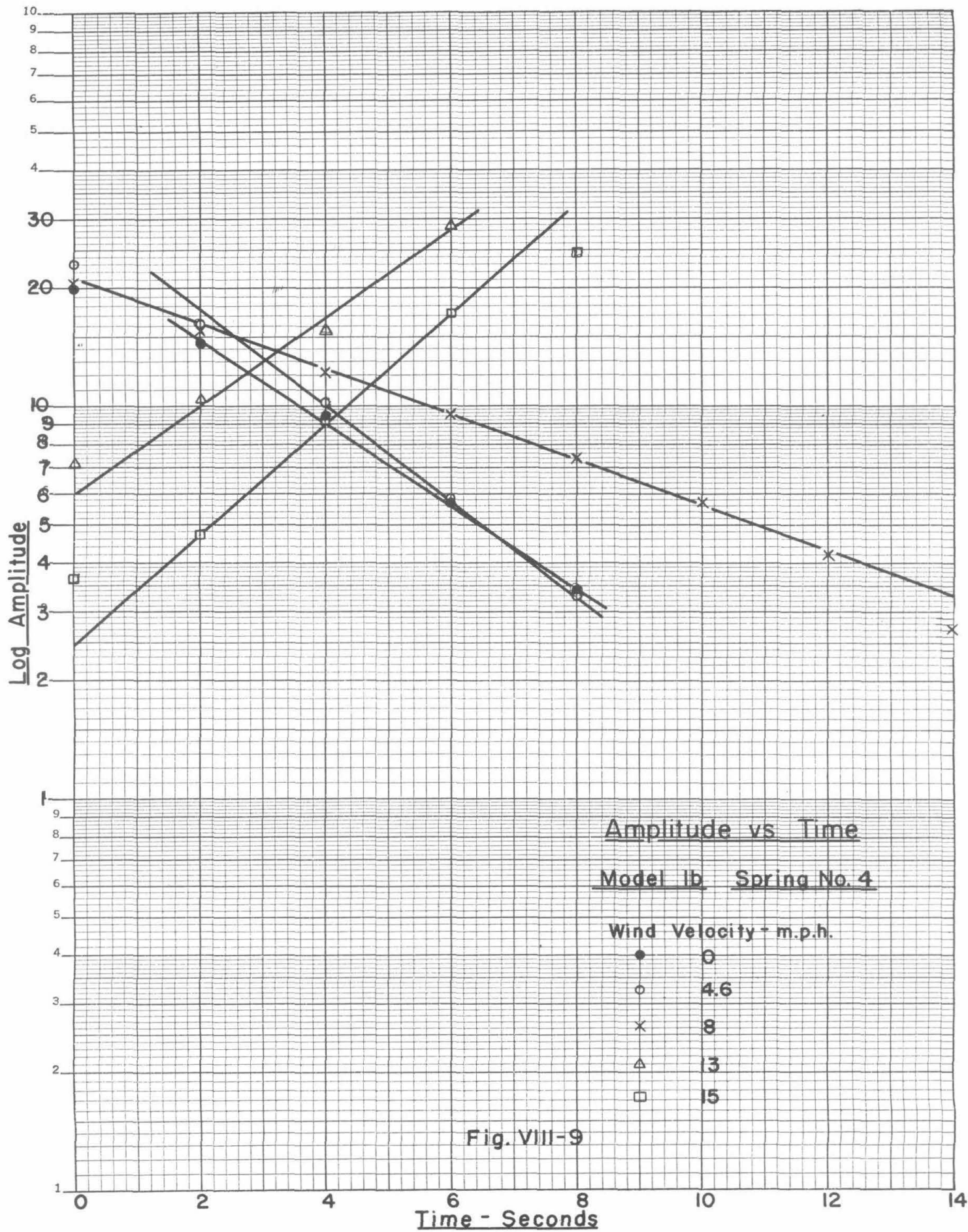
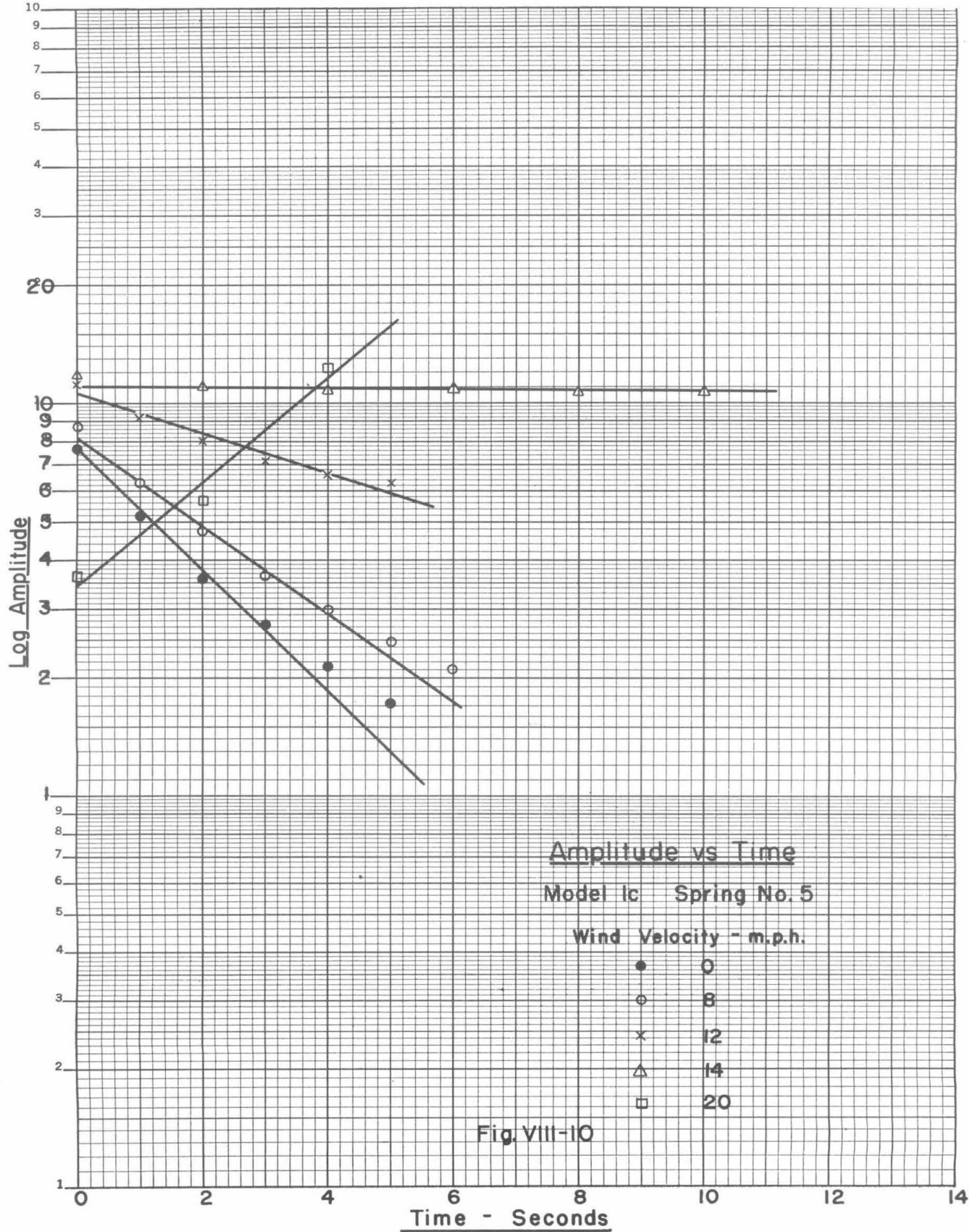


Fig. VIII-8







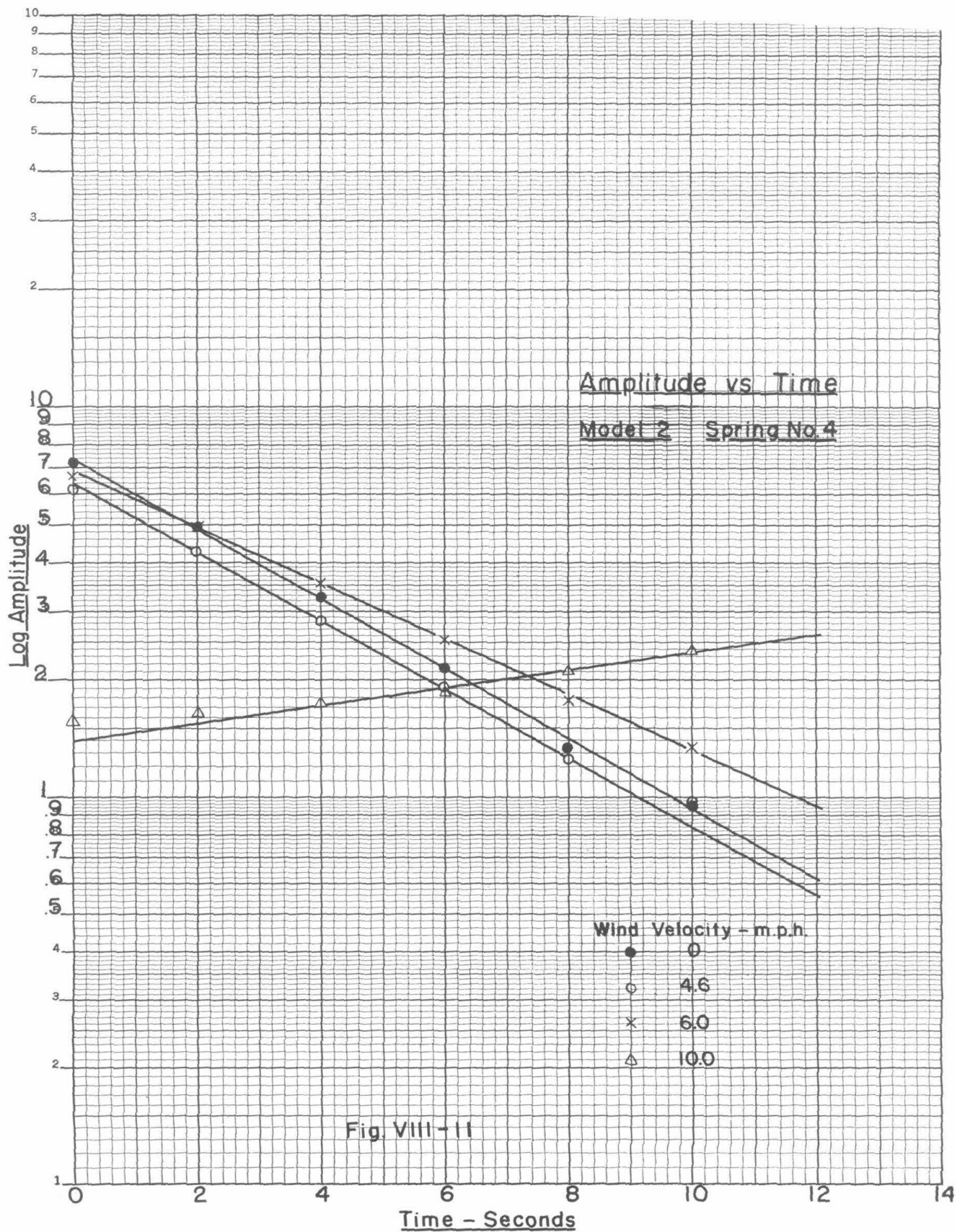


Fig. VIII-11

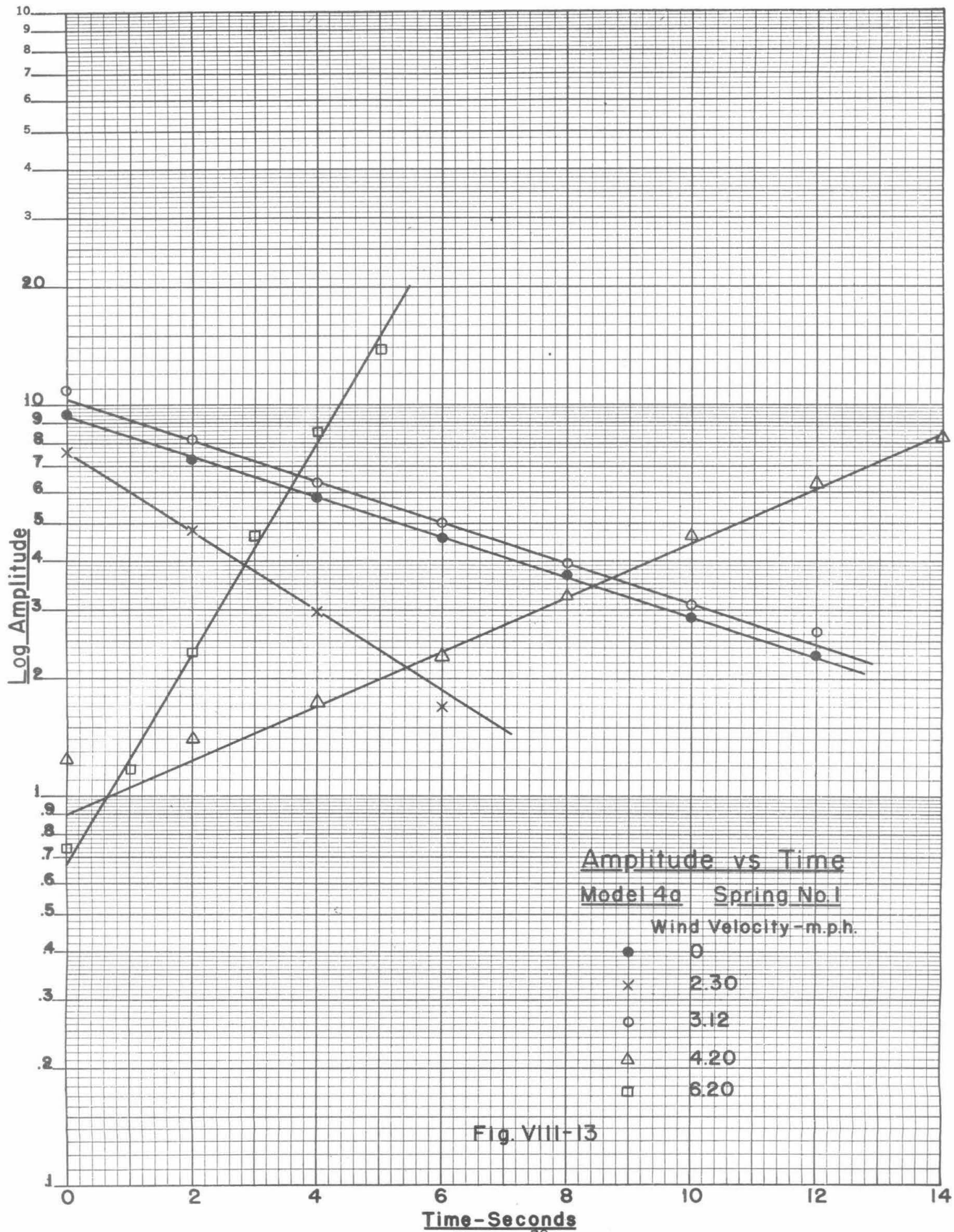
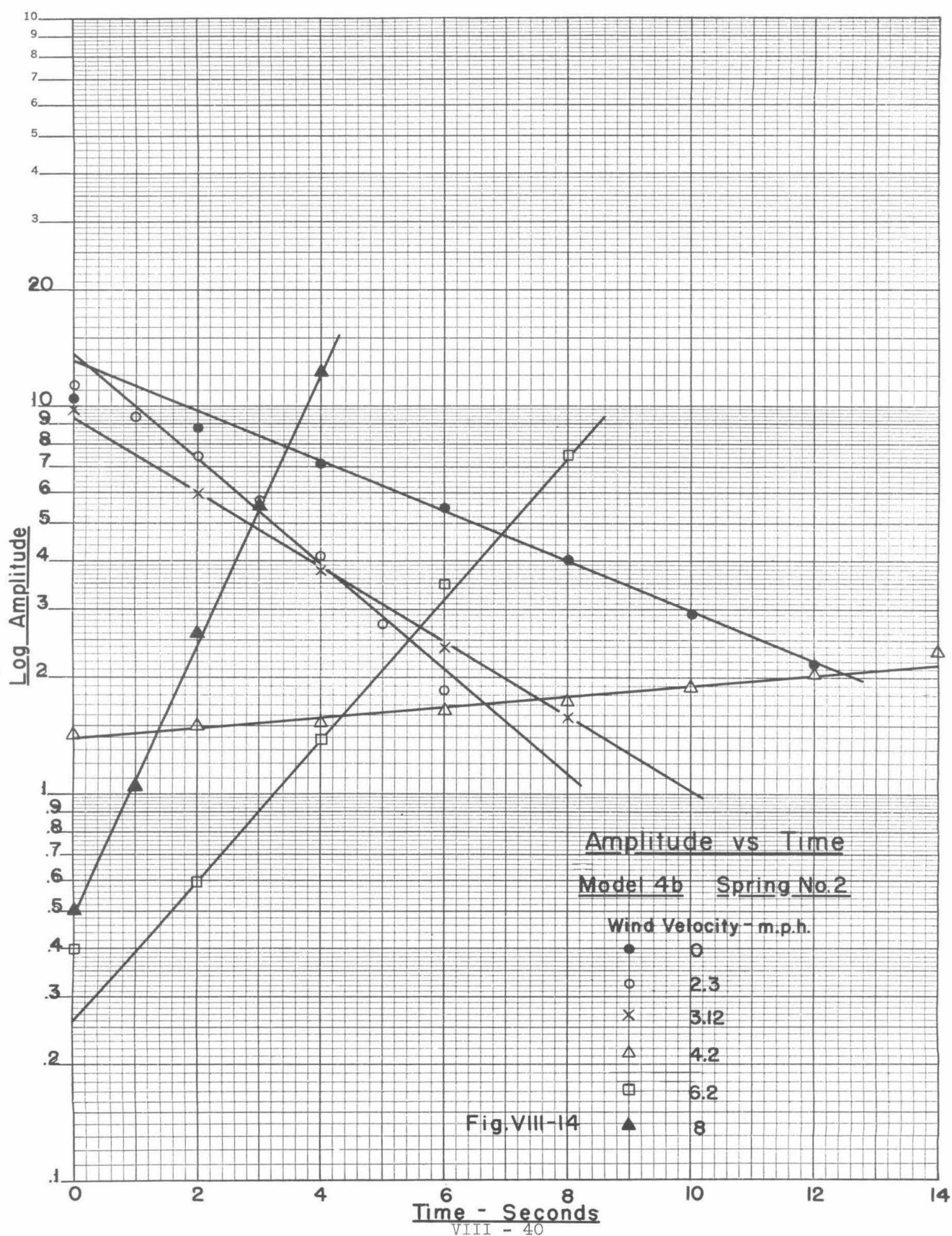
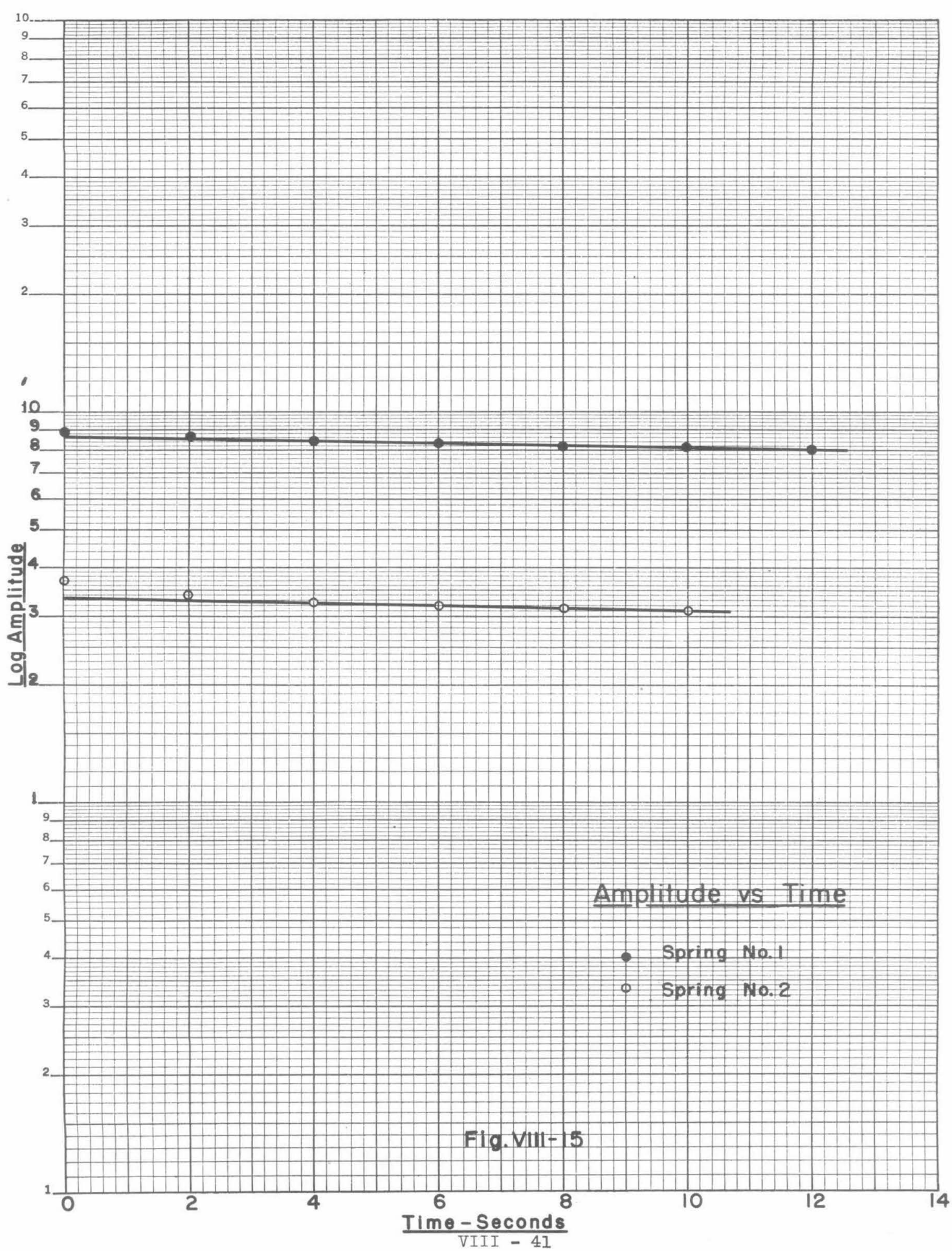


Fig. VIII-13







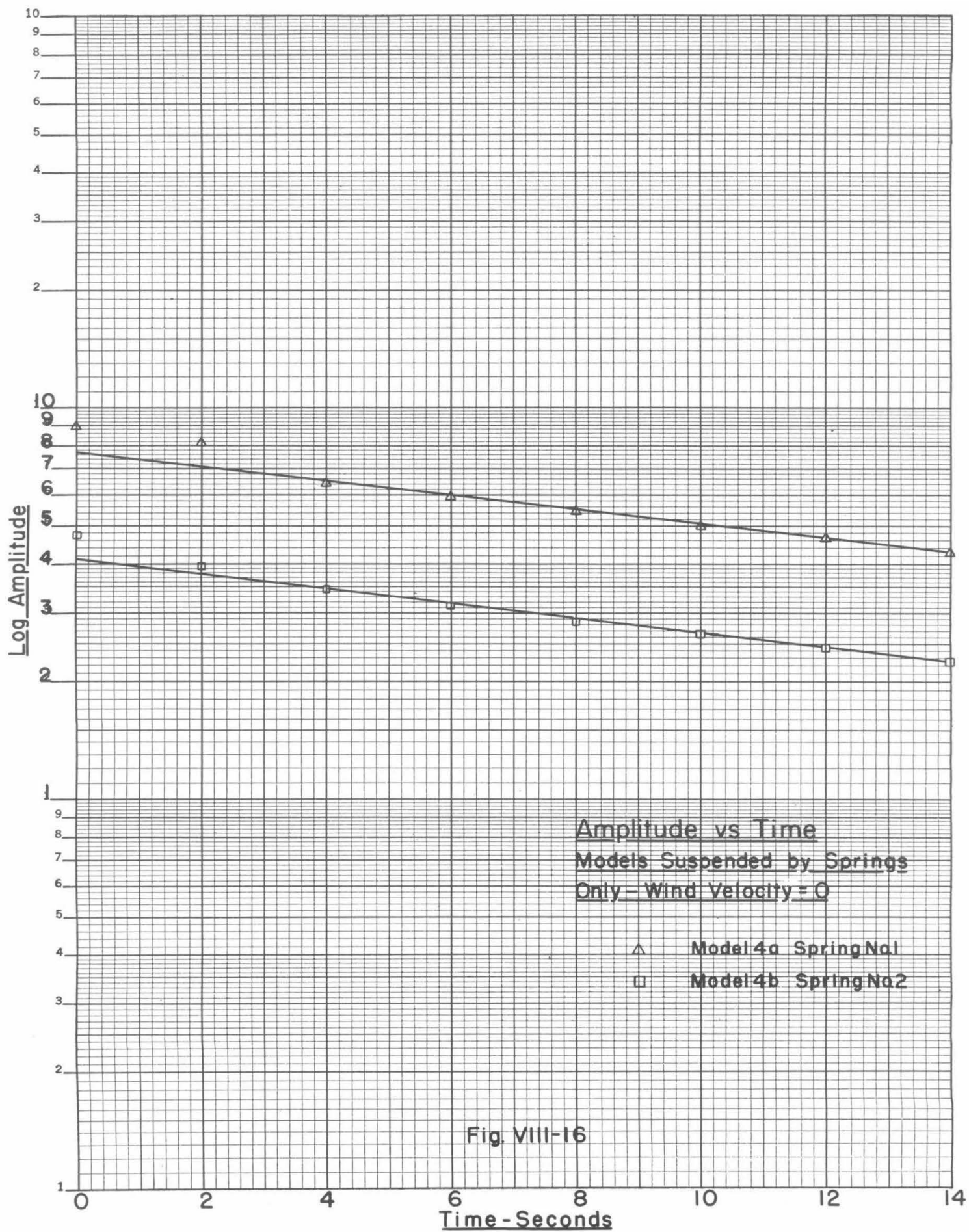


Fig. VIII-16



FIG. VIII-17  
DRAG CHARACTERISTICS  
OF VARIOUS SECTIONS  
FROM GALCIT TESTS

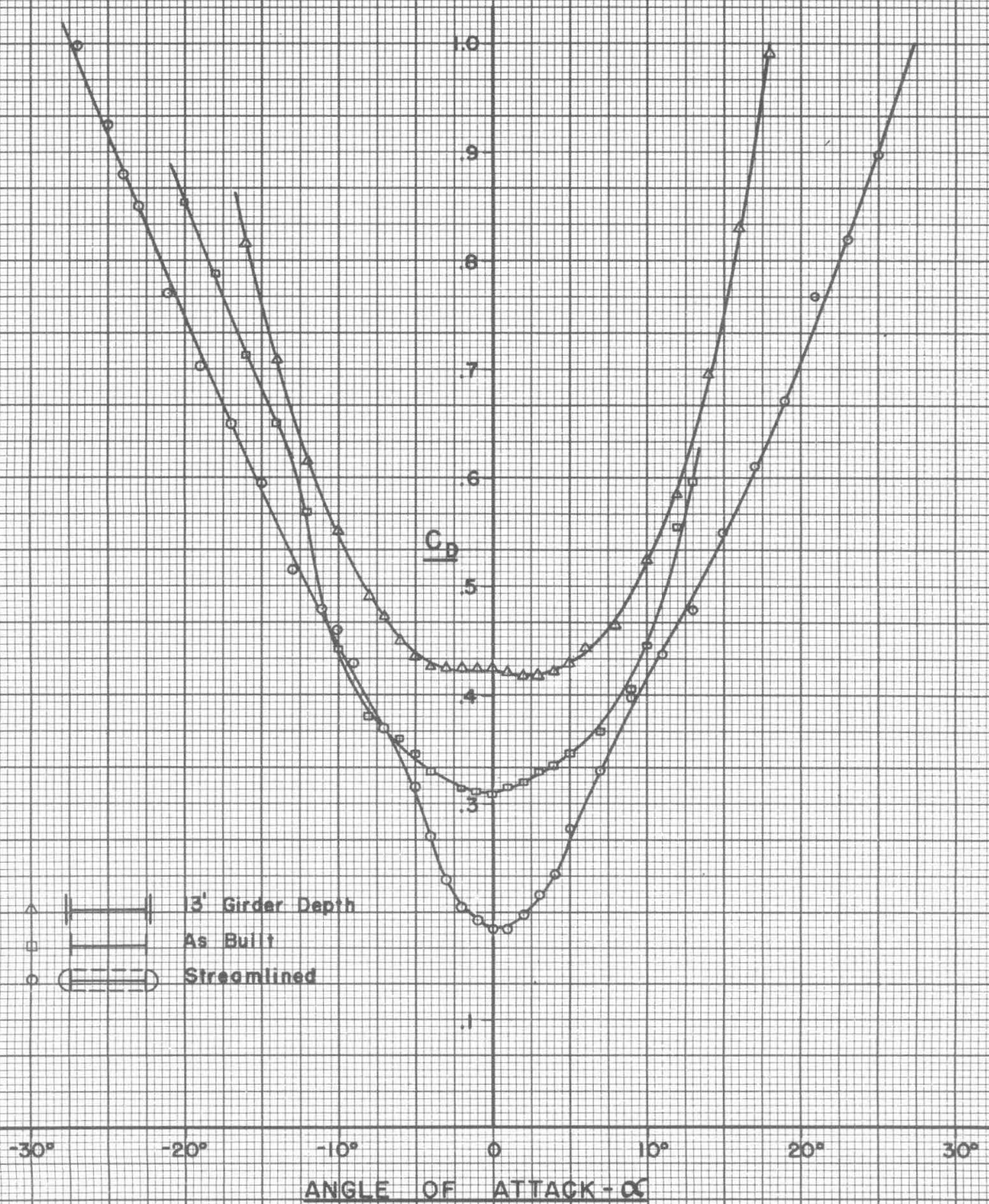
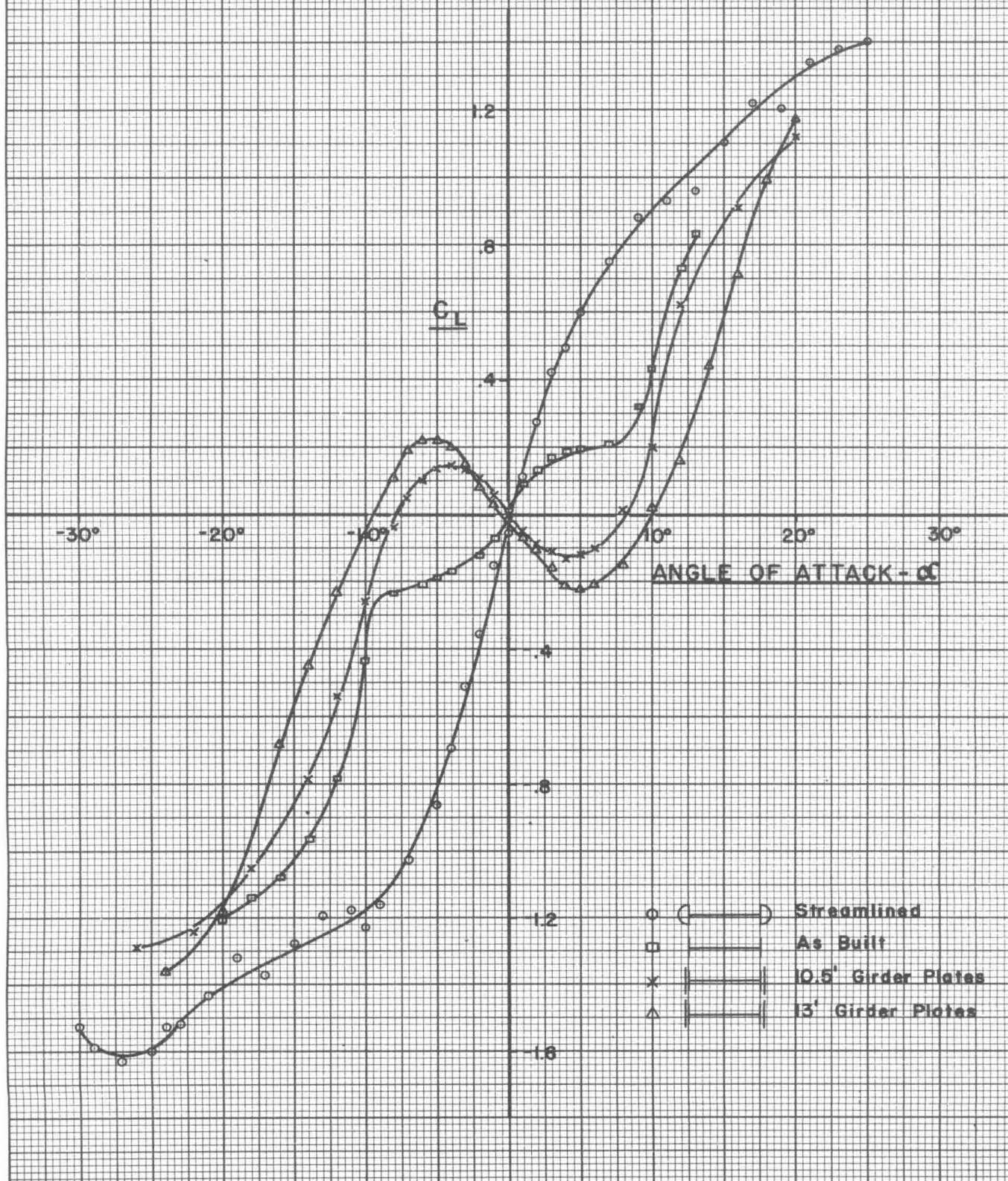


FIG. VIII-18  
LIFT CHARACTERISTICS  
OF VARIOUS SECTIONS  
FROM GALCIT TESTS





**FIG. VIII-19**  
**MOMENT CHARACTERISTICS**  
**OF VARIOUS SECTIONS**  
**FROM GALCIT TESTS**

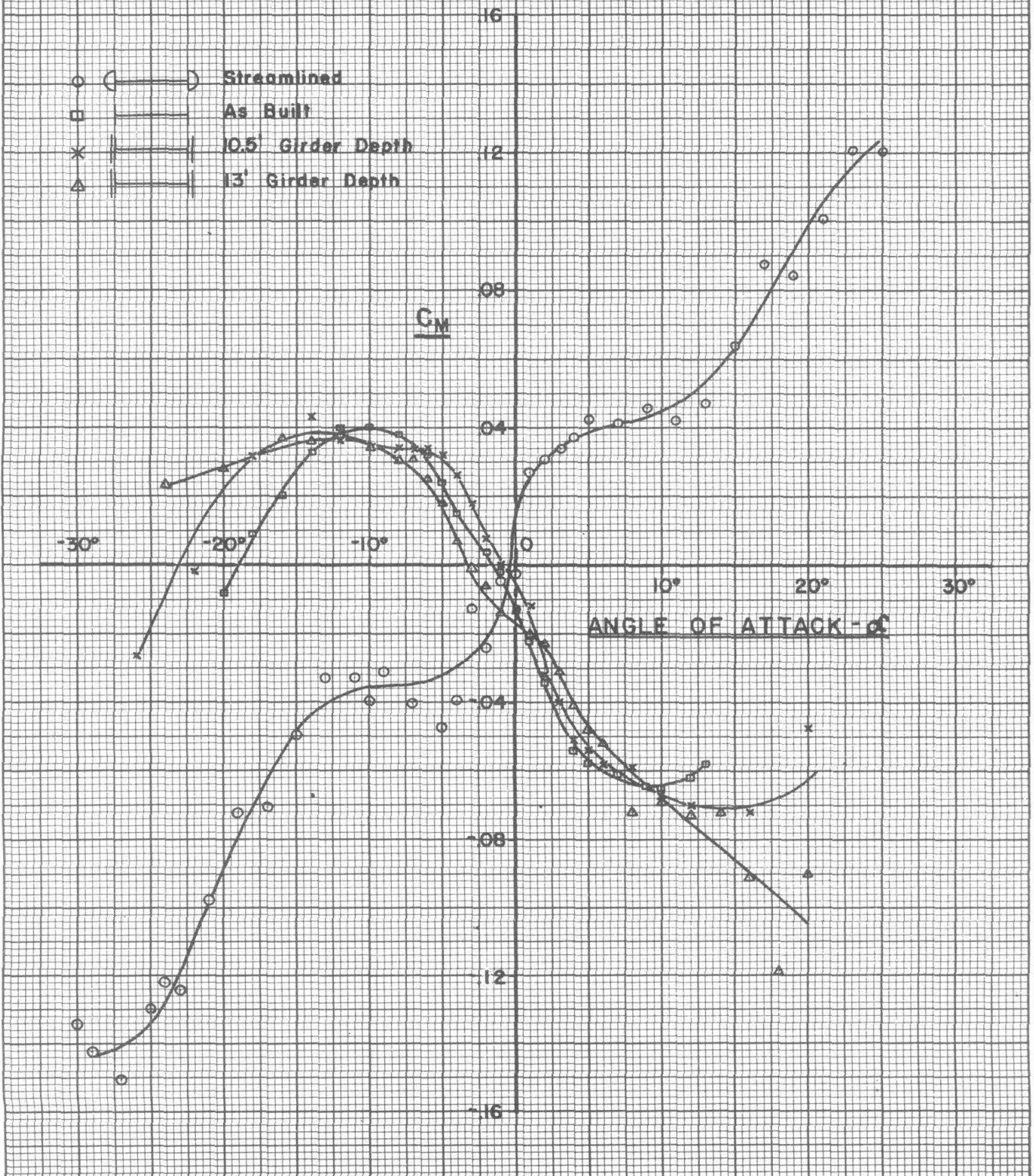
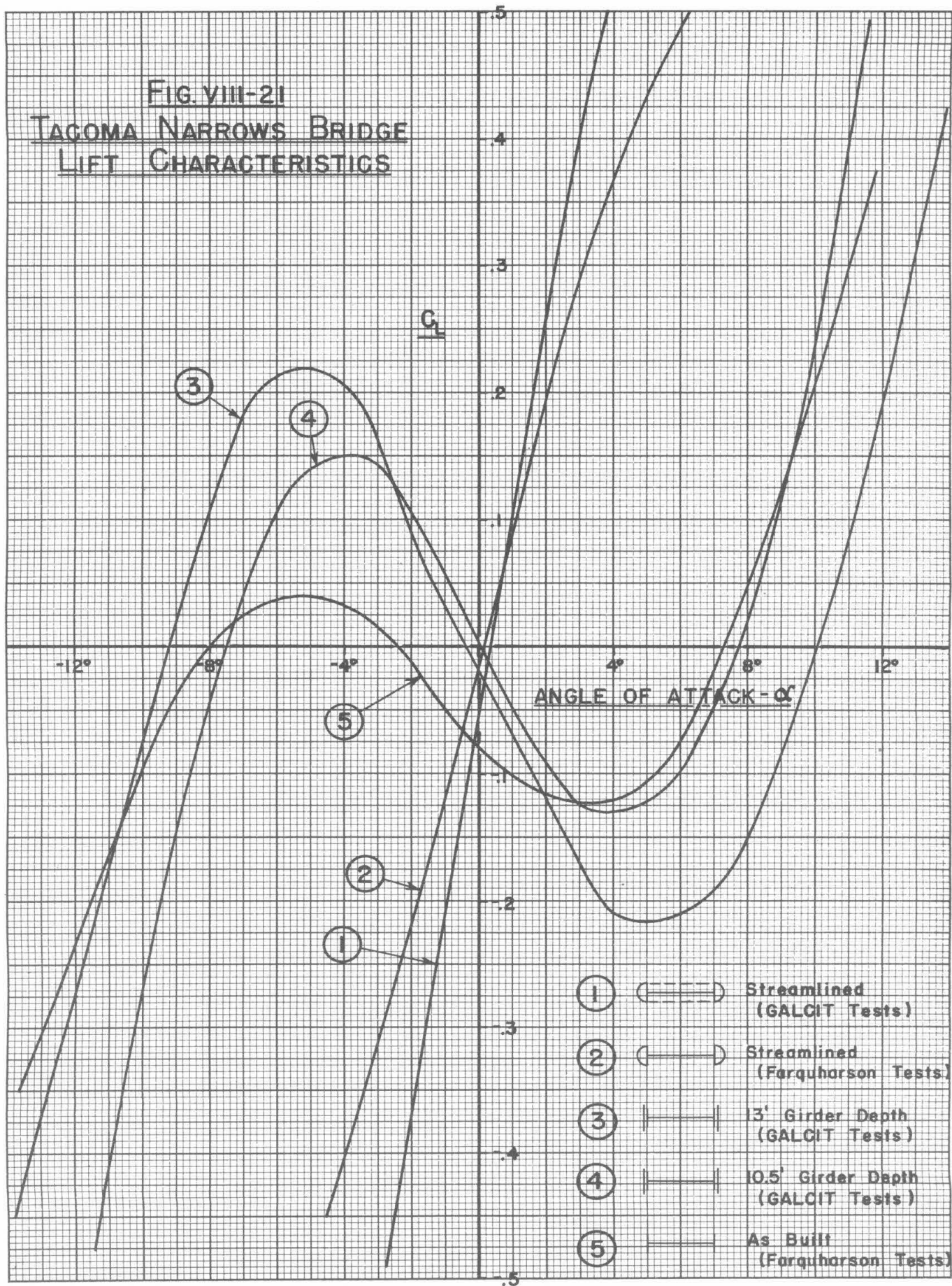
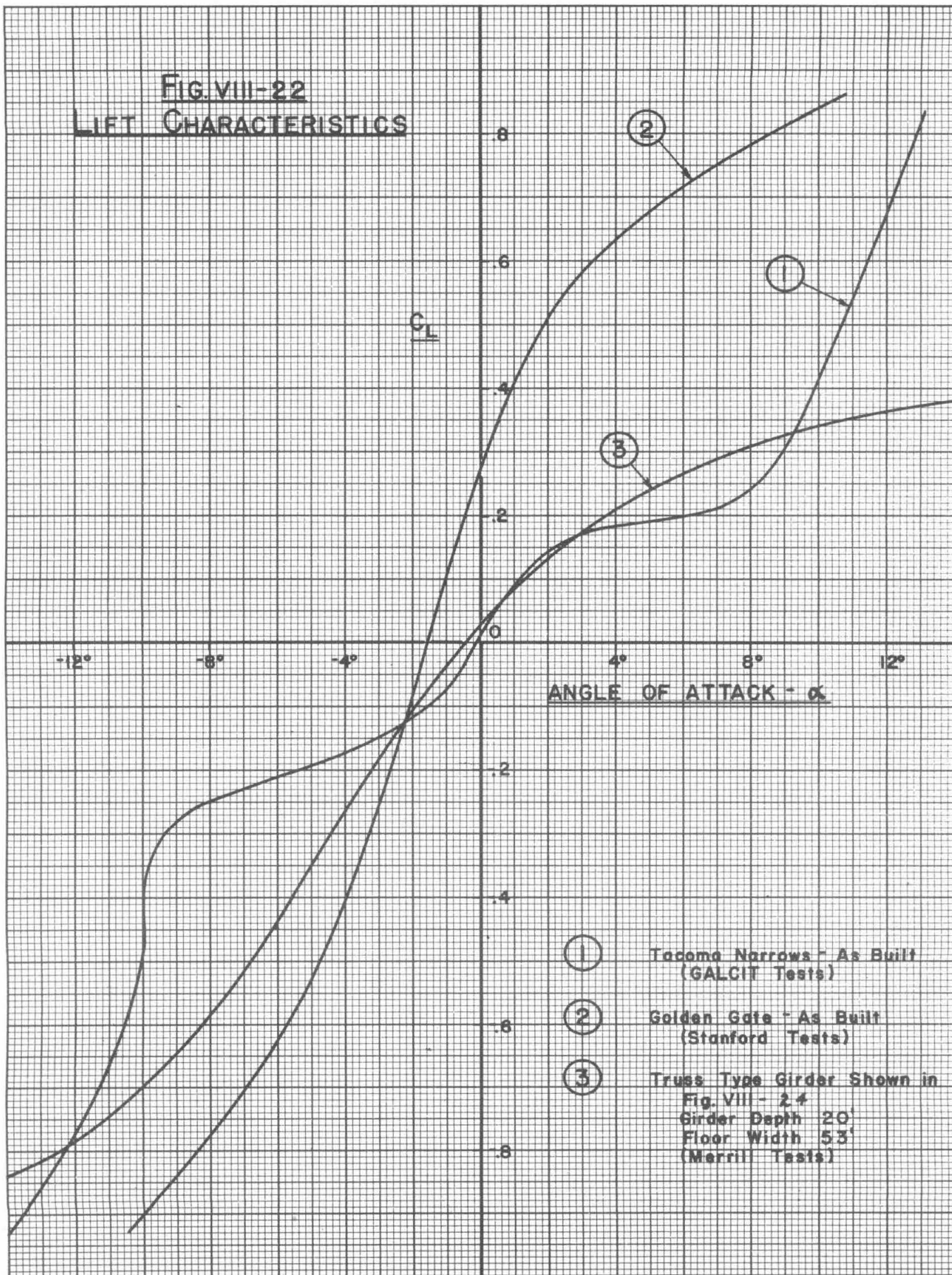


FIG. VIII-21  
TACOMA NARROWS BRIDGE  
LIFT CHARACTERISTICS





**FIG. VIII-22**  
**LIFT CHARACTERISTICS**





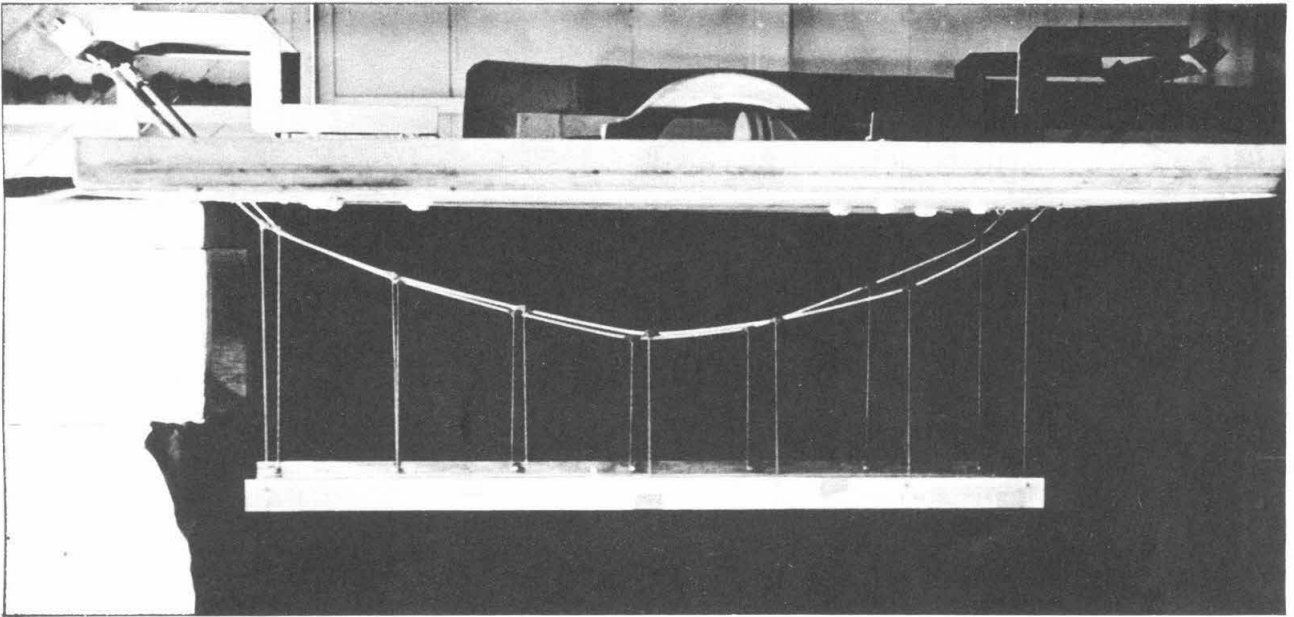
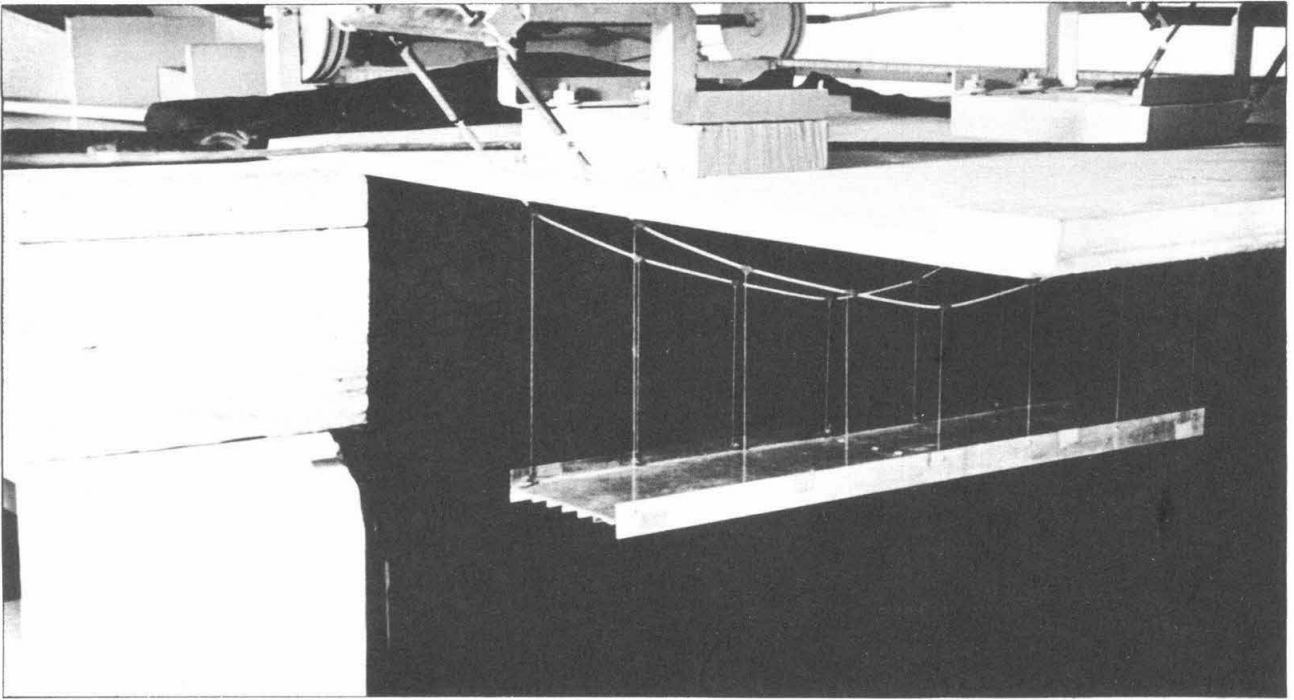


Fig. VIII-23. Model of the Tacoma Narrows Bridge  
suspended in the wind tunnel for dynamic test.

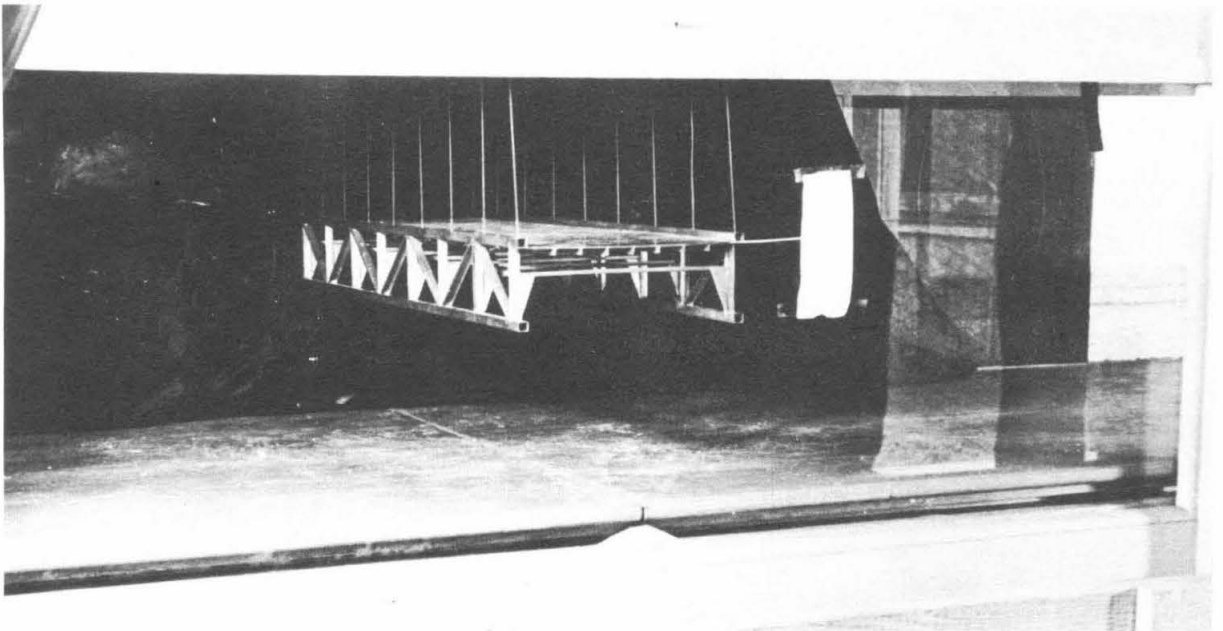
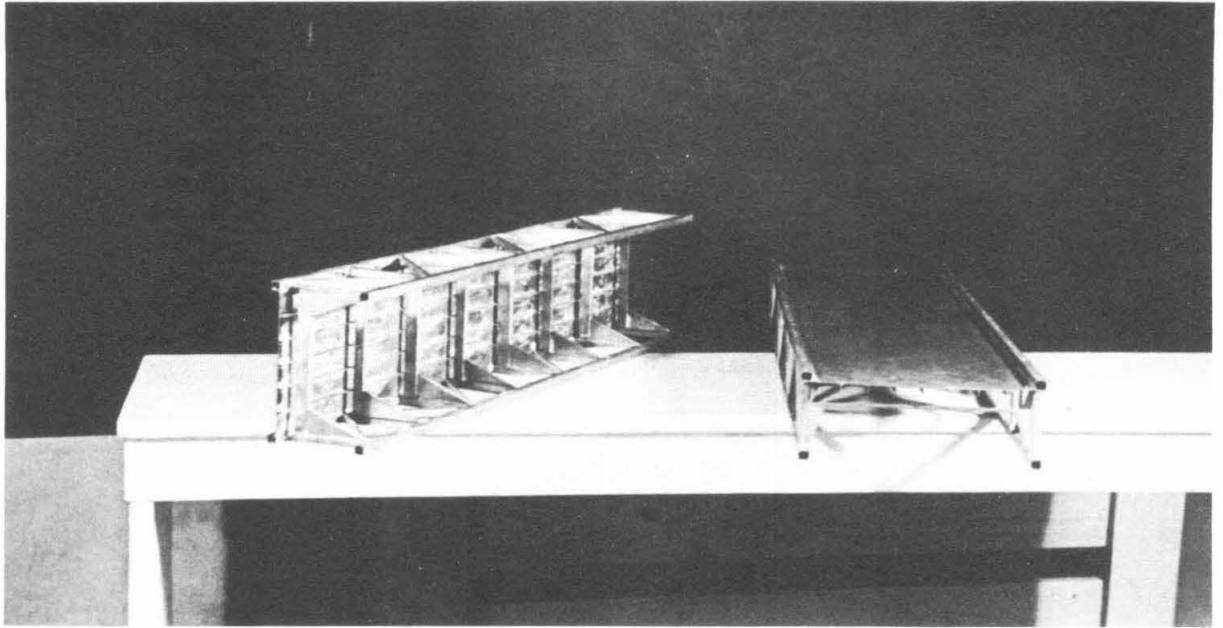


Fig. VIII-24. Models with deep girders.

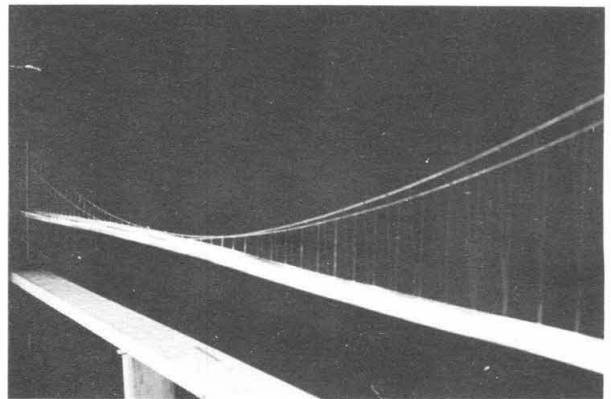
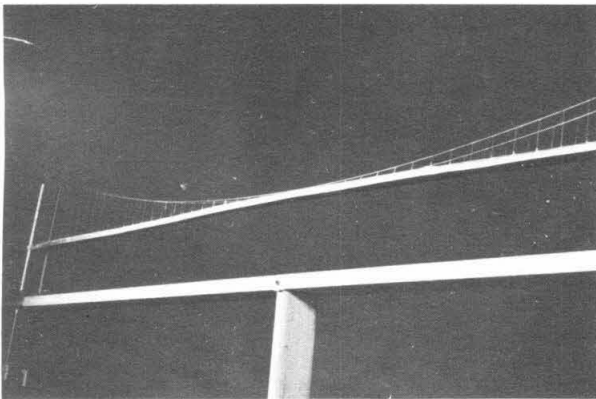
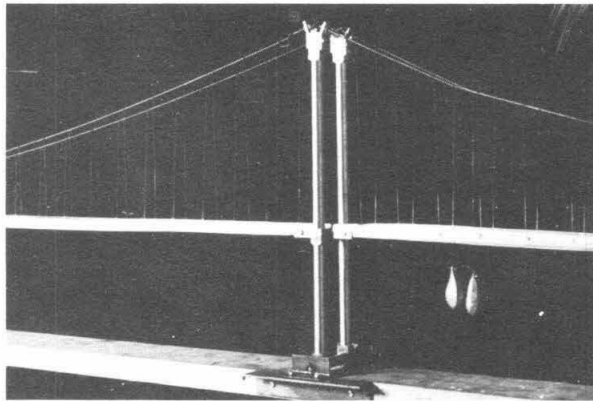


Fig. VIII-25. Scale model in wind tunnel.

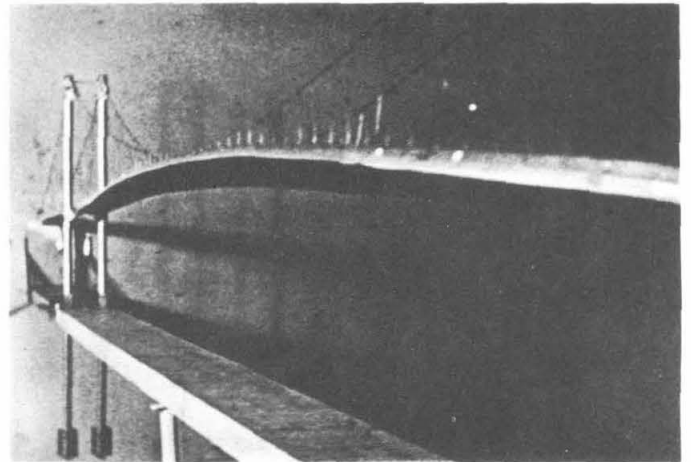
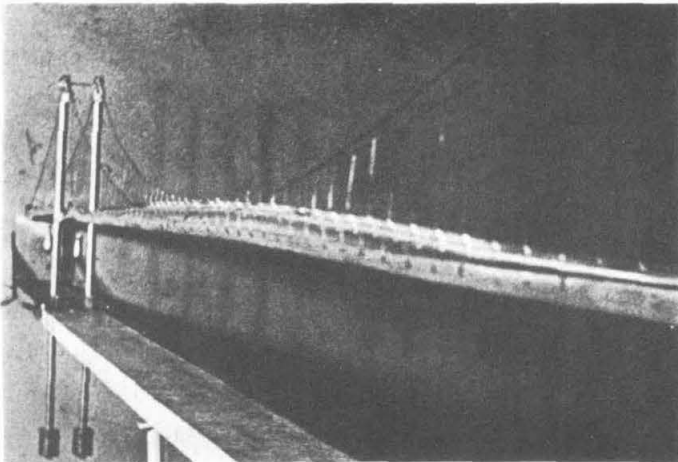
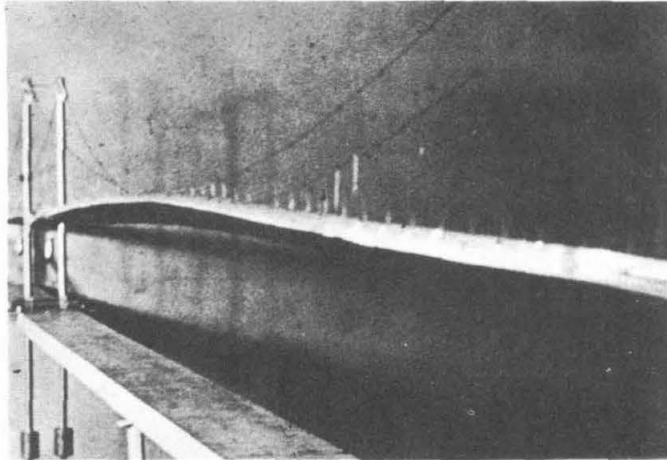


Fig. VIII-26. Scale model in torsional oscillation  
at a wind velocity of 5.7 miles per hour.

## APPENDIX IX

### OBSERVATIONS OF MOTIONS OF GOLDEN GATE BRIDGE WIND STORMS OF

FEBRUARY 9, 1938 and FEBRUARY 11, 1941

BY MR. R. G. CONE, CHIEF ENGINEER

#### 1. STORM OF FEBRUARY 9, 1938

About 1:00 P.M. on the afternoon of February 9, 1938, a wind of unusual high velocity was blowing through the Golden Gate with the direction normal to the axis of the Bridge as nearly as could be determined. The force of the wind was so strong that it was impossible to stand erect on the sidewalk or on the roadway of the Bridge.

I drove to the San Francisco tower in a closed car and was able to open the door on the leeward side and get out on the roadway. By crouching and standing in the lee of the west leg of the San Francisco tower, I was able to cross the roadway. I sighted along one of the offsets in the tower to the Marin shore and saw that the center of the Bridge was deflected between eight and ten feet from its normal position and was holding this deflected position. I also observed that the suspended structure of the Bridge was undulating vertically in a wavelike motion of considerable amplitude. These undulations were fairly rapid, in the neighborhood of 20 to 30 vibrations per minute. Because of their rapidity I could only estimate their amplitude but it appeared to me that the stiffening truss was being distorted as much as two feet vertically in 300 feet of bridge. The wave motion appeared to be a running wave similar to that made by cracking a whip. The truss would be quiet for a second and then in the distance one could see a running wave of several nodes approaching. The force of this wave was taken up in the movement of the expansion joint and the rocker arms at the top.

While this movement was going on one of the electricians, Mr. F. L. Pinkham, came by driving a truck. I motioned to Mr. Pinkham to stop and get out of the truck. He did so and attempted to climb over the curb slightly north of the tower but the force of the wind blew him back over the curb and down onto the roadway. He crawled over to where I was standing in the protection of the tower and I asked him to observe the movements and actions of the Bridge, telling him I wanted a witness to substantiate what I had seen since the oscillations and deflections of the Bridge were so pronounced that they would seem unbelievable. Mr. Pinkham stood with me for some time

observing the Bridge.

I then decided to try and secure a record of some of the deflections on film and returned to the office for my camera. Returning with the camera a few minutes later, I noticed that the roadway had stopped oscillating but it was still deflected out of line. I then went to the top of the tower to see if I could get a record of the deflection in the camera but by that time the wind had diminished so that the deflection was not pronounced enough to show clearly in the pictures.

Signed: R. G. Cone

San Francisco, California  
January 7, 1941

## 2. STORM OF FEBRUARY 11, 1941

### Direction of Wind:

From southwest at an angle of approximately 45 degrees with the axis of the Bridge.

### Velocities at top of San Francisco Tower:

|           |           |
|-----------|-----------|
| 1:00 P.M. | 48 m.p.h. |
| 1:15 "    | 60 m.p.h. |
| 2:50 "    | 62 m.p.h. |
| 3:10 "    | 60 m.p.h. |
| 3:30 "    | 60 m.p.h. |

### First Series Readings:

12:45 to 1:15 P.M. interrupted by heavy rain obscuring sights. Transit was set up in shelter of east leg of San Francisco tower on a line 50 inches east of the east edge of the top chord. 12:45 P.M. foresight on same point Marin Tower.

### Deflections laterally -

|            |          |
|------------|----------|
| 12:45 P.M. | 1.75 ft. |
| 12:50 P.M. | 2.30 ft. |
| 1:00 P.M.  | 2.90 ft. |
| 1:05 P.M.  | 4.9 ft.  |
| 1:15 P.M.  | 4.0 ft.  |

During this period vertical oscillations were observed simultaneously at three points along the span by sighting on the stadia hairs in the transit. The period of these oscillations varied from 5 1/2 seconds to 6 1/2 seconds for the full amplitude up and down. At the quarter point and at the third point (San

Francisco) the magnitude of the oscillations was from 9 inches to 20 inches as determined from using a 5-inch casting on the chord for a measure.

Second Series Readings:

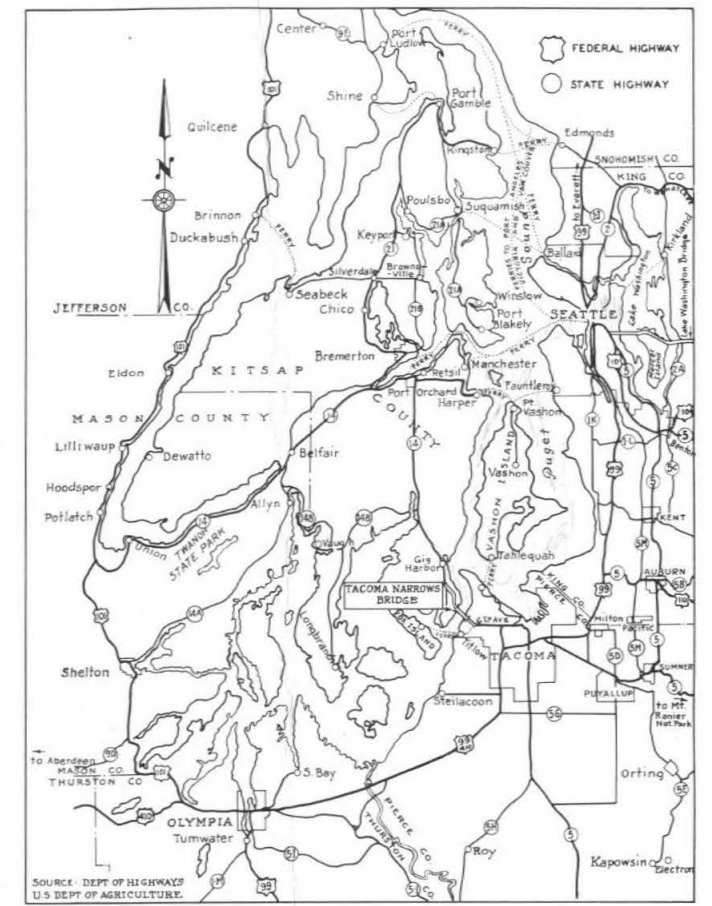
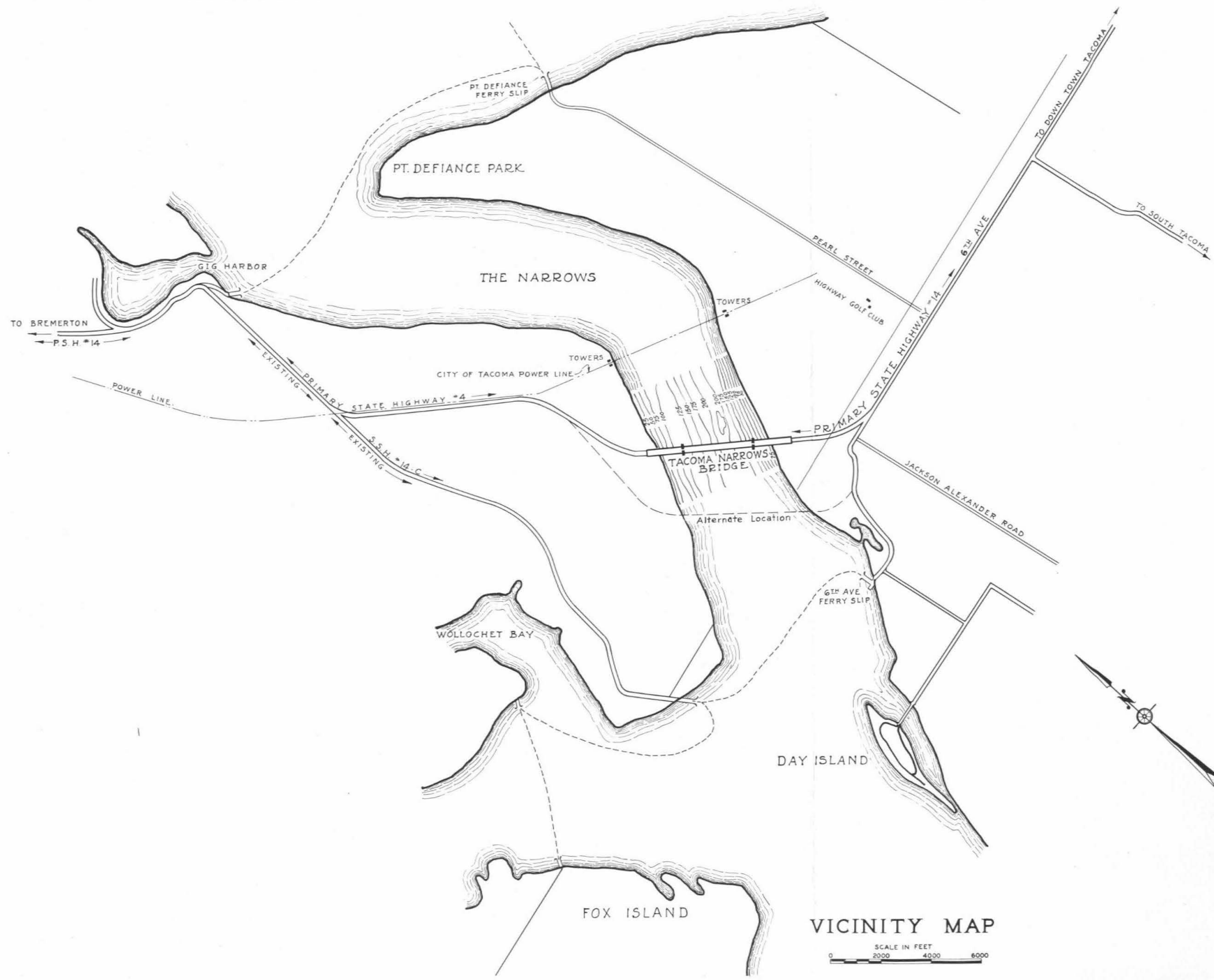
2:50 P.M. to 3:30 P.M. - Wind southwest.

| <u>Lateral Deflection</u> | <u>Velocity</u> | <u>Oscillation</u><br><u>Quarter Point</u> | <u>Oscillation</u><br><u>Center</u> |
|---------------------------|-----------------|--|-------------------------------------|
| 2:50 P.M. - 5.6 ft.       | 62 m.p.h.       | 2.0 ft.                                    | 1.6 ft.                             |
| 3:10 P.M. - 4.6 ft.       | 60 m.p.h.       | 1.8 ft.                                    |                                     |
| 3:15 P.M. - 2.9 ft.       | 60 m.p.h.       | 1.8 ft.                                    |                                     |

The period of the oscillations were in the neighborhood of 8 seconds for the full up and down movement.

R. G. CONE

March 14, 1941



LOCATION MAP

SCALE IN MILES  
0 1 2 3 4 5 6 7 8 9 10

VICINITY MAP

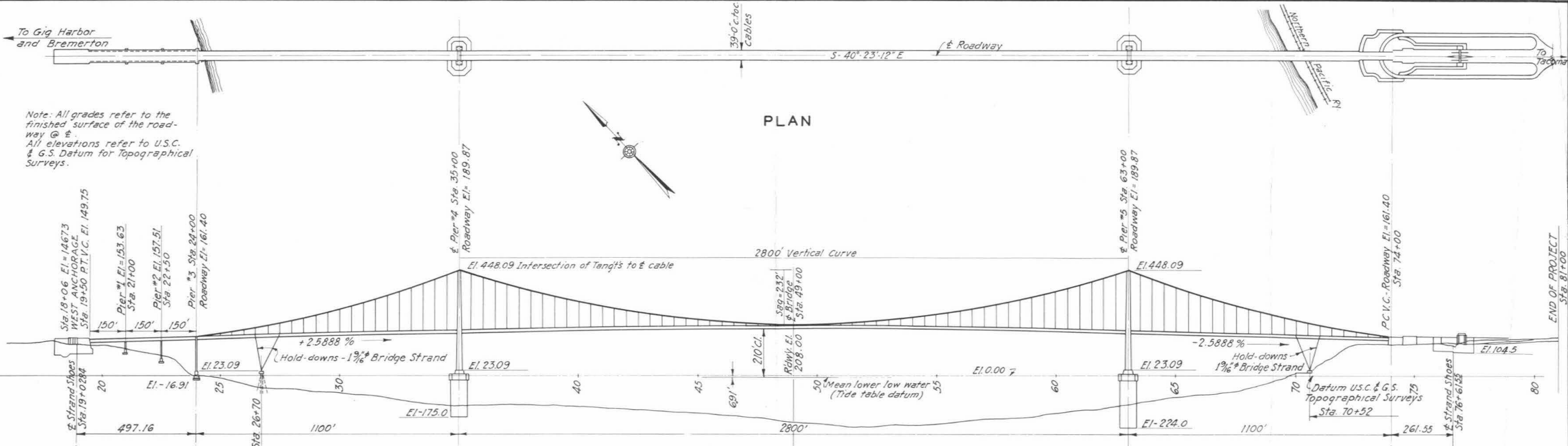
SCALE IN FEET  
0 2000 4000 6000

**TACOMA NARROWS BRIDGE**  
P.W.A. DOCKET NO. WASH 1870 F  
BRIDGE LOCATION

SCALE AS SHOWN

MARCH 1941 DRAWING 1





Cable  
19 Strands  
332-No.6 Wires per st  
6308 Wires per Cable  
Cable Diam.-17 1/8"

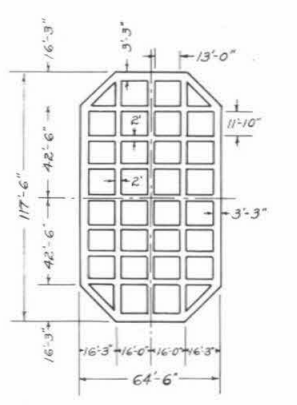
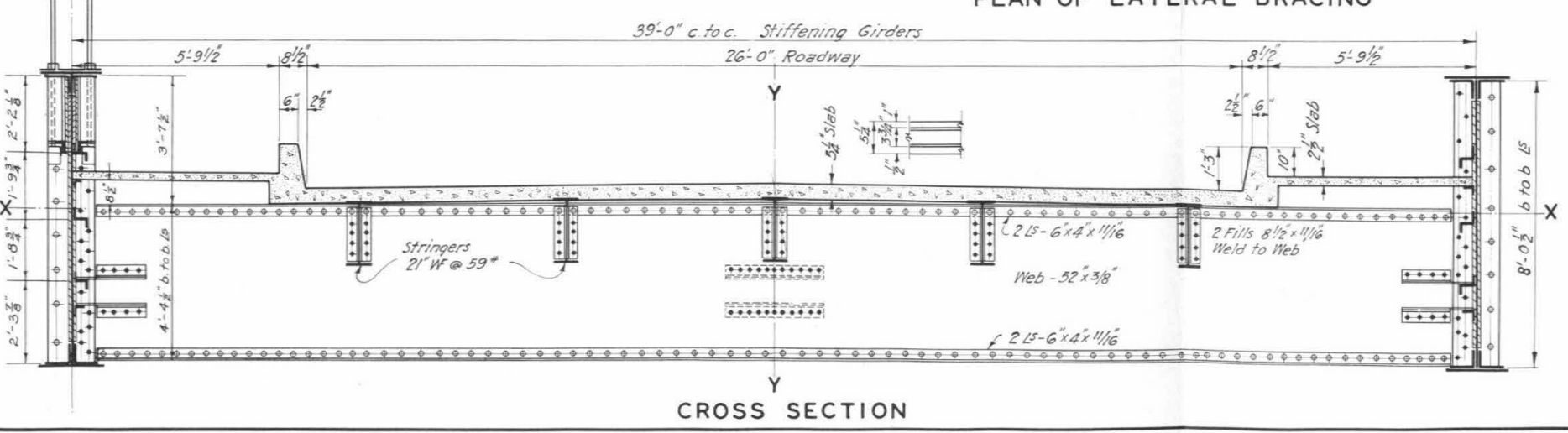
|                   |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
|-------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| No. of Panels     | 3    | 1    | 5    | 26   | 5    | 1    | 3    | 1    | 3    | 2    | 5    | 45   | 5    | 2    | 3    | 1    | 3    | 1    | 5    | 26   | 5    | 1    | 3    |
| 8" WF Beams w/ft. | 40   | 35   | 33   | 33   | 33   | 35   | 40   | 48   | 40   | 35   | 33   | 33   | 33   | 35   | 40   | 48   | 40   | 35   | 33   | 33   | 35   | 40   |      |
| Material          | Sil. | Sil. | Sil. | Car. | Sil. | Sil. | Sil. | Sil. | Sil. | Sil. | Sil. | Sil. | Sil. | Sil. | Sil. | Sil. | Sil. | Sil. | Sil. | Car. | Sil. | Sil. | Sil. |
| 1" Rivets         | 20   | 16   | 16   | 12   | 16   | 16   | 20   | 20   | 20   | 16   | 16   | 12   | 16   | 16   | 16   | 20   | 20   | 20   | 16   | 12   | 16   | 16   | 20   |

ELEVATION

25'-7" 42 Panels @ 25'-0" = 1050'-0" 20'-7" 19'-8" 5'-4" 4'-5" 110 Panels @ 25'-0" = 2750'-0" 5'-4" 4'-5" 19'-8" 20'-7" 42 Panels @ 25'-0" = 1050'-0" 22'-2 9/16" 2'-9 7/16"

Suspender Connections at Alternate Panel Points

5 Stringers, expansion seats every 150 ft.



**TACOMA NARROWS BRIDGE**

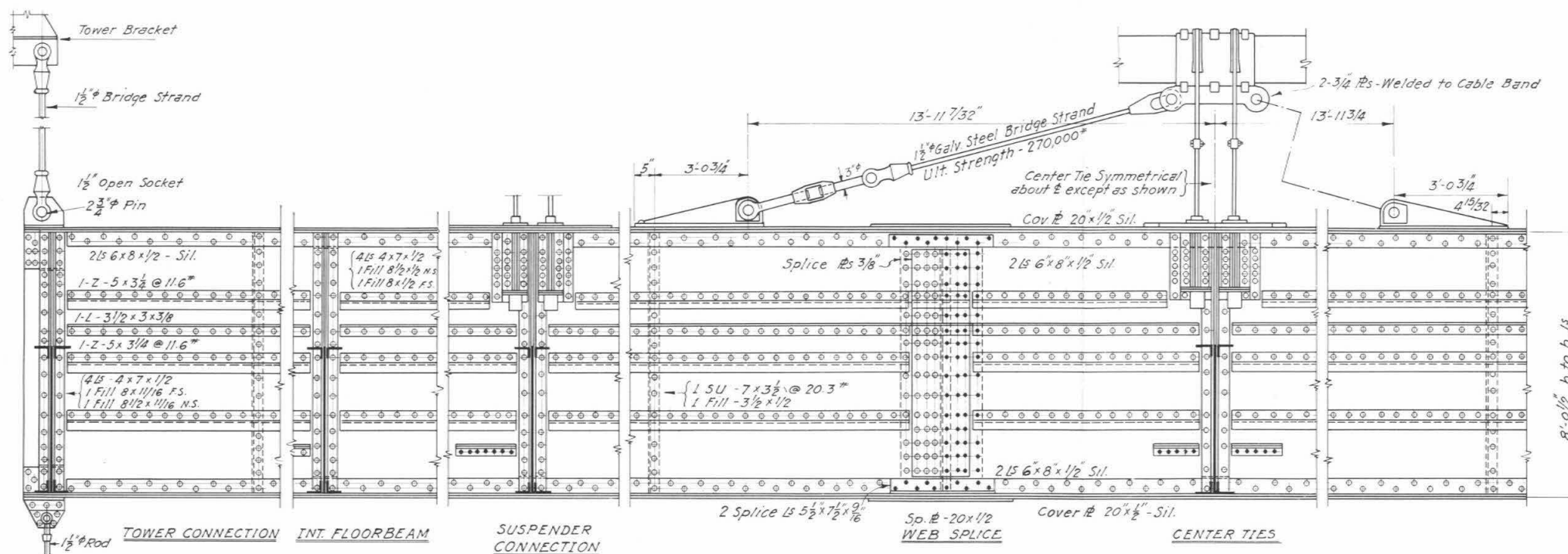
P. W. A. DOCKET NO. WASH 1870 F

ELEVATION, PLAN AND CROSS SECTIONS

FOR PLAN AND ELEVATION  
SCALE IN FEET

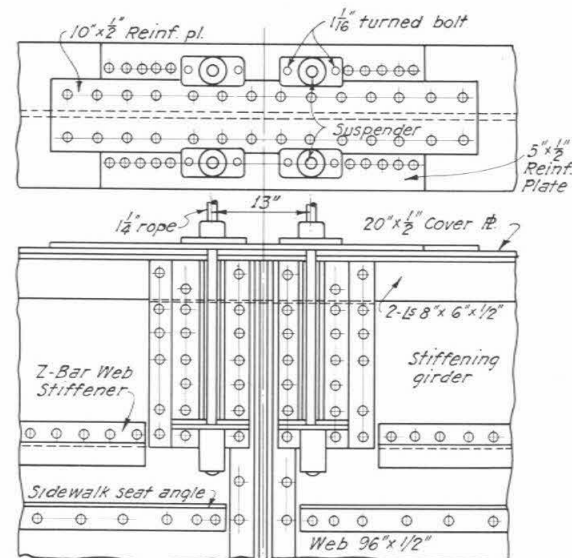
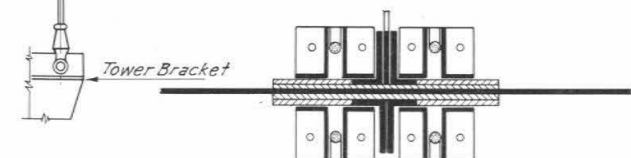
200 100 0 200 400

MARCH 1941 DRAWING 2

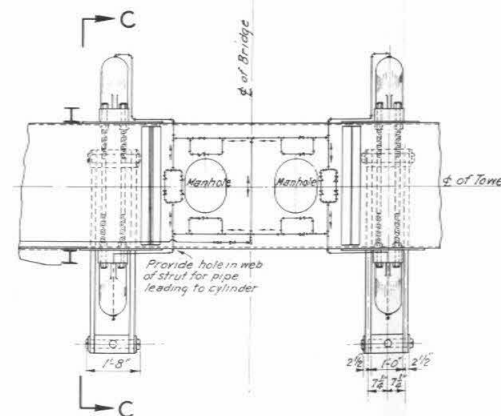
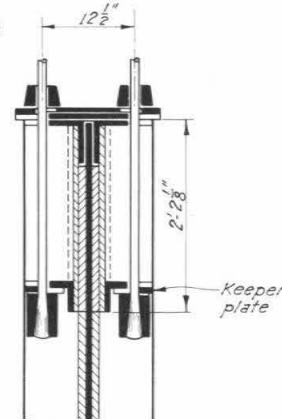


TYPICAL DETAILS OF STIFFENING GIRDERS

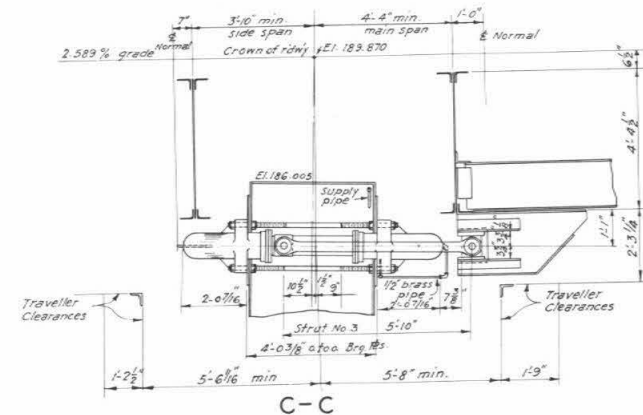
SCALE IN FEET  
2 1 0 2



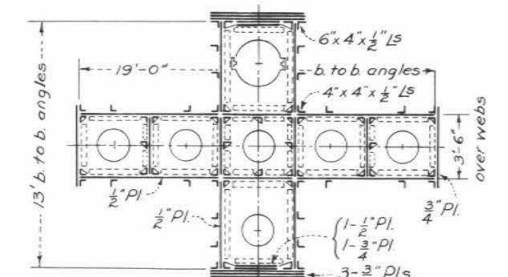
DETAIL OF SUSPENDER ATTACHMENT TO GIRDER



PLAN OF HYDRAULIC JACKS



SIDE OF TOWER FACE OF TOWER



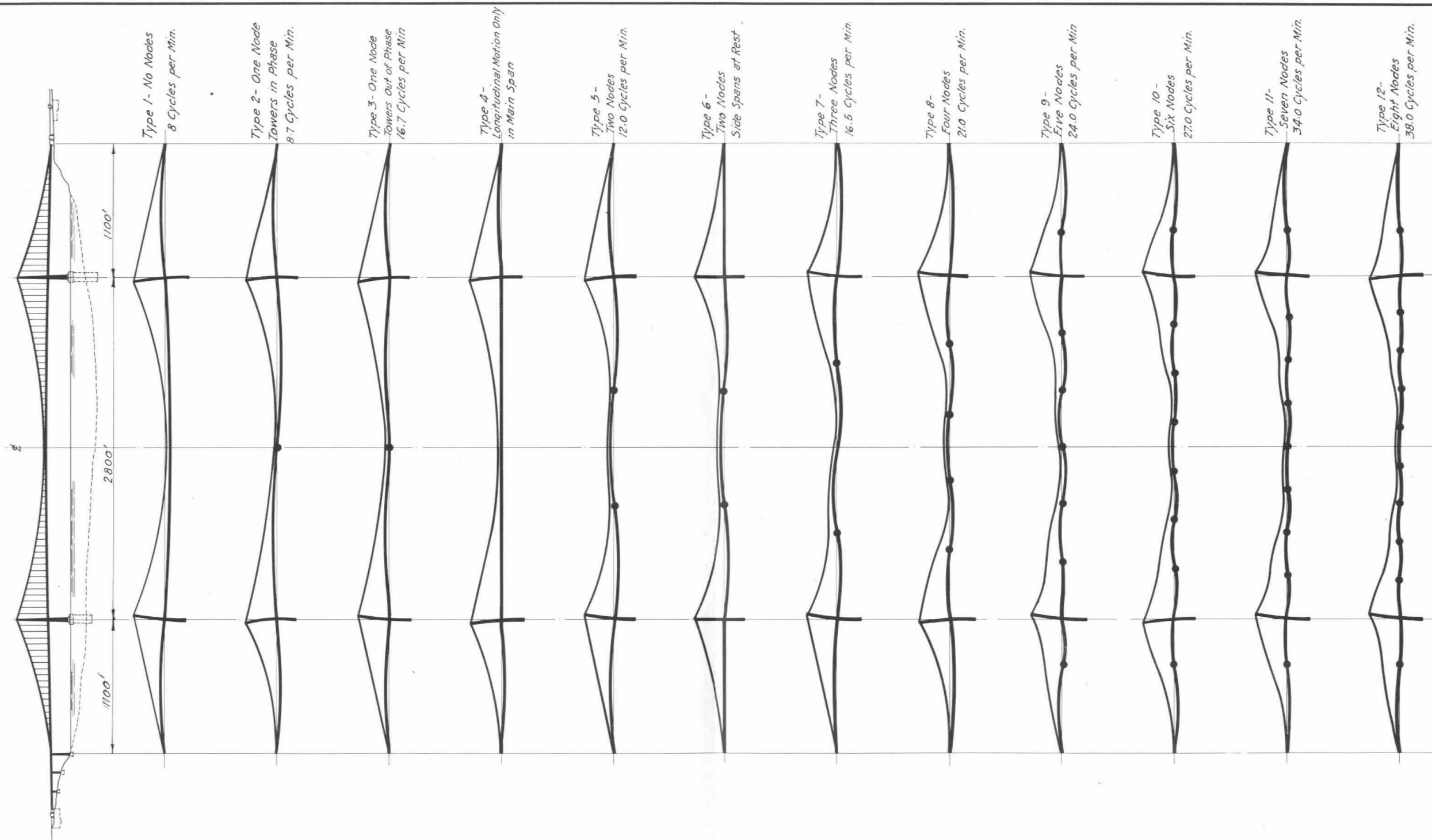
SECTION OF TOWER POST AT BASE

TACOMA NARROWS BRIDGE  
P.W.A. DOCKET NO. WASH 1870 F  
TYPICAL DETAILS

SCALE AS SHOWN

MARCH 1941

DRAWING 3



NOTE:  
The various Types, 1-12, of oscillations shown on this drawing are reproduced from information furnished by Professor F. B. Farquharson of the University of Washington.

## TACOMA NARROWS BRIDGE

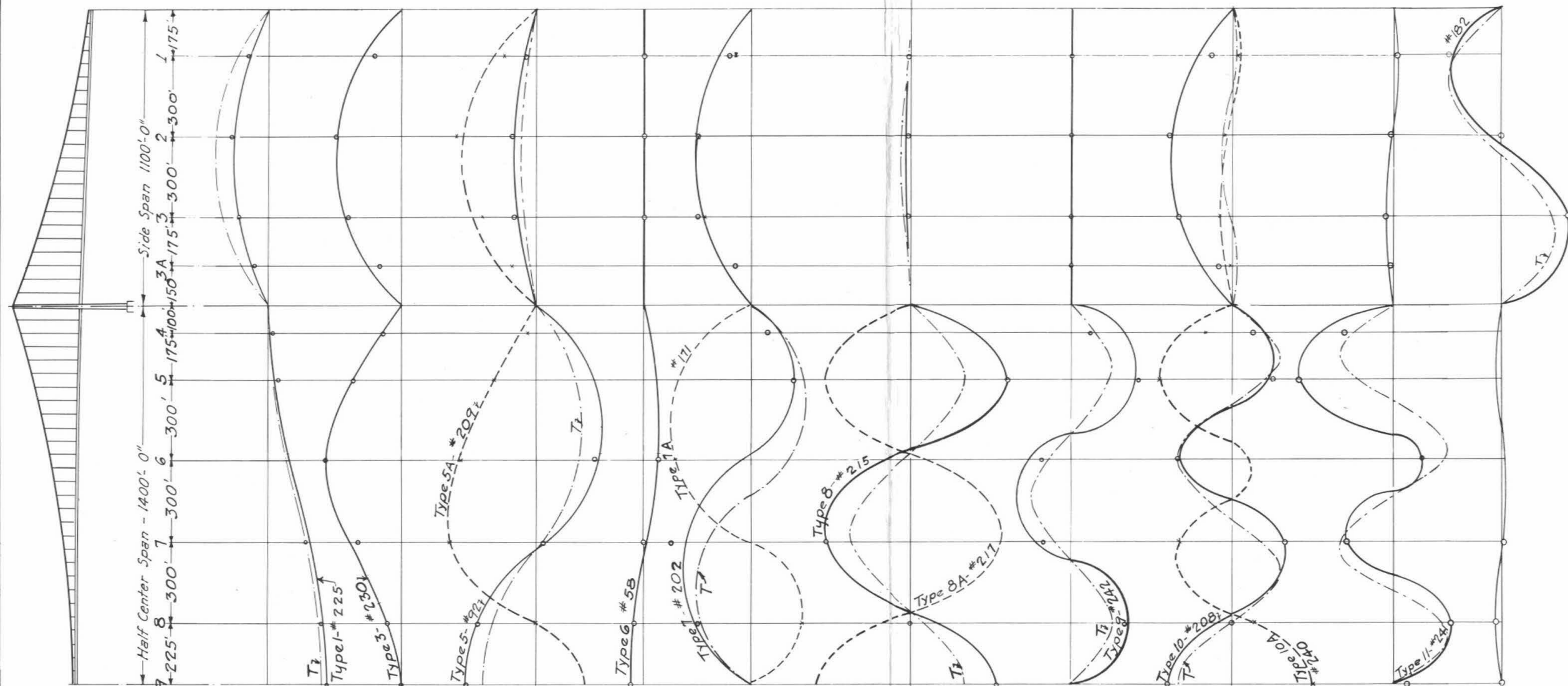
P.W.A. DOCKET NO. WASH 1870 F

MODES OF OSCILLATIONS

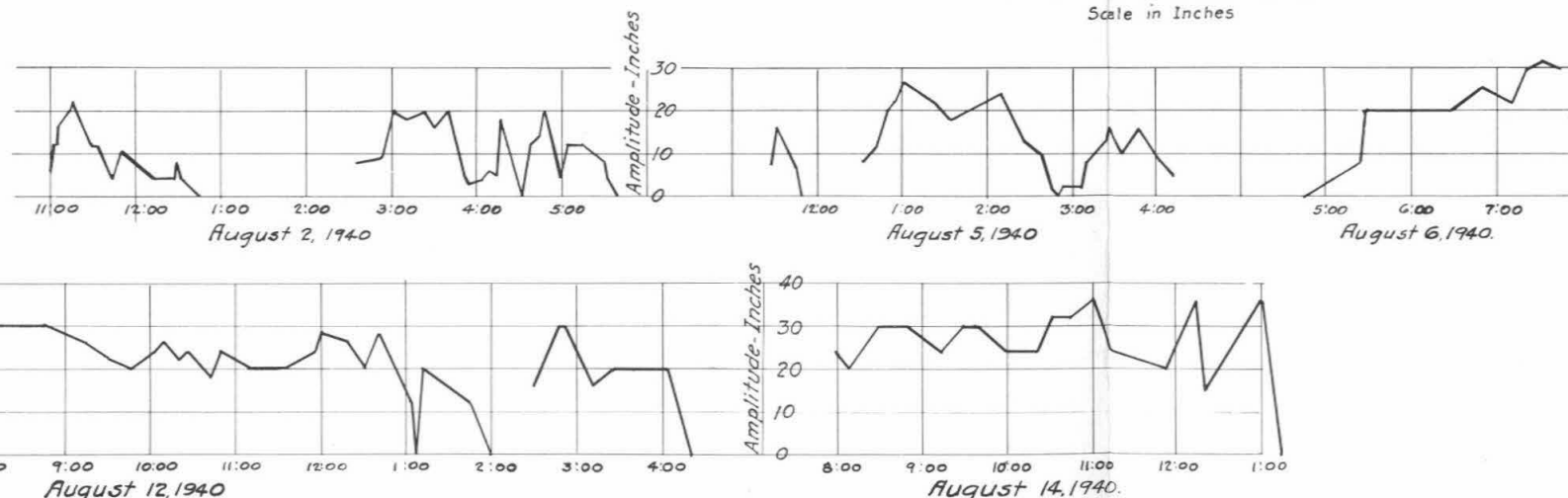
NO SCALE

MARCH 1941

DRAWING 4



Target No.  
Target Spacing



Note:  
Numbers are Observation Numbers from Tables 1 and 2  
Types refer to Mode of Oscillation Drawing #4  
"T" indicates Theoretical Oscillations with assumed maximum amplitude of 20" for the various types.  
o-Indicates observations for oscillations shown by full lines.  
x-Indicates observations for oscillations shown by dashed lines.

## TACOMA NARROWS BRIDGE

P.W. A. DOCKET NO. WASH 1870 F

TYPICAL OSCILLATIONS

SCALE OF AMPLITUDES IN INCHES  
0 10 20 30

MARCH 1941 DRAWING 5

VARIATIONS OF AMPLITUDES WITH TIME



